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Convex Restrictions for Outage Constrained MU-MISO Downlink under Imperfect Channel State Information

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***Abstract*—**In this paper, we consider the MU-MISO downlink scenario, under imperfect channel state information (CSI). The main issue in imperfect CSI is to keep the probability of each user achievable outage rate below the given threshold level. Such a rate outage constraints present significant and analytical challenges. There are many probabilistic methods are used to minimize the transmit optimization problem under imperfect CSI. Here, decomposition based large deviation inequality and Bernstein type inequality convex restriction methods are used to perform the optimization problem under imperfect CSI. These methods are used for achieving improved output quality and lower complexity. They provide a safe tractable approximation of the original rate outage constraints. Based on these method implementations, performance has been evaluated in the terms of feasible rate and average transmission power. The simulation results are shown that all the two methods offer significantly improved outage quality and lower computational complexity.

***Keywords*—**Imperfect channel state information, outage probability, multiuser- multi input single output.

I. Introduction

IN wireless communication, to leverage the new technique linear precoding. It has been recognized as a practically powerful method such as capable of controlling the quality of service (QoS) and developing system throughput [1], [2]. Basically, linear precoding methods assume knowledge of the downlink channels at the transmitter side, or simply CSI, and using perform interference management. It is common to assume perfect CSI. However, such an assumption is measured optimistic for several reasons [3]. Initially, in the time division duplex (TDD) setting, where there is reciprocity between the uplink and downlink channels, CSI is obtained by uplink channel evaluation. Such as, noise and limited training will introduce errors into the obtained CSI. Secondly, in the frequency division duplex (FDD) setting, where users approximation the downlink channels and inform the transmitter by rate-limited quantized CSI feedback. The acquired CSI is beset by quantization errors, in addition to the channel estimation errors, if the perfect CSI becomes outdated where the user mobility speed is faster than the CSI update speed. In common, imperfect CSI can lead to QoS outages. So, we consider the case of imperfect CSI and investigate how CSI error effects may be mitigated through relevant system designs. In reality, the topic is has received a great deal of attention lately. In today researches focuses on achievable rate analyses. Wherein the aim is, to study how performance depends on the CSI errors and to obtain the perfect design of channel estimation and CSI feedback schemes. There are more than a few works in this way, where optimal resource allocation for downlink/uplink training is studied [4], [5]. The achievable rates of that scheme under imperfect CSI is much challenged. In fact, many of the existing works fix the linear precoder to be the quite simple zero-forcing (ZF) beam former and analyze the subsequent ergodic achievable rate performance for obtain a more tractable problem. This perfectly assumes that the system is able to perform coding diagonally a many number of differently faded systems [4], [5]. There are few results on the outage rate metric, which is annoyed by the scenario of one-frame coding over a slowly fading environment. Most results in this work only apply to the single-user multiple-input single-output (MISO) scenario [6], [7]. In the downlink of such setups, which is a multiuser MISO scenario, precoding methods can be applied to boost the performance. The idea is to pre-determine signals at the transmitter, and mitigate the channel-induced interference. These techniques naturally demand that CSI is supplied at the transmitter. However, as explained in the provision of perfect CSI is often an involved task in wireless systems.

At present, the CSI error models considered in two different design approaches. The first is the worst-case robust approach, in which the model CSI errors are assumed to lie within a bounded set, and the goal is to design the precoder. So that it is robust against the worst-case QoS under the given CSI error model. Such an approach has some notable contributions includes the robust second-order cone program (SOCP) methods [8], [9], the robust minimum-mean-square-error (MMSE) methods [9], [10], and semi definite relaxation [11], [12].

The second approach assumes a probabilistic CSI error model. For example, the Gaussian model optimizes the precoder design with respect to the average QoS under that model. Such an average robust approach aims at good average performance, as different to the good worst-case performance required by the worst-case robust approach. This approach is often used to solve stochastic optimization problems [13], [14]. Hence, we to find approximate solutions that are efficiently computable and can give good approximation accuracies. For instance, the works we develop convex

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restrictions, or safe tractable approximations, of outage-based QoS constrained precoder optimization problems. There are also study outage-based power allocation methods under a fixed precoder structure [16], [17].

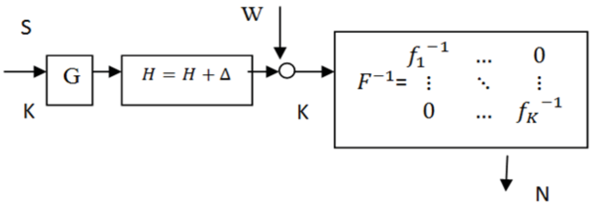

Fig. 1 Block diagram of the downlink multiuser MISO system

The main problem of interest is the transceiver optimization with the goal of minimizing the total transmit power subject to predefined users' QoS targets. This paper considers outage-based precoder optimization. Specifically, the scenario of interest is the multiuser MISO downlink, and the Gaussian CSI error model is adopted. We focus on a rate outage constrained problem, in which the goal is to optimize users' signal covariance matrices for total transmit power minimization while satisfying achievable rate outage constraints. We develop two novel convex restriction methods for the aforementioned rate outage constrained problem using probabilistic techniques. Furthermore, all two methods involve convex conic optimization problems that can be efficiently solved by an interior-point method (IPM). We use simulations to demonstrate that the presented methods perform better than the one developed in [14], [15], in terms of both computational complexity and solution quality.

A. Notations

We use a to represent vector and A to represent matrices. R^n and C^n stand for the sets of n-dimensional real and complex vectors. S^n and H^n stand for the set of n*n real symmetric matrices and complex Hermitian matrices, respectively. T and H represent the transpose and conjugate transpose. Tr(A) denote the trace. I_n denote the n x n identity matrix. ||.|| and $||.||_F$ denote the vector Euclidean norm and matrix Frobenius norm, respectively. E{.}, Prob{.} and exp(.) denote the statistical expectation, probability function and exponential function.

II. Problem Formulation

We consider a multiuser MISO downlink scenario, where in a multi-antenna base station sends independent messages to a number of single-antenna users over a quasi-static channel.

Let N_t denote the number of antennae at the base station, and K the number of users. The received signal of user i, $i = 1, \dots, K$, is modeled as $y_i(t) = h_i{}^H x(t) + v_i(t)$, where $h_i \in C^{N_t}$ is the channel of user i; $x(t) \in C^{N_t}$ is the transmit signal from the base station; is noise with distribution $CN(0, \sigma_i{}^2)$.

Under the above system setup, the achievable rate of user i may be formulated as

$$R_i = \log_2(1 + \frac{h_i{}^H S_i h_i}{\sum_{k \neq i} h_i{}^H S_k h_i + \sigma_i{}^2}), i = 1, \dots, K \tag{1}$$

The problem of interest here is to design the signal covariance matrices $\{S_i\}_{i=1}{}^K$ via a rate constrained formulation. The rate constrained problem (under perfect CSI) is formulated as

$$\mathbf{min}_{S_i, \dots, S_K \in H^{N_t}} \sum_{i=1}^{K} \boldsymbol{T_r(S_i)} \tag{2a}$$

$$\boldsymbol{s.t. R_i > r_i, i = 1, \dots, K,} \tag{2b}$$

$$\boldsymbol{S_1, \dots, S_K \geq 0} \tag{2c}$$

where each $r_i \geq 0$ is a pre-specified constant and describes the system's requirement on user i's information rate. The aim of the rate constrained problem is to find a set of signal covariance matrices such that the system's rate requirements are met using the smallest possible total transmission power.

To formulate the rate constrained problem under imperfect CSI, it is essential to first describe the CSI error model. In the imperfect CSI case, the actual channel of each user can be represented by $h_i = \overline{h}_i + e_i, i = 1, \dots, K.$ where $\overline{h}_i \in h^{N_t}$ is the presumed channel at the base station, and $e_i \in C^{N_t}$ is the channel error vector. We adopt the commonly used Gaussian channel error model. Consider the following probabilistically robust design formulation:

$e_i \sim CN(0, C_i)$

A. Rate Outage Constrained Problem

Given rate requirements and maximum tolerable outage probabilities $\rho_i, \dots, \rho_K \in (0,1)$, solve

$$\min_{S_1, \dots S_K \in H^{N_t}} \sum_{i=0}^{K} T_r(S_i) \tag{3a}$$

$$s.t. Prob_{h_i \sim CN(h_i, C_i)}\{R_i \geq r_i\} \geq 1 - \rho_i, i = 1, \dots, K \tag{3b}$$

$$S_1, \dots, S_K \geq 0 \tag{3c}$$

The above rate outage constrained problem emphasizes service fidelity, a feasible solution to problem (3) guarantees that under CSI errors, each user i. The rate outage constrained problem (3) is not known to be computationally tractable. The main challenge lies in the rate outage probability constraints in (3b), which do not admit simple closed-form expressions.

III. System Model

Our approach for tackling the rate outage constrained problem (3) is to track a convex restriction approach, also known as safe tractable approximation in the chance constrained optimization journalism [18]. The idea is to develop convex and efficiently computable upper bounds on the rate outage probabilities in (3b).

$$\boldsymbol{f(Q, r, s) \leq \rho} \tag{4}$$

$$=> Prob\{\boldsymbol{e^H Q_i e + 2Re\{e^H r\} + s \geq 0\} \geq 1 - \rho}. \tag{5}$$

Hence, the constraint (5) gives a convex restriction or safe approximation of the generally intractable probabilistic constraint (6). Returning to the rate outage constrained problem (3), we note that the rate outage constraints in (3b) can be expressed as

$$\boldsymbol{Q_i} = \boldsymbol{C_i}^{\frac{1}{2}}(\frac{1}{\gamma_i}\boldsymbol{S_i} - \sum_{k\neq i}\boldsymbol{S_k})\boldsymbol{C_i}^{1/2} \tag{6a}$$

$$\boldsymbol{r_i} = \boldsymbol{C_i}^{1/2}(\frac{1}{\gamma_i}\boldsymbol{S_i} - \sum_{k\neq i}\boldsymbol{S_k})\overline{\boldsymbol{h}}_{i,} \tag{6b}$$

$$\boldsymbol{s_i} = \overline{\boldsymbol{h}}_{\boldsymbol{i}}^{\boldsymbol{H}}\left(\frac{1}{\gamma_i}\boldsymbol{S_i} - \sum_{k\neq i}\boldsymbol{S_k}\right)\overline{\boldsymbol{h}}_{\boldsymbol{i}} - \boldsymbol{\sigma_i}^2, \boldsymbol{\gamma_i} = \boldsymbol{2}^{\gamma_i} - \boldsymbol{1} \tag{6c}$$

In formulating the rate outage constrained problem (3), we follow an information theoretic development, where the achievable rates to be optimized are based on the assumption of vector-Gaussian encoded transmit signals.

A. Bernstein Type Inequality

The probabilistic constraint (5) can be approximated in a conservative fashion using robust optimization techniques [19], [20]. The following feasibility problem is a convex restriction of (5).

Method I

Find Q,r,s,x,y

$$\text{s.t. } \boldsymbol{Tr(Q)} - \sqrt{\boldsymbol{2\ln(1/\rho)}}.\boldsymbol{x} + \boldsymbol{\ln(\rho)}.\boldsymbol{y} + \boldsymbol{s} \geq \boldsymbol{0},$$

$$\sqrt{||\boldsymbol{Q}||_F^2 + \boldsymbol{2r^2} \leq \boldsymbol{x}},$$

$$\boldsymbol{yI_n} + \boldsymbol{Q} \geq \boldsymbol{0},$$

$$\boldsymbol{y} \geq \boldsymbol{0}.$$

Applying Method I to the rate outage constrained problem (3), we obtain the following convex restriction problem is,

$$\min_{S_i\in H^{N_t}, x_i, y_i \in R} \sum_{i=0}^{K} T_r(S_i)$$

$$\text{s.t.} Tr(Q_i) - \sqrt{2\ln\left(\frac{1}{\rho_i}\right).x_i + \ln(\rho_i).x_i + \ln(\rho_i).y_i + S_i \geq 0,}$$
$$\text{i=1,….,K} \tag{7}$$

$$\left\|\begin{matrix} vec(Q_i) \\ \sqrt{2r_i} \end{matrix}\right\| \leq x_i, i, \dots, K,$$

$$y_i \mathrm{I}_{N_t} + Q_i \geq 0, i = 1, \dots, K,$$

$$\mathrm{y}_1, \dots, \mathrm{y}_\mathrm{K} \geq 0, \mathrm{S}_1, \dots, \mathrm{S}_\mathrm{K} \geq 0$$

The convex restrictions derived using method I can be formulated as semi definite programs (SDPs) and hence are polynomial-time solvable.

B. Decomposition Based Large Deviation Inequality

In this section, we propose another convex restriction method. The method is based on the following large deviation inequality for complex quadratic forms.

Method II

Let $v > 1/\sqrt{2}$ be such that $\overline{\theta}v = \sqrt{\ln 1/\rho}$, where $\overline{\theta} = 1 - 1/(2v^2)$. Then the following feasibility problem is a convex restriction of (5):

Find Q,r,s,x,y,

$$\text{s.t. } T_r(Q) + s \geq 2\sqrt{\ln 1/\rho}\,.(x+y),$$

$$\frac{1}{\sqrt{2}}||r|| \leq x,$$

$$v||Q||_F \leq y.$$

The above convex restriction constrains only SOC constrains, it can be solved more efficiently than the convex restriction obtained using Method I. Applying Method II to the rate outage constrained problem (3), we obtain the following convex restriction problem is,

$$\min_{S_i\in H^{N_t x_i, y_i \in R_i}} \sum_{i=1}^{K} T_r(S_i)$$

$$s.t. T_r(Q_i) + s_i \geq 2\sqrt{\ln 1/\rho_i}\,.(x_i + y_i), i = 1, \dots, K \tag{8}$$

$$\frac{1}{\sqrt{2}}||r_i|| \leq x_i, i = 1, \dots, K,$$

$$v_i||vec(Q_i)|| \leq y_i, i = 1, \dots., K,$$

$$S_1, \dots., S_K \geq 0.$$

IV. Results and Discussion

In this section presents simulation results to illustrate the performance of these two convex restriction methods for management the rate outage constrained problem (3). We assume that the users' noise powers are identical and given by $\sigma_1{}^2 = \cdots = \sigma_K{}^2 \triangleq \sigma^2$. We fix $\sigma^2 = 0.01$, unless specified. The outage specifications for all users are also set the same $\rho_1 = \cdots.. = \rho_K \triangleq \rho$. The convex restriction formulations are solved by the conic optimization solver SeDuMi, implemented through the parser software CVX.

We start the simulation with simple case of $N_t = K = 5$; i.e., five antennae at the base station, and 5 users. The CSI errors are spatially correlated and have standard circularly symmetric complex Gaussian distributions; that is $C_1 = \cdots = C_K = \sigma_e{}^2 I_N$, where $\sigma_e{}^2 \geq 0$ denotes the error variance. We set $\sigma_e{}^2 = 0.01$. The outage probability requirement is set to $\rho = 0.01$, which is corresponding to having a 90% or higher chance of satisfying the rate requirements.

In Fig. 2, where the feasibility rates of various methods are plotted against the SINR requirements γ. The two presented methods yield feasibility rate higher than that of the probabilistic SOCP method. Decomposition based large deviation inequality method has the best feasibility rate performance. As can be seen from Fig. 3, decomposition based large deviation inequality method yields the best average transmission power performance followed the probabilistic SOCP method.

Figs. 4 and 5 show that the performance under spatially correlated Gaussian CSI errors with Nt=5, K=5, p=0.1. As we can see from the figures, decomposition based large deviation inequality method offers best performance over the other methods.

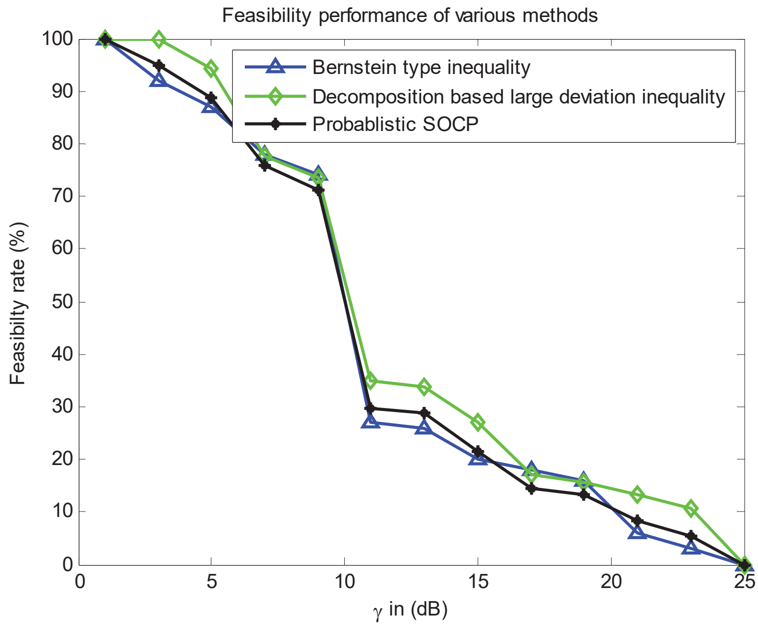

Fig. 2 Feasibility rate performance of the various method

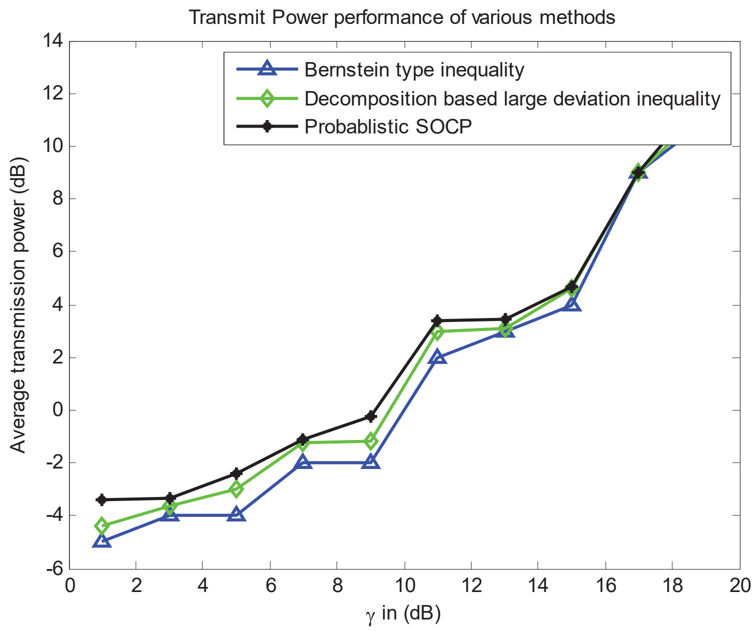

Fig. 3 Transmit power performance of the various method

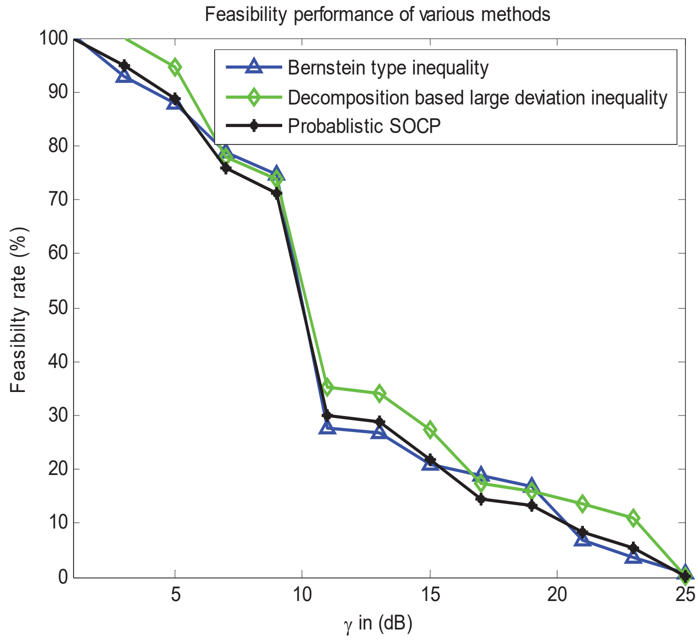

Fig. 4 Feasible rate performance under spatially correlated gaussian CSI error

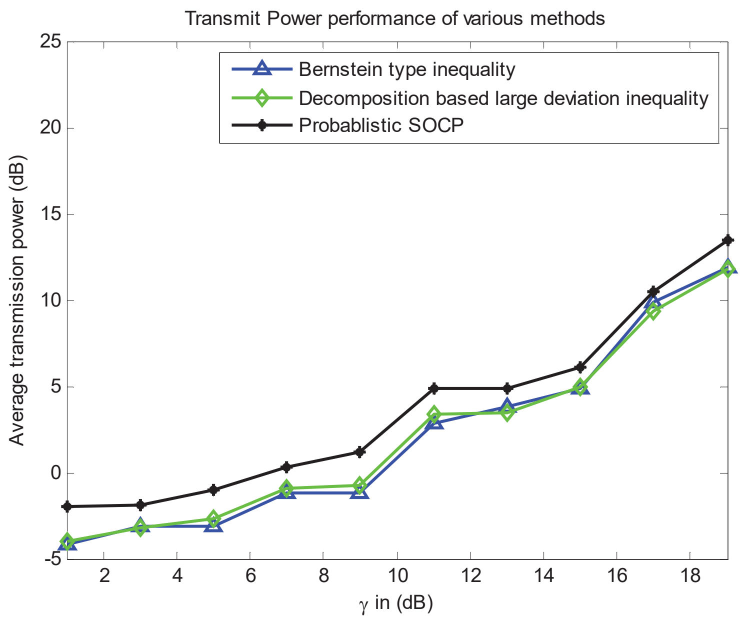

Fig. 5 Transmission power performance under the spatially correlated Gaussian error

V. Conclusion

In this work, we have formulated convex restriction problem under the MU-MISO downlink scenario with Gaussian CSI errors and studied a rate outage constrained optimization problem. This rate outage probability constraints problem, which are difficult to process computationally. To tackle these constraints, we presented two methods-namely, decomposition based large deviation inequality and Bernstein type inequality-for obtaining efficiently computable convex restrictions of the probabilistic constraints. Then we carried out performance analysis to study the complexity and relative tightness of these methods. Simulation results indicate that all two methods provide good approximation to the rate outage constrained problem.

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