International Journal of Electrical, Electronic and Communication Sciences
ISSN: 2517-9438
Vol:10, No:9, 2016

# An Eigen-Approach for Estimating the Direction-of Arrival of Unknown Number of Signals

Dia I. Abu-Al-Nadi, M. J. Mismar, T. H. Ismail

***Abstract*****—**A technique for estimating the direction-of-arrival (DOA) of unknown number of source signals is presented using the eigen-approach. The eigenvector corresponding to the minimum eigenvalue of the autocorrelation matrix yields the minimum output power of the array. Also, the array polynomial with this eigenvector possesses roots on the unit circle. Therefore, the pseudo-spectrum is found by perturbing the phases of the roots one by one and calculating the corresponding array output power. The results indicate that the DOAs and the number of source signals are estimated accurately in the presence of a wide range of input noise levels.

***Keywords*****—**Array signal processing, direction-of-arrival, antenna arrays, eigenvalues, eigenvectors.

## I. Introduction

ONE of the main applications of adaptive antenna array is to suppress the unwanted signals while receiving the desired signal from a known direction. Therefore, to suppress the unwanted signals, the DOA of the source signals must be estimated. Many signal processing techniques are proposed to estimate the DOA of the source signals [1]. From these, the eigen-approach methods have received wide attention because of their relatively high resolution [1]-[3]. However, all eigen-structure methods need exact number of sources to separate signal subspace and noise subspace. Practically, in most applications, the number of sources and their directions are unknown. Therefore, it is required to estimate the number of sources as well as their corresponding DOA simultaneously. Hence, development of techniques to estimate the DOA without knowing the number of signal sources apriori, is very much appealing [4]-[6].

Many modified methods of the eigen-structure are proposed to overcome this limitation. From these methods, MUSIC-like method has been proposed to estimate the DOA without using the subspace decomposition and the number of sources is not required for direction finding [4].

The random search optimization algorithms are proposed to estimate DOA as the DOA estimation is a highly nonlinear optimization problem. In particular, the genetic algorithms (GA) and the modified particle swarm optimization (PSO) algorithms can be used directly or with other methods to estimate the DOA of the signals [6], [7]. Also, a new DOA and power estimation method was proposed without knowing the number of source signals [6].

In this paper, the DOA estimation method relies on the fact that the array factor with the coefficients of the eigenvector corresponding to the minimum eigenvalue of the autocorrelation matrix yields the minimum output power of the array. Also, it is known that the array polynomial with the coefficients of the eigenvector corresponding to the minimum eigenvalue of the autocorrelation matrix possesses roots lying on the unit circle [8]. Thus, the roots of the polynomial of the array factor with the eigenvector corresponding to the minimum eigenvalue estimates a possible source direction on the unit circle. From the locations of the roots, the pseudo-spectrum is evaluated by shifting the phases of the roots and calculating the array output power.

## II. Problem Formulation

A Uniform Linear Antenna (ULA) array consisting of $M+1$ isotropic elements with half-wavelength $(\lambda/2)$ inter-element spacing between elements is considered. Assuming $I$ signals with different angular directions, $\theta_i$, are impinging at the ULA array. The received signal at each array element consists of the narrowband source signals contaminated with additive white Gaussian noise. It can be expressed as

$$\mathbf{x}(k) = A(\Theta)\mathbf{s}(k) + \mathbf{n}(k) \tag{1}$$

where $\mathbf{A}(\boldsymbol{\Theta}) = [\mathbf{a}(\theta_1)\,\mathbf{a}(\theta_2)\cdots\mathbf{a}(\theta_I)]$ is the matrix formed by the $I$ steering vectors along the source direction. For a direction $\theta_i$, the steering vector is given by

$$\mathbf{a}(\theta_i) = [1 \quad e^{j(kd\sin\theta_i)} \quad e^{j(2kd\sin\theta_i)} \cdots e^{j(Mkd\sin\theta_i)}]^T \tag{2}$$

The vector $\mathbf{s}(k)$ represents the $k^{th}$ sample of a narrowband source signals and $\mathbf{n}(k)$ represents the $k^{th}$ sample of an additive Gaussian noise vector to be uncorrelated across array elements and have equal power of $\sigma^2$. The array covariance matrix can be expressed without considering the effect of snapshots as

$$\mathbf{R_{xx}} = \mathbf{A}(\Theta)\mathbf{R_{ss}}\mathbf{A^H}(\Theta) + \sigma^2\mathbf{I_{(M+1)}} \tag{3}$$

Dia I. Abu-Al-Nadi is with the Department of Electrical Engineering, The University of Jordan, Amman-Jordan (e-mail: dnadi@ju.edu.jo).
M.J. Mismar is with the Department of Elecrical Engineering, Princess Sumaya University for Technology, Amman, Jordan (e-mail: m.mismar@psut.edu.jo).
T.H.Ismail is with the Department of Electrical Engineering, Zarqa University, Zarqa, Jordan (e-mail: tghanim@zu.edu.jo).

where $\mathbf{R_{ss}} = E[\mathbf{ss^H}]$ is the source correlation matrix and $\mathbf{I_{(M+1)}}$ is $(M+1)\times(M+1)$ identity matrix. With $\mathbf{w} = [w_1 \; w_2 \; \ldots \; w_m \; \ldots \; w_M]^T$ as the weight vector of the array elements, the output signal of the array is

$$\mathbf{y}(k) = \mathbf{w}^H \mathbf{x}(k) \tag{4}$$

With the assumption that the signals are stationary and ergodic processes, the average output power of the array can be estimated as the time average correlation of $K$ snapshots by

$$P_y = \mathbf{w}^H \mathbf{R_{XX}} \mathbf{w} \tag{5}$$

where

$$\mathbf{R_{XX}} = \frac{1}{K}\sum_{k=1}^{K} \mathbf{x}(k)\mathbf{x}(k)^H \tag{6}$$

and the vectors $\mathbf{x}(k)$ and $\mathbf{w}$ are the received data samples and the weights of the ULA array elements, respectively.

Generally, if the nulls of the array factor are rotated on the unit circle to coincide with the angular directions of the source signals, the output of the ULA array will be the noise power only, i.e.,

$$P_0 = \mathbf{w}_0^H \mathbf{R_{XX}} \mathbf{w}_0 \tag{7}$$

where $\mathbf{w}_0$ is the weight vector which eliminates the signals at the array output. Assuming uncorrelated noise of zero mean and $\sigma^2$ variance, the output noise power is estimated as

$$P_0 = \mathbf{w}_0^H \mathbf{R_{XX}} \mathbf{w}_0 = \sigma^2 \mathbf{w}_0^H \mathbf{w}_0 \tag{8}$$

The optimization problem is formulated as:

$$\underset{\mathbf{w}}{\text{Minimize}} \quad \mathbf{w}^H \mathbf{R_{XX}} \mathbf{w} \tag{9}$$

Subject to:

$$\mathbf{w}^H \mathbf{w} = 1$$

The autocorrelation matrix $\mathbf{R_{XX}}$ is a Toeplitz matrix and the above optimization problem can be solved using a Lagrange multiplier as:

$$\mathbf{w}^H \mathbf{R_{XX}} \mathbf{w} + \lambda\,(1 - \mathbf{w}^H \mathbf{w}) \tag{10}$$

Differentiating with respect to $\mathbf{w}$, the solution will be

$$\mathbf{R_{XX}} \mathbf{w} = \lambda \mathbf{w} \tag{11}$$

Therefore, the solution w is an eigenvector of the matrix $\mathbf{R_{XX}}$ with λ the corresponding eigenvalue. Pre-multiplying the previous equation by w, we obtain

$$\mathbf{w}^H \mathbf{R_{XX}} \mathbf{w} = \lambda\, \mathbf{w}^H \mathbf{w} = \lambda \tag{12}$$

Hence, to minimize our objective function, one should choose the eigenvector $\mathbf{w_0}$ that corresponds to the minimum eigenvalue $\lambda_{min}$ as a solution, and the corresponding minimum out power will be $\lambda_{min}$ as well. That means $\mathbf{P_0} = \lambda_{\min} = \mathbf{w}_0^H \mathbf{R_{XX}} \mathbf{w}_0$ is the eigen-approach solution for constrained optimization problem. If $\mathbf{I \le M}$, then $\mathbf{w_0}$ will be the eigenvector that corresponds to the minimum eigenvalue, $\boldsymbol{\lambda_{min}}$, which represents the power of the noise; i.e.,

$$\mathbf{P_0} = \lambda_{\min} = \sigma^2$$

## III. DOA Estimation

It was shown in [8] that roots of the polynomial made by $\mathbf{w_0}$ will lay on the unit circle. Let $w_{0m}$ represent the $m^{th}$ element of the eigenvector $\mathbf{w_0}$, then, the array factor of array can be written as

$$\begin{aligned} AF(\theta) &= \sum_{m=0}^{M} w_{0m}\, e^{-jm\pi\sin(\theta)} \\ &= \prod_{m=1}^{M} c\left(e^{-j\pi\sin(\theta)} - e^{-j\pi\sin(\phi_m)}\right) \end{aligned} \tag{13}$$

The above equation indicates that the roots of the array polynomial must coincide with direction of the signals as the solution was obtained for the minimum output power.

The process of DOA estimation can be obtained by calculating the pseudospectrum at the directions corresponding to roots locations. The pseudospectrum is calculated by perturbing each root of the array factor and calculating the corresponding output power of the array. Large variation of the output power will occur if the roots coincide with an actual direction of a signal; otherwise, this root is not in the direction of any of the source signals.

The procedure for estimating DOA is explained as:

1. The eigenvalues and the eigenvectors of $\mathbf{R_{XX}}$ should be computed.
2. Choose the eigenvector, $\mathbf{w_0}$, corresponding to the minimum eigenvalue $\boldsymbol{\lambda_{min}}$ (the power of the noise). Use (13) to calculate the angles of the roots on the unit circle, $\phi_m^{'}$ ($m = 1, 2, \ldots, M$).
3. For $m$=1,2,…$M$, perturbing the root on the unit circle (by $+\frac{\pi}{2}$ if $\phi_m^{'}$ is negative and $-\frac{\pi}{2}$ if $\phi_m^{'}$ is positive) such that $(\pi\sin(\phi_m^{'}) \Rightarrow \pi\sin(\phi_m^{'} \pm \frac{\pi}{2})$ enables computing the

corresponding weight vector $\mathbf{w}_m$ using (13). The array output power at the estimated $m^{th}$ root is

$$P'_m = \mathbf{w}_\mathbf{m}^\mathbf{H} \mathbf{R}_{\mathbf{XX}} \mathbf{w}_\mathbf{m} \tag{14}$$

4. A pseudospectrum is obtained at the array root locations by plotting $P_m$ versus $\hat{\theta}_m$, where

$$P_m = P'_m - P_0 \tag{15}$$

When $P_m > 0$, then a signal exists and the estimated angle is

$$\hat{\theta}_m = \phi'_m \tag{16}$$

## IV. Computer Simulation and Discussion

For simulations, a ULA array with eight elements and half-wavelength inter-element spacing ($\lambda/2$) is chosen to implement the proposed technique. The number of roots in this case will be seven. This means that this structure can be used to estimate up to seven DOAs of signal sources. An eigenvalue problem is formed to minimize a constrained optimization problem where the minimum eigenvalue $\lambda_{\mathbf{min}} = \sigma^2$ is the minimum energy and its corresponding eigenvector is $\mathbf{w_0}$. The location of the roots on the unit circle are calculated by (13) while the angles of arrival are estimated by using (8) and (14)-(16).

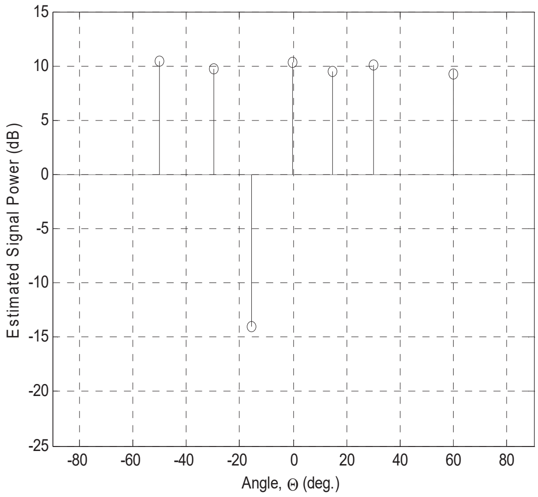


Fig. 1 The pseudospectrum estimate of six signals that are impinging at the angles -50°, -30°, 0°, ,15, 30°, and 60° with power level of 10 dB for each signal. (Number of array elements = 8, noise level = 0dB, and 100 snapshots)

The method was validated by having six source signals impinging at the array with angles (-50, -30, 0, 15, 30 and 60) and with signal-to-noise ratio (SNR) of 10 dB for all of them. The input noise level is 0 dB and 100 snapshots were taken to estimate the correlation matrix. The resulted pseudospectrum is shown in Fig. 1. The actual signals can be estimated by comparing the level of power of each value in the pseudo-spectrum to the estimated level of noise. The location of each power level on the x-axis are the corresponding estimated DOA. The power levels and their corresponding estimated DOA of the six signals are presented in Table I. In conclusion, the method was able to estimate both the angle and the level of power for each source signal accurately, even for the situation of closely spaced signals and without knowing the number of signals apriori.

To study the effect of varying the signal-to-noise ratio (SNR) on the performance of the algorithm, one source signal impinging at 30° is chosen. The estimated angle, $\hat{\theta}_i$, and the corresponding estimated power level, $P_i$, for different SNR with 100 snapshots are reported in Table II. It can be claimed that this method of DOA estimation is insensitive to SNR variation.

TABLE I
Estimated DOA Angles, $\hat{\theta}_i$, and Their Power Levels, Using the Presented Algorithm

| $\theta_i$ | $P_i$ (dB) | $\hat{\theta}_i$ |
|---|---|---|
| -50° | 10.43 | -49.997° |
| -30° | 9.69 | -29.64° |
| 0° | 10.35 | -0.02° |
| 15° | 9.57 | 14.88° |
| 30° | 10.06 | 29.96° |
| 60° | 9.21 | 60.04° |

Number of signals = 6, number of elements = 8, noise level = 0dB, and 100 snapshots.

TABLE II
Estimated Angle, $\hat{\theta}_i$, against Various SNR Levels for One Signal Impinging at 30°

| SNR (dB) | $\hat{\theta}_i$ |
|---|---|
| 10.04 | 29.71° |
| 6.85 | 30.04° |
| 0.76 | 30.04° |
| -2.63 | 29.94° |

Number of array elements = 8, noise level = 0dB and 100 snapshots.

## V. Conclusion

In this work, the DOA without knowing number of source signals is estimated using the eigen-approach. The eigenvector that corresponds to the minimum eigenvalue of the autocorrelation matrix is first calculated. If the number of the source signals is less than the number of the array elements, then the array output power is equal to the noise power only when the coefficients are the eigenvector corresponding to the minimum eigenvalue. Also, this array factor yields a polynomial with roots laying on the unit circle. The pseudo-spectrum is evaluated by moving the phases of the roots one by one on the unit circle and calculating the corresponding array output power. A source signal exists when the array pseudo-spectrum power is greater than the noise power at the root direction. The results show that the number of source signals and the directions of arrival are estimated accurately in the presence of a wide range of input noise levels.

## REFERENCES

[1] L. Godara, "Application of Antenna Arrays to Mobile Communications, Part II: Beam-Forming and Direction-of-Arrival Considerations", IEEE Proceedings of the IEEE, vol. 85 (8), pp. 1195-1245, 1997.

[2] B. Liao, G. S. Liao & J. Wen, "A Method for DOA Estimation in the Presence of Unknown Nonuniform Noise", Journal of Electromagnetic Waves and Applications, Volume 22, Issue 14-15, pp. 2113-2123, 2008

[3] M. Li and Y. Lu , "Maximum likelihood DOA estimation in unknown colored noise fields", IEEE Trans. on Aerospace and Electronic Sys. Vol. 44 ( 3 ) pp. 1079 – 1090, July 2008

[4] Zhang, Y. and B. P. Ng, "MUSIC-Like DOA estimation without estimating the number of sources," *IEEE Trans. on Signal Processing*, 58(3), 1668-1676, 2010.

[5] Q. C. Zhou, H. T. Gao, and F. Wang, "A high resolution DOA estimation method without estimating the number of sources", Progress in Electromagnetics Research C, vol. 25, pp. 233–247, 2012.

[6] M. J. Mismar and T. H. Ismail, "DOA and Power Estimation by Controlling the Roots of the Antenna Array Polynomial", Progress in Electromagnetics Research M, Vol. 46, 193–201, 2016.

[7] J. Hung. "Modified particle swarm optimization structure approach to direction of arrival estimation", Applied Soft Computing, vol. 13, pp. 315–320, 2013.

[8] J. Makhoul, "On the Eigenvectros of Symmetric Toeplitz/Matrices", IEEE Trans. On Acoustics, Speech and Signal Processing, vol. ASSP.29, no. 4, pp. 868-872, Aug.1981.