International Journal of Business, Human and Social Sciences
ISSN: 2517-9411
Vol:10, No:11, 2016

Financial Portfolio Optimization in Electricity Markets: Evaluation via Sharpe Ratio

F. Gökgöz, M. E. Atmaca

***Abstract*—**Electricity plays an indispensable role in human life and the economy. It is a unique product or service that must be balanced instantaneously, as electricity is not stored, generation and consumption should be proportional. Effective and efficient use of electricity is very important not only for society, but also for the environment. A competitive electricity market is one of the best ways to provide a suitable platform for effective and efficient use of electricity. On the other hand, it carries some risks that should be carefully managed by the market players. Risk management is an essential part in market players' decision making. In this paper, risk management through diversification is applied with the help of Markowitz's Mean-variance, Down-side and Semi-variance methods for a case study. Performance of optimal electricity sale solutions are measured and evaluated via Sharpe-Ratio, and the optimal portfolio solutions are improved. Two years of historical weekdays' price data of the Turkish Day Ahead Market are used to demonstrate the approach.

***Keywords*—**Electricity market, portfolio optimization, risk management in electricity market, Sharpe ratio.

I. Introduction

ELECTRICITY plays a crucial role in industry, commerce, agriculture, transformation, and the development of countries. It is an indispensable part of the daily lives of people and society. Furthermore, it has great effect on the environment depending on the source. Primary energy sources like coal, oil, natural gas, shell gas, nuclear energy, solar, wind, and hydropower are used to generate electricity. Some of them have adverse effects on environment. Meanwhile, world primary energy consumption is estimated to increase by 36% between 2011 and 2030 [1]. In addition, there are many stakeholders including electricity generators, transmission companies, regulatory bodies, dispatchers, consumers, industries, etc., comprising the electricity industry. Taking into account above mentioned facts, it is understood that electricity generation is a very strategic and important industry. Competitive electricity market environments provide a convenient stage for effective and efficient use of electricity. In addition, appropriate risk management techniques should be used by market players to compensate risks arising from the market.

F. Gökgöz, Ph.D. is with Ankara University, Siyasal Bilgiler Fakültesi, Cemal Gürsel Bulvarı, 06590 Cebeci, Ankara, Turkey (phone: +90-312-5951220; fax: +90-312-3197738; e-mail: fgokgoz@ankara.edu.tr).

M. E. Atmaca is Ph.D. candidate in Ankara University and he is with Electricity Generation Co. Inc., Nasuh Akar mah. Türkocağı cad. No: 2/F-1 06520 Bahçelievler, Çankaya, Ankara, Turkey (e-mail: meteemin.atmaca@euas.gov.tr).

Portfolio optimization used in finance is one of the effective decision tools based on risk management. Tradeoff between risk and profit is the main focus of this approach. According to the classical portfolio theory, the total risk of the portfolio can be decreased via diversification of assets, and expected to converge the market risk level [2]–[5]. In the 1950s, Modern Portfolio Theory was introduced to finance literature by H. M. Markowitz, who is the recipient of the 1990 Nobel Memorial Prize in Economic Sciences, which demonstrated that the classical portfolio approach is not a systematic way of managing risks in the portfolio because the co-movements of assets are not taken into account, as the theory only focuses on a number of assets in the portfolio [6]–[8]. The presence of a positive high correlation between assets can produce less effective risky portfolios and decrease the effect of diversification [9]. Portfolio optimization is considered as risk control technique and is described as the allocation of portfolio risky assets based on their relative risk and benefit: try to maximize return while minimizing risk or minimize risk while maximizing the return [2], [10].

The theory demonstrated by Markowitz was improved by Sharpe in 1964 and by Linther in 1965, respectively and separately [8], [11]–[13]. Markowitz's approach is based on mean-variance optimization and produces an efficient frontier that provides minimum risk for a given level of return under predefined constraints. There are two other important methods also: Down-side risk and Semi-variance risk approaches. They are mainly concentrating on negative deviations of returns from expected return of distribution. They are also called as Lower Partial Moments (LPM), which is described as the special case of Bernell Stones's Generalized Risk Measure. Down-side risk is taking into account first order of deviation from expected returns and Semi-variance risk is taking into account second order of deviation from expected returns [14]–[17]. Recently, some other down-side based studies have also contributed to the theory, including the optimization of portfolios by applying a copula based extension of conditional value at risk, and the multi-objective portfolio considering the dependence structure of asset returns [18],[19].

This paper aims to provide a theoretical background for the improvement of portfolio optimization results obtained using the Mean-variance, Down-side, and Semi-variance methods. Sharpe ratio is used for performance evaluation and improvement of the portfolio optimization results. Besides obtaining efficient frontier for each method, adjusted utility functions that include risk aversion constant representing of investors risk aversion level, are used to find optimal portfolio solutions. The performance of these optimal portfolio solutions are analyzed by using Sharpe ratio and improved by determining optimal interval for investors' risk aversion constants to obtain better Sharpe performance portfolios.

This paper is organized as follows: Section II introduces the theoretical background for portfolio optimization theory and Sharpe ratio, Section III provides a brief glance into the Turkish Electricity Market, Section IV introduces risky asset and risk-free assets in electricity market. Data, methodology and assumptions regarding an empirical case study and the results of the study are also demonstrated in this section. Finally, the conclusion section lists all the important findings and offers future directions for further studies.

II. Portfolio Optimization Theory

A. Modern Portfolio Theory & Mean-Variance Optimization

Classical portfolio theory claims that a sufficient level of diversification can reduce the total risk of a portfolio. In the stock market environment, investors can manage their risks by investing in different stocks from diverse industries, treasury bills or different currencies. Diversification can reduce the total risk of a portfolio to a certain degree, while on the other hand, co-movements of assets can negatively affect this process and classical portfolio theory does not take this issue into consideration [5].

Modern Portfolio Theory (MPT) takes account of co-movement/correlation of risky assets. Considering the co-movements/correlation of risky assets satisfies the ability to construct a portfolio that has the same expected return and is less risky than a portfolio constructed ignoring these factors [2], [10], [17], [20].

It is difficult for an investor to know exactly what the assets' risk and return will be in the future. The main problem for an investor in this case is portfolio selection to determine the weighting percentage of assets in it [2]. H. M. Markowitz published the article, "*Portfolio Selection*" in the Journal of Finance in 1952. That article is assumed to be the first milestone of MPT [7], [8]. Markowitz argued that the portfolio selection process can be divided into two stages: the first stage ends with beliefs about the future performance of securities, while the second stage ends with a choice of a portfolio. His paper is mainly concern about that second stage [7]. The theory demonstrated by Markowitz was improved and amplified by Sharpe in 1964 and by Linther in 1965, respectively and separately [8], [11]–[13]. With the addition of risk-free asset by Sharpe and Linther to portfolio optimization model, they improved the capital market line and developed Capital Asset Pricing Model (CAPM) [21].

Markowitz's approach is based on mean-variance optimization and it produces an efficient frontier that provides minimum risk for a given level of return or maximum return for given level of risk under predefined constraints. The main assumptions of the theory are listed as follows:

1) Investors have all the information and understand the market in the same way.
2) All investors are risk averse.
3) No transaction costs or taxes.
4) While taking investment decisions, investors are taking account only of expected returns, standard deviations, and co-variance of risky assets.
5) Returns on assets have normal distributions [22].

With the help of the normal distributions of assets' return, return distribution of alternative portfolios can be estimated by using only their means and variances [23]. The efficient frontier mentioned previously consists of efficient portfolios on it, and is produced by Mean-variance optimization as illustrated in Fig. 1, where an efficient portfolio is the only portfolio that offers the highest return at the same level of risk. So as seen in Fig. 1, the upper part of the efficient set is known as the efficient frontier.

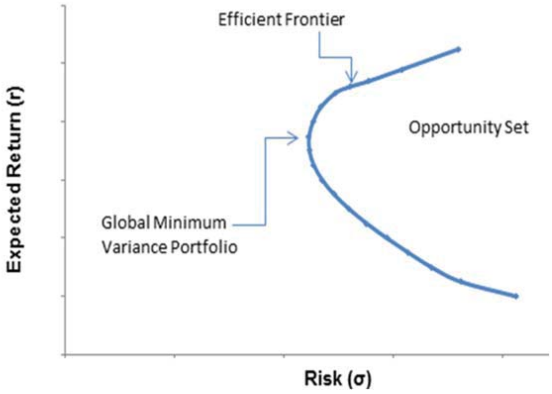

Fig. 1 Efficient frontier for mean-variance optimization model

The basic form of the Mean-variance optimization model with "n" risky assets includes three essential constraints as:

- Non-negativity condition for assets' portfolio weightings.
- Expected return of the portfolio will be equal to a target return.
- Total weights of risky assets are equal to 1.

Expected return and variance of a portfolio are described as:

$$E(r_p) = \sum_{i=1}^{N} X_i r_i \tag{1}$$

$$\sigma_p^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} X_i X_j \sigma_{ij} \tag{2}$$

where N describes a number of investable risky assets in the portfolio opportunity set, X_i denotes weights of i^{th} asset in the portfolio while r_i denotes expected return on them. σ_{ij} describes the covariance between assets. Object function and constraints for Mean-variance optimization are set as:

$$Min.(\sigma_p^2) = \sum_{i=1}^{N} \sum_{j=1}^{N} X_i X_j \sigma_{ij} \tag{3}$$

s.t.

$$\sum_{i=1}^{N} X_i r_i = r_{target} \tag{4}$$

$$\sum_{i=1}^{N} X_i = 1 \tag{5}$$

$$X_i \geq 0, \forall X_i \in [i = 1,2, \ldots, N] \tag{6}$$

where r_{target} describes the target portfolio return for the minimization of the portfolio's variances. The solution of this problem produces efficient portfolios on the efficient set. The upper part of this set is known as the efficient frontier, as seen in Fig. 1. To reach the optimal solution, the utility function that represents an investor's risk aversion level should be determined first. Utility function for mean-variance optimization is determined as a quadratic function and produces indifferences curves of investors. The utility value never changes along the curve, so it is also called as the indifferences curve. The touchpoint between the differences curve and efficient frontier is produced by optimal portfolio solution as seen in Fig. 2.

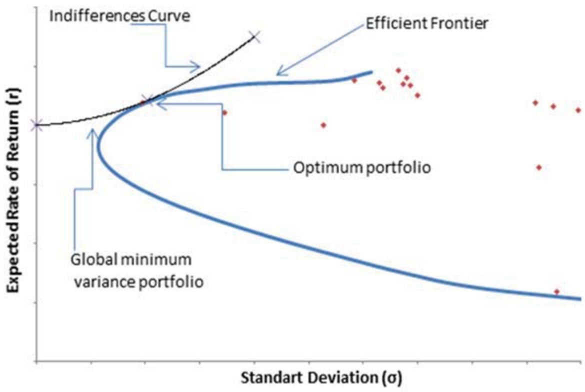

Fig. 2 Efficient frontier for mean-variance optimization model

Utility function includes the terms variance of a portfolio σ_p^2 expected return of a portfolio $E(r_p)$, and the risk aversion constant of an investor A [2], [9], [10], [20], [23]–[26]. The equation set that provides the maximum utility value for the optimal portfolio solution is set up as:

$$Max.(U) = E(r_p) - 1/2A\sigma_p^2 \tag{7}$$

s.t.

$$\sum_{i=1}^{N} X_i = 1 \tag{8}$$

$$X_i \geq 0, \forall X_i \in [i = 1,2,\dots,N] \tag{9}$$

where $E(r_p)$ and σ_p^2 can be seen in (1) and (2). If necessary, the common upper investment limit ψ_i for risky assets can also be determined as an additional constraint in the following form:

$$X_i \leq \varphi_i, \forall X_i \in [i = 1,2,\dots,N] \tag{10}$$

Depends on the need, the addition of risk-free asset, customizing the upper investment constraints for each risky asset, lending and borrowing, and other different issues can easily be modelled too [2], [9], [17], [20].

B. Semi-Variance Portfolio Optimization

Semi-variance and Down-side risk approaches are described as the special moments of Lower Partial Moments (LPM). In Mean-variance optimization, one of the big major criticisms is that it concentrates on positive and negative deviation from expected return at the same time. Actually, only negative deviations from expected returns are producing the loss for portfolios. Semi-variance and Down-side account only for the left-hand side of the return distribution as a risk [14].

Semi-variance is described as the second order moment of LPM. It takes into account square of left-hand side deviations from the expected return, and is formulized as [14], [16], [20]:

$$LMP_2(\tau : r) = \int_{-\infty}^{\tau} (\tau - r)^2 dF(r) \tag{11}$$

The basic form of the Semi-variance optimization model for N risk assets can be determined as:

$$Min. \sum_{j=1}^{M} p_j \left(d_j^-\right)^2 \tag{12}$$

s.t.

$$\sum_{i=1}^{N} r_{ij} X_j = r_j,\ \forall r_j \in [j = 1,2,\dots,M] \tag{13}$$

$$\sum_{i=1}^{N} X_i = 1 \tag{14}$$

$$\sum_{j=1}^{M} p_j \, . \, r_j = r_{target} \tag{15}$$

$$d_j^- = max\left[0, -(r_j - r_{target})\right] \tag{16}$$

$$X_i \geq 0, \forall X_i \in [i = 1,2,\dots,N] \tag{17}$$

M denotes a scenario number, d_j represents jth scenario's negative deviation from expected return, and p_j represents the probabilities of jth scenario. The solution of the equation set produces the efficient frontier for Semi-variance optimization. All scenarios are taken into consideration in the solution, so according to Mean-variance it is a little bit complex. To find the optimal portfolio solution, the utility function should be maximized depending on the relative risk aversion level of the investors [2], [9], [20], [24], [27]. The utility function for Semi-variance optimization is as:

$$Max. U_{Semi-variance} = E(r_p) - 1/2ALPM_2(\tau : r) \tag{18}$$

s.t.

$$\sum_{i=1}^{N} X_i = 1 \tag{19}$$

$$X_i \geq 0, \forall X_i \in [i = 1,2,\dots,N] \tag{20}$$

where $E(r_p)$ and $LPM_2\,(\tau{:}r)$ can be seen in (1) and (11). Again common upper investment limit ψ_i for risky assets can be determined by adding other constraints as seen in (10).

C. Down-Side Portfolio Optimization

Down-side risk is also concentrating on deviations of below-target returns like Semi-variance. It is the first order of LPM. It is similar to Semi-variance but it takes direct deviation from target return into consideration, not the square of them [14], [16], [17], [20].

Being the first order LPM, Down-side can be formulized as:

$$LMP_1(\tau:r) = \int_{-\infty}^{\tau}(\tau - r)dF(r) \tag{21}$$

The basic form of the Down-side optimization model for *N* risk assets can be determined as:

$$Min. \sum_{j=1}^{M} p_j(d_j^-) \tag{22}$$

s.t.

$$\sum_{i=1}^{N} r_{ij}X_j = r_j,\ \forall r_j \in [j = 1,2, \dots, M] \tag{23}$$

$$\sum_{i=1}^{N} X_i = 1 \tag{24}$$

$$\sum_{j=1}^{M} p_j \cdot r_j = r_{target} \tag{25}$$

$$d_j^- = max[0, -(r_j - r_{target})] \tag{26}$$

$$X_i \geq 0, \forall X_i \in [i = 1,2, \dots, N] \tag{27}$$

As can be seen from (22)–(27), all the constraints are the same but the object functions are different with the Semi-variance. As to the utility approach of Down-side, utility function to reach optimal portfolio solution is as:

$$Max. U_{Down-side} = E(r_p) - 1/2ALPM_1(\tau:r) \tag{28}$$

s.t.

$$\sum_{i=1}^{N} X_i = 1 \tag{29}$$

$$X_i \geq 0, \forall X_i \in [i = 1,2, \dots, N] \tag{30}$$

where $E(r_p)$ and $LPM_1\ (\tau{:}r)$ can be seen in (1) and (21). Again common upper investment limit ψ_i for risky assets can be determined by adding to other constraints as seen in (10).

D. Sharpe Ratio for Performance Evaluation

There are more than one performance measurement approaches in portfolio performance measurement. Depending on the point of view and need of the investor, these different approaches can be used separately or together in the performance evaluation of portfolios. The most common are the Sharpe Ratio (reward to variability) and the Treynor Ratio (reward to volatility) [28]. Jensen, Information Ratio (IR), and Omega and Sortino are the other important performance measurement indicators [29]. But, this paper focuses only on the Sharpe Ratio.

The Sharpe Ratio is very well known and widely used performance indicator in finance literature. It is one parameter the risk/return measurement method and is also referred to as the reward to variability. It is calculated by the division of adjusted returns of the portfolio based on the risk-free rate (residual return) to standard deviation of the portfolio itself. It can be shown as:

$$RVAP_p = \frac{r_p - r_f}{\sigma_p} \tag{31}$$

One of the important aims of this paper is the optimization and improvement of the optimal portfolio solutions, which are obtained from different portfolio optimization approaches, using the Sharpe Ratio.

III. Turkish Electricity Market

Turkey is considered an important developing country, and was listed as the 18th biggest economy in the world in 2014. It also has an important geopolitical position between Asia and Europe.

Installed capacity in Turkey was only about 408 MW at the beginning of 1950s and the total amount of electricity generation was 789.5 GWh [30]. Public ownership and a vertically integrated structure in the electricity industry continued until 1984, at which time a reform programme was initiated [20]. The programme gained momentum after the Electricity Market Law entered into force in 2001 [31]. The total installed capacity of Turkey has reached 74,627 MW as of May, 2016 [32]. Deregulation and construction process in the electricity market is continuing [33]. The related regulatory body of Turkey took a decision to decrease the limit of eligible/free customers (3600 kWh) at the end of 2015 and the market openness ratio reached over 85% after the decision [34].

After 2001, which was the establishment date of the energy sector's regulatory body, many developments were put into practice. The Turkish electricity market structure consists of an ancillary services market operated by a Transmission System Operator, a balanced market for real time balancing of load imbalances, a day-ahead market as a spot market, and an intra-day and Over The Counter (OTC) for bilateral contracts. Hourly uniform marginal pricing mechanisms are used in the spot markets with daily (24 hours) settlement periods. Supply and demand lines are produced by linear interpolation and final clearing price is produced by the intersection of these lines. Turkey is aiming to adopt a European market model [20]. Market participants can also tender block and flexible offers, but hourly and block offers have priority against flexible offers. In the day-ahead market, all offers for each hour of next day are gathered 11-35 hours before real consumption time [33]. While determining the final uniform clearing price, transmission system constraints are taken into consideration and applied to all market participants.

IV. Data, Methodology and Results of Study

The day-ahead electricity market structure came into effect after December 2011 in Turkey. Within the scope of this study, the hourly weekdays day-ahead electricity prices for a two year period between April 28, 2014 and April 24, 2016 are used. All Turkish Liras price values for each hour (520 data for each of the 24 hours) are converted to Euros (€) using the daily exchange rates of the Central Bank of the Republic of Turkey. There are very high level of positive correlations between the weekdays' same hourly electricity market prices (see Table I). The average value of the correlation between weekdays' same hourly electricity market prices is equal to 0.7320.

Average hourly electricity prices for a given period of time can be seen in Fig. 3. This shape is very characteristic and moreover it has a very high positive correlation with the consumption curve that reflects the consumption characteristics of customers.

TABLE I
CORRELATION MATRIX FOR WEEKDAYS

ρ	Mon	Tue	Wed	Thu	Fri
Mon	1	0.7974	0.7181	0.6592	0.617
Tue	0.7974	1	0.8024	0.7316	0.7096
Wed	0.7181	0.8024	1	0.7882	0.738
Thu	0.6592	0.7316	0.7882	1	0.7582
Fri	0.6170	0.7096	0.738	0.7582	1

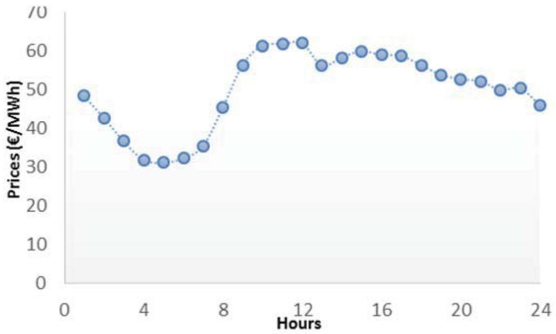

Fig. 3 Average hourly prices

Different from the stock market, rate of returns are calculated by using previously experienced approaches that applied in this field: Market prices are normalized by the generation cost for electricity [2], [9], [17], [20], [26]. In the real environment, the generation cost for electricity depends on the efficiency of the power plant, maintenance and operation conditions, weather conditions, and the quality of human resources, etc. Changes in generation costs can happen on a daily and or seasonal basis, but to simplify it and concentrate on the main scope of this study it is assumed constant (30 €/MWh). In fact, this kind of information is assumed strictly confidential for electricity generators, as this is commercially sensitive information. The hourly rate of returns for the day-ahead electricity market are calculated using the above mentioned generation cost as:

$$r_{n,m} = (a_{n,m} - C_k)/C_k \, , (m = 1,2, \ldots ,520) \quad (32)$$

$$\vec{r_1} = \begin{bmatrix} r_{1,1} \\ r_{1,2} \\ \vdots \\ \vdots \\ r_{1,520} \end{bmatrix}, \vec{r_2} = \begin{bmatrix} r_{2,1} \\ r_{2,2} \\ \vdots \\ \vdots \\ r_{2,520} \end{bmatrix}, \ldots \ldots \ldots, \vec{r_{24}} = \begin{bmatrix} r_{24,1} \\ r_{24,2} \\ \vdots \\ \vdots \\ r_{24,520} \end{bmatrix} \quad (33)$$

where $r_{n,m}$ indicates the hourly rate of returns against $a_{n,m}$ hourly weekdays' spot prices for n^{th} hour of m^{th} day for the given two year period. C_k is assumed constant and equal to 30 €/MWh. The rate of return vectors for n^{th} hour represents r_n as seen in (33). Mean and standard deviation for hourly electricity prices are demonstrated below (see Table II).

TABLE II
HOURLY ELECTRICITY PRICES AND DISTRIBUTION CONSTANTS

Hour	Mean €/MWh	Standard Deviation	Skewness	Kurtosis	VaR (95%)
1	48.39	13.38	-0.240	0.816	22.16
2	42.77	14.91	-0.617	1.214	13.54
3	36.75	16.00	-0.294	0.337	5.39
4	31.94	16.48	-0.154	-0.295	-0.37
5	31.16	15.93	-0.235	-0.165	-0.06
6	32.30	14.98	-0.184	0.274	2.94
7	35.42	15.34	-0.393	0.635	5.35
8	45.31	13.20	-0.413	1.747	19.43
9	56.37	12.67	-0.428	0.165	31.54
10	61.34	11.39	-0.774	1.183	39.01
11	61.72	11.78	-0.535	2.199	38.64
12	61.98	11.34	-0.662	0.016	39.74
13	56.25	14.24	-0.449	-0.182	28.33
14	58.31	13.11	-0.422	-0.513	32.61
15	59.96	12.50	-0.462	-0.550	35.45
16	58.96	12.68	-0.386	-0.616	34.11
17	58.73	12.96	-0.355	-0.651	33.33
18	56.33	14.59	-0.157	-0.425	27.75
19	53.86	13.63	-0.135	-0.415	27.14
20	52.69	12.58	0.152	-0.503	28.03
21	52.20	12.05	0.300	-0.511	28.58
22	49.88	13.02	0.266	-0.466	24.37
23	50.39	13.76	-0.100	-0.164	23.43
24	45.99	15.37	-0.294	0.392	15.87

The average rate of returns and related standard deviations for risky assets are calculated as:

$$\bar{r_n} = 1/520 \left(\sum_{m=1}^{520} r_{n,m}\right) \quad (34)$$

$$\sigma_n = \sqrt{\sum_{m=1}^{520} \left(r_{n,m} - \bar{r_n}\right)^2 / (520 - 1)} \quad (35)$$

A. Case Study

As mentioned in the first paragraph of this section, day–ahead hourly weekdays' electricity market prices of the Turkish electricity market for a two year period are assumed for our data set. For the application of risk/return based approaches (Mean-variance, Down-side risk, Semi-variance etc.), each of the 24 hours of a day is assumed as a separate risky asset. Bilateral contracts and forward contracts under the guarantee of a clearing house or other similar mechanism are assumed as risk free assets for the electricity market [2], [17], [20], [26], [33]. The main assumptions of study are listed as:

- Investor has a one-day investment horizon (electricity selling).
- Generation cost of electricity is constant (30 €/MWh) during the analyzed interval and the one-day investment horizon.
- Market is deep enough and not affected by the amount of electricity offered by the investor.

International Journal of Business, Human and Social Sciences
ISSN: 2517-9411
Vol:10, No:11, 2016

- Rate of return for a risk-free asset is assumed as 10%.
- Bids can be divided into infinitesimal parts.
- All bids will be bought by market.
- Investors are rational and prefer less risky portfolio at the same level of return, and highest return at the same level of risk.
- There is no congestion for transmission.
- Generation units are 100% available to provide proposed electricity.
- Rate of returns have normal distribution.
- Generation units are flexible to operate at every level of generation without efficiency lost.

The credentials and parameters of the empirical case study are demonstrated in the following table (see Table III).

TABLE III
CASE STUDY CREDENTIALS

Topic	Case Value
Installed capacity	100 MWe
Total available electricity energy	500 MWh
Investment period	1 day (weekday)
Generation cost	30 €/MWh
Risk-free rate of return	10%
Bilateral contract price	33 €/MWh
Weekdays for electricity selling	Monday, Tuesday, Wednesday, Thursday, Friday (5 days)
Market Data	Turkish day-ahead electricity Spot market prices (from April 28, 2014 to April 24, 2016)
Number of risky assets	24
Number of risk-free assets	1
Upper investment constraint	10% of available electricity
Optimization Methods	Mean-variance, Semi-variance, Down-side
Performance Method	Sharpe Ratio

TABLE IV
RISKY ASSETS BASED ON GENERATION COST

Hour	Mean Return 100%	Standard Deviation	Hour	Mean Return 100%	Standard Deviation
1	0.6131	0.4462	13	0.8749	0.4747
2	0.4256	0.4970	14	0.9437	0.4371
3	0.2251	0.5334	15	0.9986	41.67
4	0.0648	0.5495	16	0.9655	42.26
5	0.0386	0.5309	17	0.9575	43.20
6	0.0765	0.4993	18	0.8778	48.62
7	0.1808	0.5115	19	0.7953	45.43
8	0.5104	0.4401	20	0.7563	41.94
9	0.8790	0.4223	21	0.7399	40.16
10	1.0448	0.3798	22	0.6628	43.39
11	1.0574	0.3926	23	0.6798	45.87
12	1.0659	0.3781	24	0.5331	51.24

The questions are: "What is the weighting percentage of the available electricity for sale on the market to achieve maximum profit while minimizing risk?"; "Can we improve the performance of optimum portfolios obtained by using the Mean-variance, Down-side, and Semi-variance methods?"

Hourly electricity market prices are converted to the rate of return vectors using (32)-(35). The 24 risky assets have been produced (see Table IV). A covariance matrix (24x24), which is mostly used in Mean-variance optimization calculations, is also produced using the return vectors of each asset.

The mathematical structure of Mean-variance (MV), Semi-variance (SV), and Down-side (DS) methods for 24 risky assets have been successfully modelled and the results of the analysis were obtained in MATLAB. In Fig. 4, the efficient frontiers obtained by three methods are demonstrated.

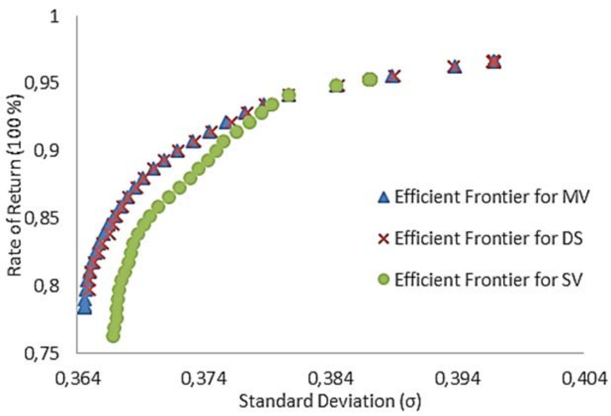

Fig. 4 Efficient frontiers

The efficient frontiers are formed from the optimal portfolio solutions of the related methods. The optimal portfolios have a minimum risk value for a given target return or maximum return for given target risk level. Depending on the methodology, all three methods produce their own frontiers. Although the methods are different, they produce very close efficient frontiers as seen in Fig. 4. When the Sharpe performance of the efficient frontiers are measured, it has been seen that frontier portfolios are reaching their maximum Sharpe value at very close points. Figs. 5 and 6 are demonstrating the variation of the Sharpe ratio performance of the efficient frontiers' portfolios depending on their related standard deviation and rate of return, respectively.

Utility functions for MV, SV, and DS methods are described in (7), (18), and (28). All utility functions include *A* constant that represents the risk aversion level of investors. The higher values of *A* are suitable for risk averse investors, while the lower values of *A* are suitable for risk seeking investors. *A* is generally assumed 3 for normal risk averse investors. Risk seeking investors prefer values that are < 3 and risk averse investors prefer values > 3 [24].

For the given risk aversion levels, solutions that make maximum the utility value of utility functions provide optimum portfolios for investors' decision. All utility functions for MV, SV, and DS methods are optimized for different values of risk aversion constant *A* (between 0 and 20). Rate of returns, standard deviations, Sharpe ratios based on the 10% risk-free rate have been calculated for each optimal utility function portfolios' solution. In each method, the Sharpe ratio is maximized for different values of *A*. In MV, the maximum Sharpe ratios are obtained by taking *A* between 7 and 10. In DS, maximum Sharpe ratios are obtained for values of *A* more than 7 and less than 12. On the other hand, in SV maximum Sharpe ratios are obtained for values of *A* more than 10 and less than 12. Even though they are producing same Sharpe optimal portfolio result, investors should be careful while applying these methods to find optimal portfolio solutions. *A* are not representing the same risk aversion level in each method, so that application of method should be customized by taking into account this important fact. The results of the Sharpe ratio performance analysis can be seen in Fig. 7 and the optimal portfolio solutions are listed in Table V.

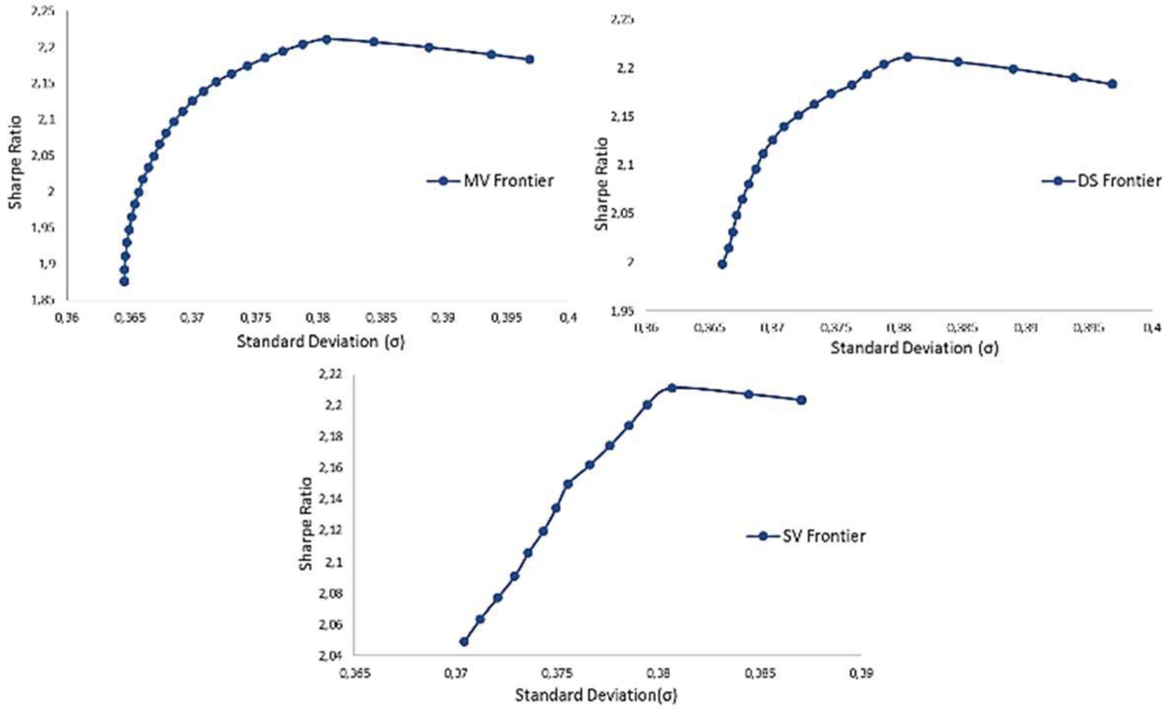

Fig. 5 Standard deviation vs Sharpe ratio for efficient frontiers' portfolios

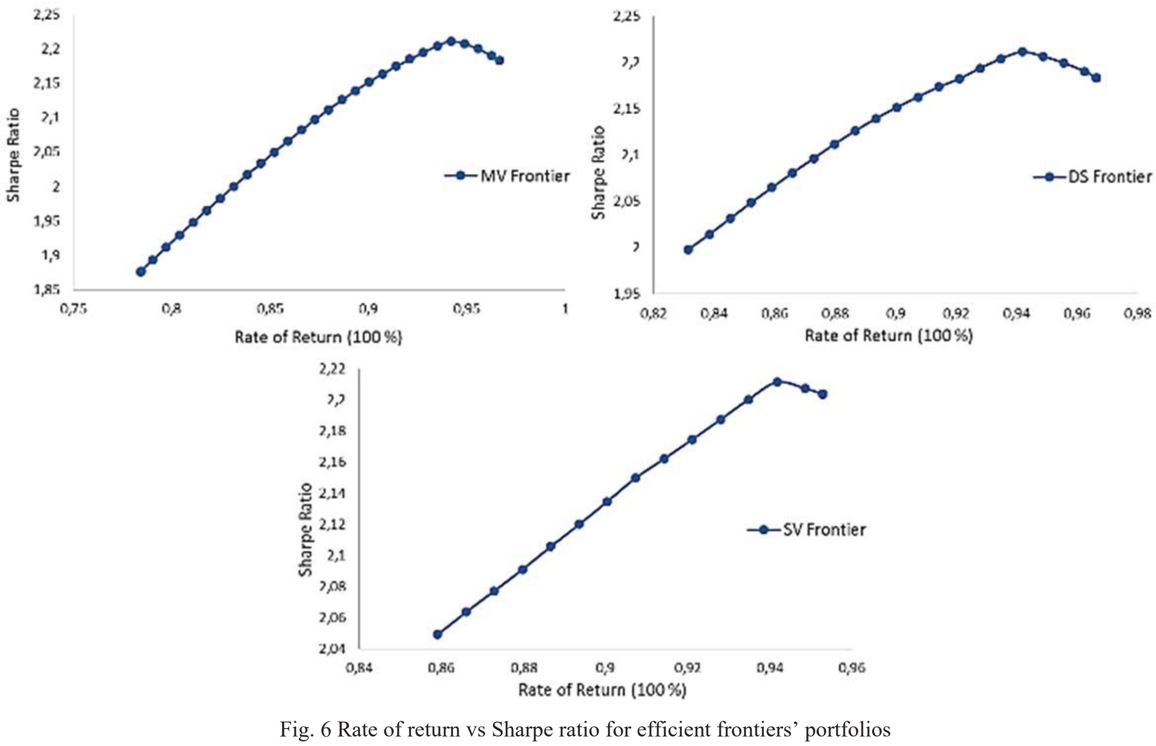

Fig. 6 Rate of return vs Sharpe ratio for efficient frontiers' portfolios

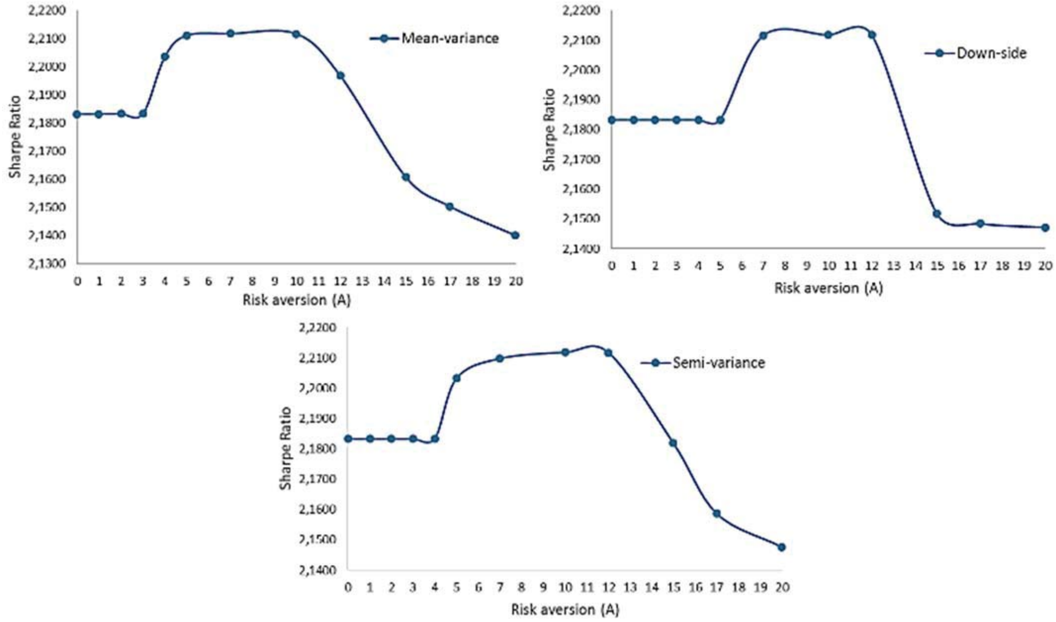

Fig. 7 Sharpe ratios for different risk aversion levels

TABLE V
SHARPE OPTIMUM PORTFOLIO SOLUTION

Hour	Weight %	Electricity MWh	Hour	Weight %	Electricity MWh
1	-	-	13	-	-
2	-	-	14	10	50.0
3	-	-	15	10	50.0
4	-	-	16	10	50.0
5	-	-	17	10	50.0
6	-	-	18	-	-
7	-	-	19	-	-
8	-	-	20	10	50.0
9	10	50.0	21	10	50.0
10	10	50.0	22	-	-
11	10	50.0	23	-	-
12	10	50.0	24	-	-

According to solution

V. CONCLUSION

In this paper, risk management through diversification is successfully applied with the help of Markowitz's Mean-variance, Down-side and Semi-variance methods. These methods have found wide application in finance literature. Two years of historical weekdays' price data of the Turkish Day Ahead Market were used for application of study. Using these data sets, risky assets have been created and determined according to an empirical electricity cost value (see Table IV).

The performance of the efficient frontier and optimal solutions obtained using the above mentioned optimization methods have been measured and evaluated via the Sharpe-ratio as seen in Figs. 5 and 6. The effective interval of the optimum risk aversion constants A that maximize the Sharpe-ratio for each methodology have been successfully determined and demonstrated for Turkish Electricity Market, as seen in Fig. 7. Finally, the same optimal portfolio solution that maximizes the Sharpe performance for Electricity Generator Company has been obtained using all methods (see Table V). It is understood that depending on the optimization methods, investors should customize their own risk aversion constant of the Utility functions to reach the maximum Sharpe-ratio performance. Otherwise sub-optimal solutions are obtained.

Suggestions for future directions of investigation can include, according to the needs, the addition of risk-free assets, customizing upper investment constraints for each risky asset, lending and borrowing, variation in generation cost, transaction costs, and other different issues that can easily be modelled. Additional performance measurement indicators like Treynor Ratio (reward to volatility), Jensen, Information Ratio (IR), Omega and Sortino can be applied for performance evaluation of the optimum portfolios and results can be compared. Short, middle, and long term characteristic values of the risk aversion constant can be analyzed for different electricity market environments.

REFERENCES

[1] BP Official Web Site, (2015, July 30), *BP Energy outlook 2030 Report*, retrieved from http://www.bp.com/content/dam/bp/pdf/Energy-economics/Energy-Outlook/BP_Energy_Outlook_Booklet_2013.pdf.

[2] F. Gökgöz, and M. E. Atmaca, "Financial optimization in the Turkish electricity market: Markowitz's mean-variance approach," *Renewable and Sustainable Energy Reviews,* vol. 16, no. 4, pp. 357-368, Jan. 2012.

[3] M. Statman, "How many stocks make a diversified portfolio?," *Journal of Financial and Quantitative Analysis,* vol. 22, no. 3, pp. 353-363, 1987.

[4] C. P. Jones, *Investments Analysis and Management*, New York, NY: John Wiley & Sons, 1999.

[5] T. E. Copeland, J. F. Weston, and K. Shastri, *Financial Theory and Corporate Policy*. USA: Pearson Addison Wesley, 2005.

[6] Nobelprize.org., (2016, May 15), *Nobel Prizes and Laureates*, retrieved from http://www.nobelprize.org/nobel_prizes/economic-sciences/.

[7] H. Markowitz, “Portfolio selection,” *Journal of Finance,* vol. 48, pp. 77-91, 1952.
[8] W. F. Sharpe, G. J. Alexander, and J. V. Bailey, *Investments*, New Jersey, USA: Prentice Hall, 1999.
[9] M. Liu, and F. F. Wu, “Managing price risk in a multimarket environment,” *IEEE Transactions on Power Systems,* vol. 21, no. 4, pp. 1512-1519, Nov. 2006.
[10] M. Liu, F. F. Wu, and Y. Ni, “A survey on risk management in electricity markets,” in *Proc. IEEE Power Engineering Society General Meeting,* Montreal, Quebec, 2006, pp. 1-6.
[11] W. F. Sharpe, “Capital asset prices: A theory of market equilibrium under condition of risk,” *Journal of Finance,* vol. 19, no. 3, pp. 425-442, Sep. 1964.
[12] J. Linther, “Security prices, risk, and maximal gains from diversification,” *Journal of Finance,* vol. 20, no. 4, pp. 587-615, Dec. 1965.
[13] J. Linther, “The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets,” *The Review of Economics and Statistics,* vol. 47, no. 1, pp. 13-37, 1965.
[14] H. Grootveld, and W. Hallerbach, “Variance vs down-side risk: is there really that much differences?,” *European Journal of Operational Research,* vol. 114, pp. 304-319, Apr. 1999.
[15] A. D. Roy, “Safety first and the holding of assets,” *Econometrica,* vol. 20, no. 3, pp. 431-449, Jul. 1952.
[16] Yu, Z. (2007). Spatial energy market analysis using the semi-variance risk measure. *Electrical Power and Energy Systems, 29*(8), 600-608.
[17] F. Gökgöz, and M. E. Atmaca, “An Optimal Asset Allocation in Electricity Generation Market for the Policy Makers and Stakeholders”, *Handbook of Research on Managerial Solutions in Non-Profit Organizations*, Ch. 21, to be published.
[18] A. Krzemienowski, and S. Szymczyk, “Portfolio optimization with a copula-based extension of conditional value-at-risk,” *Annals of Operations Research,* vol. 237, issue 1, pp. 219-236, May 2014.
[19] S. Babaei, M.M. Sepehri, and E. Babaei, “Multi-objective portfolio optimization considering the dependence structure of asset returns,” *European Journal of Operational Research,* vol 244, pp. 525-539, Jan. 2015.
[20] F. Gökgöz, and M. E. Atmaca, “Optimal Asset Allocation in the Turkish Electricity market: Down-Side vs Semi-Variance Risk Approach”, in *Proc. The 2013 International Conference of Financial Engineering of World Congress on Engineering (ICFE),* London, UK, 2013.
[21] M. H. Cohen, and V. D. Natoli, “Risk and utility in portfolio optimization,” *Physica A: Statistical Mechanics and its Applications,* vol. 324, no. 1-2, pp. 81-88, Jun. 2003.
[22] R. A. Defusco, D. W. McLeavey, J. E. Pinto, and D. E. Runkle, *Quantitative Investment Analysis*, USA: John Wiley & Sons Inc., 2004.
[23] D. King, “Portfolio optimization and diversification,” *Journal of Asset Management,* vol. 8, no. 5, pp. 296-307, Dec. 2007.
[24] M. Liu, and F. F. Wu, “Portfolio optimization in electricity markets,” *Electric Power Systems Research,* vol. 77, no. 8, pp. 1000-1009, June 2007.
[25] Z. Bodie, A. Kane, and A. J. Marcus, *Investments*, New York, USA: McGraw-Hill Irwin, 2009.
[26] M. E. Atmaca, *Portfolio optimization in electricity market,* (Thesis No: 313125), Ankara, Turkey: Council of Higher Education Thesis Center, 2010.
[27] F. Donghan, G. Degiang, J. Zhong, and Y. Ni, “Supplier asset allocation in a pool-based electricity market,” *IEEE Transactions on Power Systems,* vol. 22, no. 3, pp. 1129-1138, Aug. 2007.
[28] M. B. Karan, *Investment Analysis and Portfolio Management (Yatırım Analizi ve Portföy Yönetimi)*, Ankara, Turkey: Gazi Kitabevi, 2004.
[29] P. Bertrand, and J. Prigent, “Omega Performance Measure and Portfolio Insurance,” Journal of Banking and Finance, vol.35, no. 7, pp. 1811-1823, July 2011.
[30] H. K. Öztürk, A. Yılancı, and Ö. Atalay, “Past, Present and Future Status of Electricity in Turkey and the Share of Energy Sources,” *Renewable and Sustainable Energy Reviews*, vol. 11, no. 2, pp. 183-209, Feb. 2007.
[31] N. Bağdadioğlu, and N. Ödyakmaz, “Turkish Electricity Reform,” *Utility Policy*, vol. 17, no. 1, pp. 144-152, Mar. 2009.
[32] Energy Institute web site, (2016, May 15), *Turkish Installed Capacity Based on Fuel Type*, retrieved from http://enerjienstitusu.com/turkiye-kurulu-elektrik-enerji-gucu-mw/.
[33] F. Gökgöz, M. E. Atmaca, “Portfolio Optimization under Lower Partial Moments in Emerging Electricity Markets: Evidence from Turkey,” *Renewable and Sustainable Energy Reviews*, submitted for publication.
[34] Republic of Turkey Energy Market Regulatory Authority web site, (2016, May 15), retrieved from www.epdk.org.tr/TR/Dokuman/1680.