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Sliding Mode Control for Active Suspension System with Actuator Delay

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Abstract—Sliding mode controller for a vehicle active suspension system is designed in this study. The widely used quarter car model is preferred and it is aimed to improve the ride comfort of the passengers. The effect of the actuator time delay, which may arise due to the information processing, sensors or actuator dynamics, is also taken into account during the design of the controller. A sliding mode controller was designed that has taken into account the actuator time delay by using Smith predictor. The successful performance of the designed controller is confirmed via numerical results.

Keywords—Sliding mode control, active suspension system, actuator time delay.

I. Introduction

PASSIVE suspensions are composed of spring and damper elements and for normal operation conditions their performance is fixed. To improve ride comfort of passengers, semi-active suspensions equipped with electrorheological [1] or magnetorheological [2] dampers may be used. Further improvement is also possible by using active suspensions that include actuators providing external energy to the system. This research area has remained attractive for many years and various control strategies such as PID [3], fuzzy logic [4], H_∞ [5], sliding mode [6], fuzzy sliding mode [7], [8], backstepping control [9] have been proposed. Most of the previously mentioned studies neglect the time delays during the controller design. However, in practice it is not possible to calculate and apply the needed control action to the system without any time delay. Therefore in reality effect of time delay should be taken into account. If not, the performance of the controlled system may degrade or even cause instability of the system. Various approaches have been used in literature for the control of active suspensions with actuator delay. In [10], constrained optimization was used to calculate state feedback gains along with a scheme for stability chart strategy for quarter active suspension system. H_∞ control design has also been proposed for vehicle active suspension system with actuator delay in [11] and [12]. Sliding mode control (SMC) is a variable structure controller and it is known to be a robust control method that can be applied to non-linear and uncertain systems [13], [14]. In this study, a sliding mode controller is designed for the quarter car active suspension system where the actuator time delay is also taken into account. The Smith predictor [15] is used to cope with the actuator time delay.

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II. Vehicle Model and Controller Design

A. Classical Sliding Mode Control

Equations of motion of an affine system can be written as

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{B}\mathbf{u}. \tag{1}$$

where $f(\mathbf{x})$ is the vector of the state equations without control inputs, $\mathbf{B}$ is the control force matrix and $\mathbf{u}$ is the vector of control forces. The sliding surface was selected as

$$\sigma = \mathbf{G}\Delta\mathbf{x}. \tag{2}$$

where $\Delta\mathbf{x}$ is the difference between the reference value x_r and the corresponding state x. $\mathbf{G}$ stands for the positive sliding slopes. Lyapunov function was selected such that it is positive definite and its derivative is negative semi-definite.

$$\mathbf{V}(\sigma) = \frac{\sigma^T\sigma}{2} > 0 \tag{3}$$

$$\frac{d\mathbf{V}(\sigma)}{dt} = \frac{\dot{\sigma}^T\sigma}{2} + \frac{\sigma^T\dot{\sigma}}{2} > 0 \tag{4}$$

If $\Delta\mathbf{x} = \mathbf{x}_r - \mathbf{x}$ then $\sigma = \mathbf{G}\mathbf{x}_r - \mathbf{G}\mathbf{x}$ is obtained that can be rewritten as

$$\sigma = \Phi(t) - \sigma_a(\mathbf{x}) \tag{5}$$

where $\Phi(t) = \mathbf{G}\mathbf{x}_r$ and $\sigma_a(\mathbf{x}) = \mathbf{G}\mathbf{x}$. Then,

$$\frac{d\sigma}{dt} = \frac{d\Phi(t)}{dt} - \frac{\delta\sigma_a(\mathbf{x})}{\delta\mathbf{x}}\frac{d\mathbf{x}}{dt} \tag{6}$$

Thus, for the limit case the controller force is obtained as

$$\mathbf{u}_{eq} = (\mathbf{GB})^{-1}\left(\frac{d\Phi(t)}{dt} - \mathbf{G}\mathbf{f}(\mathbf{x})\right) \tag{7}$$

An additional term should be defined to pull the system to the surface. Thus, derivative of the Lyapunov function is selected as

$$\dot{\mathbf{V}}(\sigma) = -\dot{\sigma}^T\Gamma\sigma < 0 \tag{8}$$

where Γ is a positive definite matrix. By using 6

$$\sigma^T\dot{\sigma} = -\dot{\sigma}^T\Gamma\sigma \tag{9}$$

by using 9, 6 and 7 total control input can be written as

$$\mathbf{u} = \mathbf{u}_{eq} + (\mathbf{GB})^{-1}\Gamma\sigma \tag{10}$$

where $(\mathbf{GB})^{-1}$ equals to mass matrix. But, sometimes $\mathbf{f}(\mathbf{x})$ and $\mathbf{B}$ are not well-known. For this purpose, a low-pass filter can be used to calculate equivalent control by using the average of the total control. Finally, total control is,

$$\mathbf{u} = \hat{\mathbf{u}}_{eq} + (\mathbf{GB})^{-1}\Gamma\sigma. \tag{11}$$

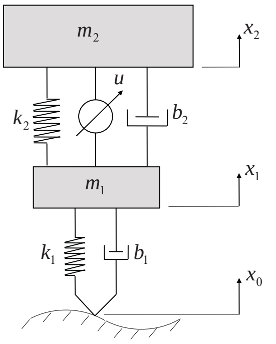

Fig. 1 Quarter car with active suspension

B. Vehicle Model

Quarter car active suspension system model, presented in Fig. 1, is used in this study. Mathematical model of the system is given by

$$m_1\ddot{x}_1 = -(k_1+k_2)x_1 + k_2x_2 - (b_1+b_2)\dot{x}_1 + b_2\dot{x}_2 + k_1x_0 + b_1\dot{x}_0 - u(t-D), \quad (12)$$

$$m_2\ddot{x}_2 = k_2x_1 - k_2x_2 + b_2\dot{x}_1 - b_2\dot{x}_2 + u(t-D). \quad (13)$$

The model has two degrees of freedom which are body bounce x_2 and displacement of the wheel x_1 that are both in vertical directions. Here, x_0 is the road surface input representing the road surface unevenness. The m_1 and m_2 represent the mass of the wheel-axle assembly and the vehicle main body, respectively; k_1 and b_1 are the stiffness and damping constants of the tire; similarly k_2 and b_2 stand for the stiffness and damping constant of the suspension spring and damper, respectively; $u(t-D)$ is the control signal with time delay D. Numerical values of the vehicle parameters are given in Table I.

TABLE I
VEHICLE PARAMETERS

$m_1 = 45\ kg$	$m_2 = 320\ kg$	$k_1 = 211180\ N/m$
$k_2 = 27000\ N/m$	$b_1 = 20\ Ns/m$	$b_2 = 935\ Ns/m$

C. Delay Compensation

The road input applied to the vehicle suspension is shown in Fig. 2. The vehicle travels over that road profile with a constant velocity of 20 m/s. In order to show the effects of the actuator delay on the system performance, the time responses of the vehicle body are presented in Fig. 3. Sliding Mode Control(SMC) was chosen as the first controller since it is a powerful control method that stability of the controlled system can be investigated easily. There are two cases for the controlled suspensions namely, the case with (w) time delay and the case without (w/o) time delay. The case without actuator delay may be thought as the desired force u is produced by the actuator immediately, that is without any time delay. On the other hand in reality it is not possible due to actuator dynamics. Therefore, the performance of this controller with time delay, $D = 35ms$ is also presented here. It is seen that the displacements grow up rapidly for the SMC case with actuator delay, that is time delay destabilized the suspension system.

Equations of motion of the quarter car suspension model are presented below in vector matrix form.

$$\frac{d\mathbf{X}}{dt} = \mathbf{A}\,\mathbf{X} + \mathbf{B}\,\mathbf{U}(t-D) + \mathbf{W}\,\mathbf{X_0}. \quad (14)$$

where $\mathbf{X} = [x_1\, x_2\, x_3\, x_4]$ is the state vector which includes the displacement x_1 and the velocity x_3 of the wheel and similarly displacement x_2 and velocity x_4 of vehicle body. $\mathbf{X_0} = [x_0\ \frac{dx_0}{dt}]^T$ is the road excitation vector, $\mathbf{U}(t-D)$ is the actuator control signal with time delay, D, and related matrices of the state equation of the vehicle model are given by

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{-(k_1+k_2)}{m_1} & \frac{k_2}{m_1} & \frac{-(b_1+b_2)}{m_1} & \frac{b_2}{m_1} \\ \frac{k_2}{m_2} & \frac{-k_2}{m_2} & \frac{b_2}{m_2} & \frac{-b_2}{m_2} \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 0 & 0 & -\frac{1}{m_1} & \frac{1}{m_2} \end{bmatrix}$$

$$\mathbf{W} = \begin{bmatrix} 0 & 0 & \frac{k_1}{m_1} & 0 \\ 0 & 0 & \frac{b_1}{m_1} & 0 \end{bmatrix}^T$$

$(\mathbf{A},\mathbf{B})$ is a controllable pair. By using the work [17], vehicle model 14 is equivalent to the following delay-free system;

$$\dot{\bar{\mathbf{X}}} = \mathbf{A}\bar{\mathbf{X}} + \mathbf{B}\,\mathbf{U}(t) + \mathbf{W}\,\mathbf{X_0}. \quad (15)$$

where

$$\bar{\mathbf{X}} = \mathbf{X} + \int_{t-\tau}^{t} e^{\mathbf{A}(t-\tau)}\mathbf{B}\mathbf{U}(\tau)d\tau. \quad (16)$$

The control rule for SMC that do not take into account the actuator time delay is given in Subsection II-A. Thus, by using the equivalent system 15 and the control input 11, we define the controller rule for the system 14 as

$$\mathbf{U} = \mathbf{U}_{eq} + (\mathbf{GB})^{-1}\Gamma\left(\mathbf{G}\mathbf{X}_r - \mathbf{G}\bar{\mathbf{X}}\right), \quad (17)$$

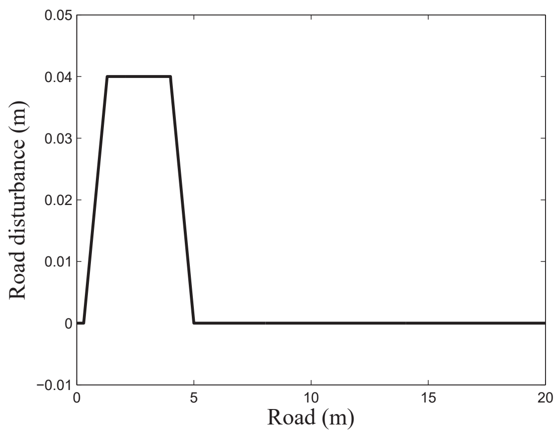

Fig. 2 The road disturbance acted on the suspension system

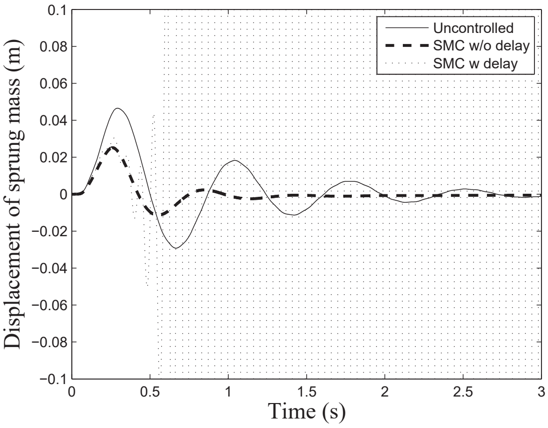

Fig. 3 The open and closed-loop response of the quarter-car model with sliding mode controller. Displacement of sprung mass with sliding mode controller without delay (dashed line); sliding mode controller with delay (dotted line); Passive system (solid line)

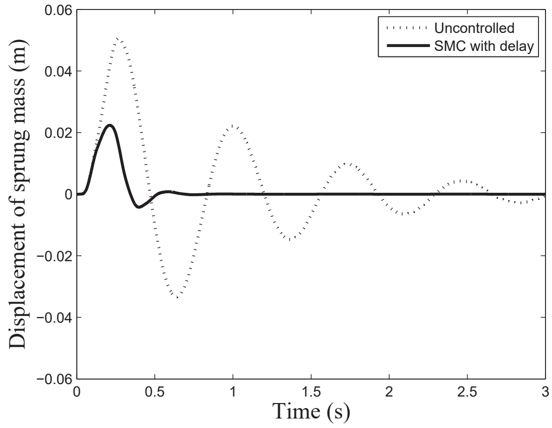

Fig. 4 The open and closed-loop response of the quarter-car model. Displacement of sprung mass with; Sliding mode controller with delay (thick solid line); Passive system (dotted line)

substituting 16 into 17 the final controller rule is defined as

$$\mathbf{U}(t) = \mathbf{U}_{eq} + (\mathbf{GB})^{-1}\Gamma\mathbf{G}\mathbf{X}_r - (\mathbf{GB})^{-1}\Gamma\mathbf{G}\left[\mathbf{X} + \int_{t-\tau}^{t} e^{\mathbf{A}(t-\tau)}\mathbf{B}\mathbf{U}(\tau)d\tau\right]. \quad (18)$$

During numerical implementation because of the last term of the control law 18, some problems described in [16] such as numerical instabilities can occur. To solve this problem, we use ordinary differential equation

$$\frac{\mathrm{d}z}{\mathrm{d}t} = \mathbf{A}\, z(t) + \mathbf{B}\, u(t) - e^{\mathbf{A}\,\mathbf{D}}\, \mathbf{B}\, u(t-D), \quad (19)$$

which has the solution as

$$z(t) = \int_{t-D}^{t} e^{\mathbf{A}\,(t-\tau)}\, \mathbf{B}\, U(\tau)\, d\tau. \quad (20)$$

Since our open-loop system is stable, 20 can be used to calculate the last term in 18.

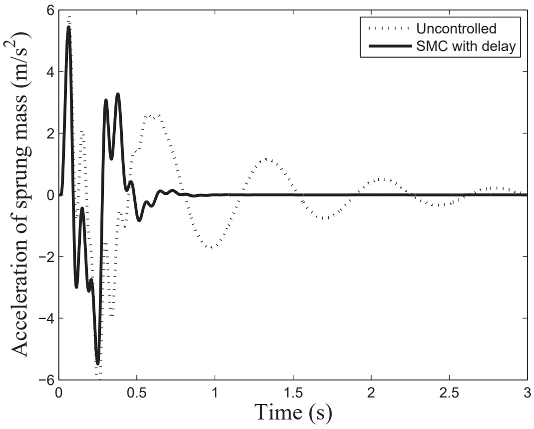

Fig. 5 Acceleration of the quarter-car model without control (dotted line); with Sliding mode controller with delay (thick solid line)

III. Numerical Results

The time responses for the vehicle body displacement are presented in Fig. 4 for the passive suspension, and active suspension with delay using designed SMC. It is seen from the same figure that the designed SMC stabilizes the system while satisfactorily suppressing the vehicle body displacements in the presence of time delay. Since acceleration of the vehicle body is also an important measure of the ride comfort, it is also presented in Fig. 5. If compared with the passive case it is seen that designed controller suppresses the acceleration of the vehicle body which means that the ride comfort is improved. The delayed control signal applied to the quarter-car suspension is shown in Fig. 6. It is seen that there is not any control effort at the beginning due to the time delay. Fig. 7 presents the time history of the dynamic tire load for the vehicle. It is seen that dynamic tire loads are not increased during ride comfort improvement. Moreover, the dynamic tire load was also reduced to a some degree indicating that road holding was also improved. Suspension travel response of the investigated vehicle active suspension system is presented in Fig. 8. It is seen from this figure that the magnitudes of the suspension travel response for the SMC case do not exceed the suspension travel response magnitudes of uncontrolled suspension system. Finally, the frequency responses for the vehicle body displacement is presented in Fig. 9. It is seen that the magnitudes for the displacements are reduced for a wide frequency range which indicates that ride comfort of the passengers is improved by the designed SMC eve in the presence of actuator time delay.

IV. Conclusion

Actuator time delays should be taken into consideration during controller design if not they may give rise to instability of the closed loop system. A sliding mode controller was designed that has taken into account the actuator time delay by using Smith predictor. Then this controller was applied to a

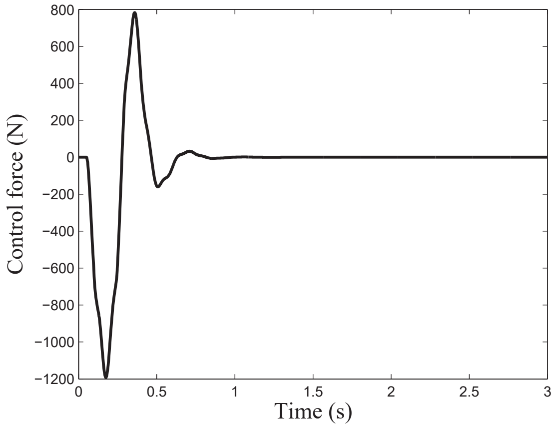

Fig. 6 Sliding mode control force applied to the quarter-car active suspension system

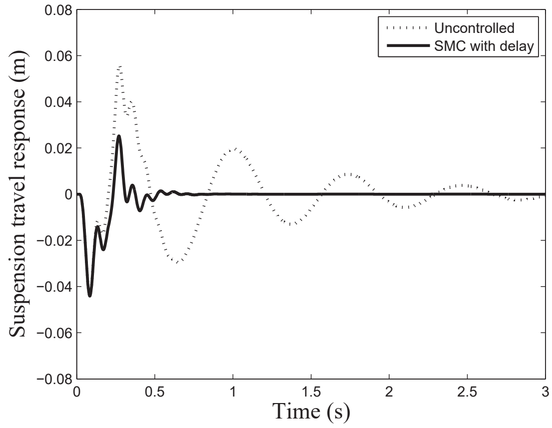

Fig. 8 Suspension travel responses of the vehicle system

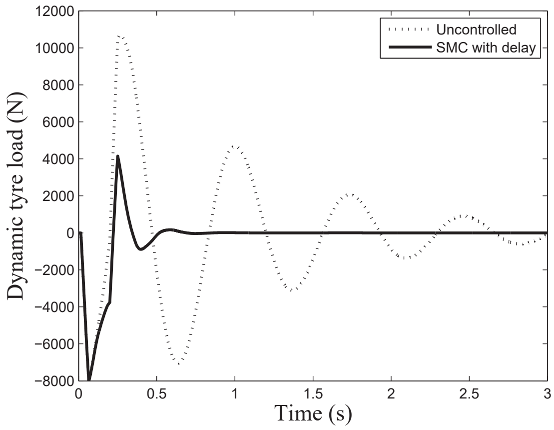

Fig. 7 Comparison of dynamic tire load of the quarter-car model without control and with Sliding mode controller.

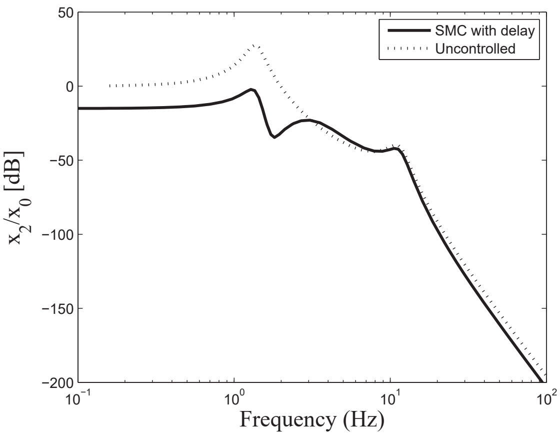

Fig. 9 Frequency Response of Sprung Mass

quarter car active suspension system with actuator time delay. The time and frequency responses have demonstrated that this controller improved ride comfort of passengers.

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