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Non-Singular Gravitational Collapse of a Homogeneous Scalar Field in Deformed Phase Space

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***Abstract*—**In the present work, we revisit the collapse process of a spherically symmetric homogeneous scalar field (in FRW background) minimally coupled to gravity, when the phase-space deformations are taken into account. Such a deformation is mathematically introduced as a particular type of noncommutativity between the canonical momenta of the scale factor and of the scalar field. In the absence of such deformation, the collapse culminates in a spacetime singularity. However, when the phase-space is deformed, we find that the singularity is removed by a non-singular bounce, beyond which the collapsing cloud re-expands to infinity. More precisely, for negative values of the deformation parameter, we identify the appearance of a negative pressure, which decelerates the collapse to finally avoid the singularity formation. While in the un-deformed case, the horizon curve monotonically decreases to finally cover the singularity, in the deformed case the horizon has a minimum value that this value depends on deformation parameter and initial configuration of the collapse. Such a setting predicts a threshold mass for black hole formation in stellar collapse and manifests the role of non-commutative geometry in physics and especially in stellar collapse and supernova explosion.

***Keywords*—**Gravitational collapse, non-commutative geometry, spacetime singularity, black hole physics.

I. Introduction

ONE of the most important questions in a gravitational theory, such as General Relativity (GR), and relativistic astrophysics is the gravitational collapse of a massive star under its own gravity at the end of its life cycle. A process in which a sufficiently massive star undergoes a continual gravitational collapse on exhausting its thermonuclear fuel, without achieving an equilibrium state such as a white dwarf or a neutron star. It is a well-established result that a gravitational collapse process, governed by the Einstein field equations with physically reasonable matter configurations, may end in a spacetime singularity [1]; this is an event where physical parameters such as the matter energy density and spacetime curvatures will diverge. [1]. However, classical GR breaks down at the very late stages of a collapse scenario, where densities and curvatures are extreme so that alternative theories of gravity could provide a suitable framework to resolve the singularity. One such possible effect is non-commutativity between spacetime coordinates, which was first proposed by Snyder [2] in an effort to introduce a short length cutoff (the non-commutativity parameter) in a Lorentz covariant way. The aim was to improve the renormalizability properties of relativistic quantum field theory. The basic idea that lies behind non-commutativity is to take into account the uncertainty in simultaneous measurements of the phase space coordinates and their conjugate momenta. The main motivation of such an interest was triggered by the works that have made the connection between non-commutativity and string and M theories [3]. Since the advent of non-commutative field theory, the interest in this area slowly but continuously made progress into the domain of gravity theories. Recent progresses in non-commutative geometry imply that, the non-commutative effects in GR may be taken into account by keeping the standard form of the Einstein tensor on the left-hand side of the field equations and introducing a modified energy-momentum tensor as a source including non-commutative parameter, on the right-hand side [4]. Since the past decades, much attention has been given into analyzing the collapse process of a spherical homogeneous scalar field, both numerically and analytically. The authors have dealt with the class of collapsing scalar field models with a non-zero potential, where the weak energy condition is satisfied by the collapse setting. It is seen that the endstate of the collapse at the classical level can be either a blackhole or a naked singularity, and that physically it is the rate of collapse that decides the final outcome of the collapse process [1]. In the herein model, we take the background spacetime as FRW model and investigate the effects of phase space deformation on the final fate of the collapse process of a homogeneous scalar field [5]. We introduce such a deformation as a noncommutativity between the canonical momenta of the scalar field and that of the scale factor. When the deformation effects are absent, the collapse scenario ends in a spacetime singularity, while in the presence such effects, the singularity is removed by a non-singular bounce. The phase space deformation effects show itself as a negative pressure that decelerates the collapse rate to finally prevent the singularity formation. Our objective is then to investigate the gravitational collapse of a minimally coupled scalar field ϕ in the presence of a specific phase-space deformation. In particular, this modification will concern the dynamical sector involving the momenta of the scale factor a and of ϕ. From a mathematical point of view, such a deformation introduces a deformed Poisson algebra, and hence makes the trajectory of the system within the phase-space to be different in comparison to the un-deformed case.

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II. Gravitational Collapse of a Homogeneous Scalar Field

Let us begin with the Lagrangian density of a scalar field which is given by

$$L=\sqrt{-g}\left[\frac{R}{2\kappa^2}-\frac{1}{2}g^{\mu\nu}\varphi_{,\mu}\phi_{,\nu}-V(\phi)\right], \tag{1}$$

where $\kappa^2=8\pi G$, R is the Ricci scalar, g is the determinant of a metric $g_{\mu\nu}$ (where the Greek indices run from zero to three) and $V(\phi)$ is the scalar field potential. The interior spacetime of the collapsing cloud is parametrized as

$$ds^2=dt^2-a^2(t)dr^2-R^2(t,r)d\Omega^2, \tag{2}$$

where $R(t,r)=ra(t)$ is the physical radius of the body, $a(t)$ is the scale factor and $d\Omega^2$ is the standard line element on a unit two-sphere. From the Lagrangian (1) we can find the corresponding Hamiltonian as

$$H=-\frac{\kappa^2}{12}a^{-1}P_a^2+\frac{1}{2}a^{-3}P_\phi^2+a^3V(\phi). \tag{3}$$

Let us now consider the ordinary phase-space structure described by the usual Poisson brackets, as

$$\{a,P_a\}=\{\phi,P_\phi\}=1, \tag{4}$$

We then get the equations of motion for the Hamiltonian (3) as

$$\begin{aligned}
\dot{a}&=\{a,H\}=-\frac{\kappa^2}{6}a^{-1}P_a,\\
\dot{P}_a&=\{P_a,H\}=-\frac{\kappa^2}{12}a^{-2}P_a^2+\frac{3}{2}a^{-4}P_\phi^2-3a^2V(\phi),\\
\dot{\phi}&=\{\phi,H\}=a^{-3}P_\phi,\\
\dot{P}_\phi&=\{P_\phi,H\}=-a^3\frac{dV}{d\phi},
\end{aligned} \tag{5}$$

whence we can easily get the equations governing the dynamics of the system as

$$\begin{aligned}
H^2&=\frac{\kappa^2}{3}\left[\frac{1}{2}\dot{\phi}^2+V(\phi)\right]\equiv\frac{\kappa^2}{3}\rho(t),\\
2\frac{\ddot{a}}{a}+H^2&=-\kappa^2\left[\frac{1}{2}\dot{\phi}^2-V(\phi)\right]=-\kappa^2p(t),
\end{aligned} \tag{6}$$

and the evolution equation for the scalar field

$$\ddot{\phi}+3H\dot{\phi}+\frac{dV(\phi)}{d\phi}, \tag{7}$$

where $H=\dot{a}/a$ is the rate of collapse. These equations admit a solution where the scale factor vanishes at a finite amount of time (we set $\kappa^2=1$ and α and β are negative constants).

$$\begin{aligned}
a(t)&=\left[a_i^{-\beta}-\alpha\beta\left(t-t_i\right)\right]^{\frac{-1}{\beta}},t_t=t_i+\frac{a_i^{-\beta}}{\alpha\beta},\\
\phi(t)&=\mp\sqrt{\frac{-2}{\beta}}\ln\left[a_i^{-\beta}-\alpha\beta\left(t-t_i\right)\right],\\
V(\phi)&=\alpha^2(3+\beta)\exp(-\sqrt{-2\beta}),
\end{aligned} \tag{8}$$

and the energy density together with Kretschmann invariant diverge

$$\begin{aligned}
\rho&=3\alpha^2\left[a_i^{-\beta}-\alpha\beta\left(t-t_i\right)\right]^{-2},\\
K&=12\left[\left(\frac{\ddot{a}}{a}\right)^2+\left(\frac{\dot{a}}{a}\right)^4\right]=\frac{24\alpha^4\left(1+\beta\left(1+\frac{\beta}{2}\right)\right)a_i^{4\beta}}{\left[1-(t-t_i)\alpha\beta a_i^{\beta}\right]^4},
\end{aligned} \tag{9}$$

Hence, we have a spacetime singularity. We subsequently show in the next section, by resorting to phase-space deformation effects, that the corresponding gravitational collapse procedure does not only culminate in the formation of a spacetime singularity but also exhibits a bouncing behavior.

III. Effects of Phase-Space Deformation on The Collapse Dynamics and Singularity Removal

In this section, inspired by the mentioned motivations in Reference [6] and also by the corrections from string theory to Einstein gravity [7], we propose to change the structure of the phase-space by introducing non-commutativity between conjugate momenta to trace the deformation implications in the gravitational collapse of a homogeneous scalar field. To retrieve a model with deformation (in the phase-space), where the calculations would allow interesting novel results, but that do not convey a mere trivial scenario, we should reasonably pick a convenient framework. Therefore, we choose to employ a dynamical deformation within the canonical conjugate momentum sector given as

$$\{P_a,P_\phi\}=\ell\phi^3, \tag{10}$$

where ℓ is called as deformation parameter. The other Poisson brackets have been left unchanged. The modified equations of motion with respect to Hamiltonian (3) are then found as

$$\begin{aligned}
\dot{a}&=\{a,H\}=-\frac{1}{6}a^{-1}P_a,\\
\dot{P}_a&=\{P_a,H\}=-\frac{1}{12}a^{-2}P_a^2+\frac{3}{2}a^{-4}P_\phi^2-3a^2V(\phi)+\ell a^{-1}\phi^3P_a,\\
\dot{\phi}&=\{\phi,H\}=a^{-3}P_\phi,\\
\dot{P}_\phi&=\{P_\phi,H\}=-a^3\frac{dV}{d\phi}+\frac{1}{6}\ell a^{-1}\phi^3P_a,
\end{aligned} \tag{11}$$

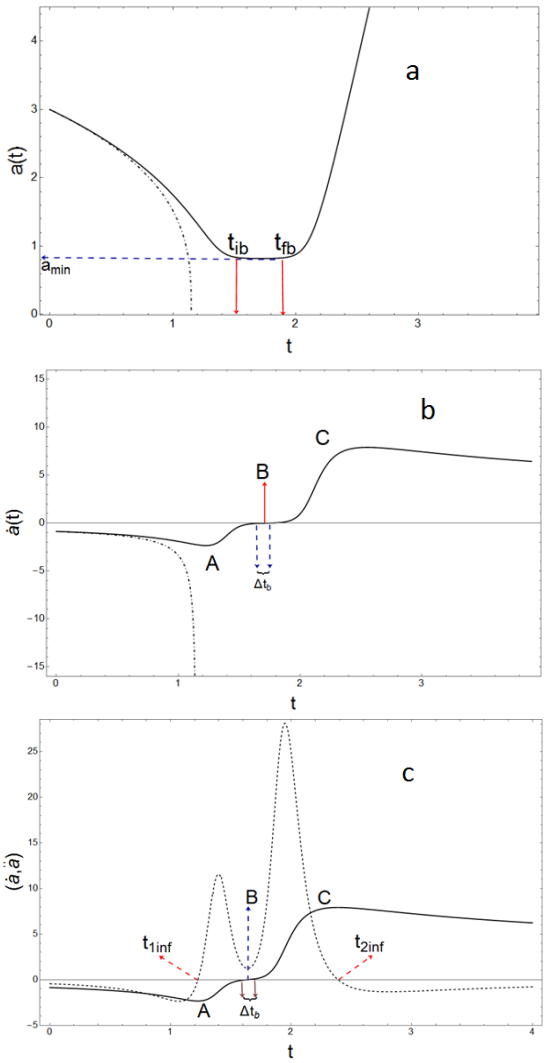

Fig. 1 (a) and (b) Time behavior of the scale factor and the speed of collapse for different values of deformation parameter, $\ell = -0.211$ (solid curve) and $\ell = 0$ (dotted-dashed curve), $\beta = -3.2$ and $\alpha = 1.1$ (c) The time behavior of $\dot{a}$ (solid curve) and $\ddot{a}$ (dashed curve) for $\ell = -0.211$. We have taken the initial values

whence after a few algebras, we get the evolution equations for the system as

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right] \equiv \frac{1}{3}\rho_{eff}, \tag{12}$$

$$2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 = -\left[\frac{1}{2}\dot{\phi}^2 - V(\phi)\right] - \frac{1}{3}\ell a^{-2}\phi^3\dot{\phi} \equiv -(p + p_d) \equiv -p_{eff}, \tag{13}$$

together with the scalar field evolution equation as

$$\ddot{\phi} + 3\left(\frac{\dot{a}}{a}\right)\dot{\phi} + \frac{dV(\phi)}{d\phi} + \ell\dot{a}\left(\frac{\phi}{a}\right)^3 = 0. \tag{14}$$

We note that the above equations reduce to the commutative case once we set $\ell = 0$. The p_d term is a pressure coming from non-commutative effects that play a crucial role in the collapse dynamics. We now investigate some aspects of the gravitational collapse, within the above framework for deformed phase-space, by means of numerical methods. We are particularly interested in probing the behavior of the scale factor, its time derivative, collapse acceleration, the scalar field evolution, and other related quantities for a potential of the same type as (8), in order to properly contrast the presence of non-commutative features in the collapse dynamics. Fig. 1 (a) presents numerically the time evolution of the scale factor for different values of the deformation parameter. The scale factor trajectories begin from the same initial value, a_i but, as the collapse proceeds, the full curve ($\ell < 0$) separates from the other one and reaches a minimum value for the scale factor at a critical epoch which lies between $t_{ib} < t_{cr} < t_{fb}$. Thus, for $t_{ib} < t < t_{cr}$, the collapse scenario proceeds much slower than $t < t_{ib}$, ceasing at t_{cr} and then entering a smooth expanding phase for $t_{cr} < t < t_{fb}$. Therefore, it is seen that for $\ell < 0$ the collapse scenario presents a soft bouncing behavior during the time interval $\Delta t_b = t_{fb} - t_{ib}$. As the dot-dashed curve shows, the scale factor vanishes when the phase space deformation effects are absent i.e., $\ell = 0$. In Fig. 2 (b) we have shown the behavior of the speed of collapse ($\dot{a}$) where we see that for $\ell < 0$ the collapse commences from $\dot{a}(t_i) < 0$ proceeding for a while in an accelerating phase until an absolute maximum value in negative direction is reached (point A). It then decelerates and halts at point B where $\dot{a}(t_{cr}) = 0$. After this epoch, the collapse regime is replaced by an accelerated expansion and continues up to the point C. This expanding phase slows down when this point is passed. Fig. 1 (c) further supports this argument: the acceleration of collapse remains negative prior to point A, where the collapse speed achieves its maximum negative value.

$$\phi(t_i) = 1.98,\ \dot{\phi}(t_i) = 1.98 \text{ and } a(t_i) = 3$$

This point corresponds to the first inflection point of acceleration curve, occurring at $t = t_{1\text{inf}}$. Thus, for $t < t_{1\text{inf}}$ the collapse proceeds in the so-called fast-reacting process while for $t_{1\text{inf}} < t < t_{cr}$ a slow-reacting regime governs. The collapse experiences a decelerating phase from points A to B (see Fig. 1 (b) with $\ddot{a}$ achieving in between a local maximum.

As time evolves, the acceleration decreases to point B, with $\dot{a}$ progressing toward less negative values (upwards), eventually being $\dot{a}=0$ and then smoothly becoming positive. This happens during the time interval Δt_b within which the bounce appears. We note that Δt_b is too small so that $\dot{a}$ changes infinitesimally and $\ddot{a} \sim cte.$ For $t > t_{fb}$, an accelerating expanding phase governs the scenario until the time $t_{2\inf}$, at which $\ddot{a}$ reaches its second inflection point, where $\dot{a}$ achieves its absolute maximum (see also point C). For $t > t_{2\inf}$ the expanding phase slows down at late times. For the case $\ell = 0$, the collapse velocity gets arbitrary large values at the singularity. In order to study the causal structure of spacetime during the dynamical evolution of the collapsing cloud, we need to investigate the time behavior of the apparent horizon which is the outermost boundary of trapped surfaces. The equation that governs the apparent horizon curve is given by [5]

$$r_{ah}(t) = \frac{1}{a(t)}\left[\frac{3}{\rho_{eff}}\right]^{\frac{1}{2}}. \qquad (15)$$

Fig. 2 shows the behavior of the apparent horizon curve where we see that when the phase space deformation effects are present the apparent horizon curve reaches a minimum value in the contracting phase; it then diverges at the bounce and converges to finite non-vanishing values at the expanding phase. The apparent horizon never gets zero values. Therefore, by choosing the boundary of the collapsing cloud suitably, the formation of the apparent horizon can be prevented. However, for $\ell = 0$, the apparent horizon converges monotonically to zero to finally cover the singularity. In this case a black hole is formed. Finally, in Fig. 3, we have plotted the behavior of scale factor for different values of the deformation parameter. It is seen that the larger the absolute value of ℓ the longer it takes for the bounce to occur. We therefore conclude that the effects of deformation within the phase-space could alter the final fate of the collapse scenario of a homogeneous scalar field so that the spacetime singularity is removed by a non-singular bounce. Since the apparent horizon is not formed, the bounce could be observable and hence providing possible chance to detect the effects of non-commutative geometry.

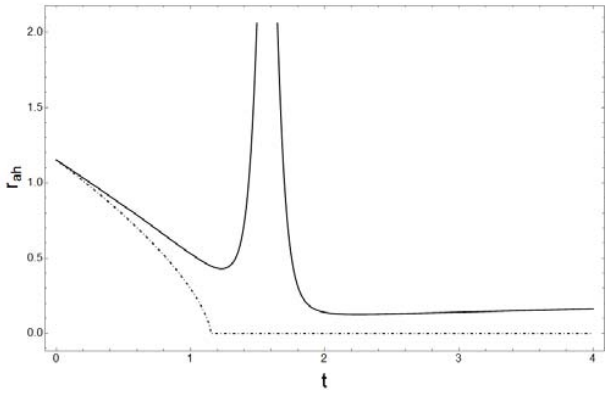

Fig. 1 The behavior of the apparent horizon for $\ell = -0.211$ (solid curve) and $\ell = 0$ (dotted-dashed curve), $\beta = -3.2$ and $\alpha = 1.1$. Lower panel: The time behavior of $\dot{a}$ (solid curve) and $\ddot{a}$ (dashed curve) for $\ell = -0.211$. We have taken the initial values $\phi(t_i) = 1.98$, $\dot{\phi}(t_i) = 1.98$ and $a(t_i) = 3$

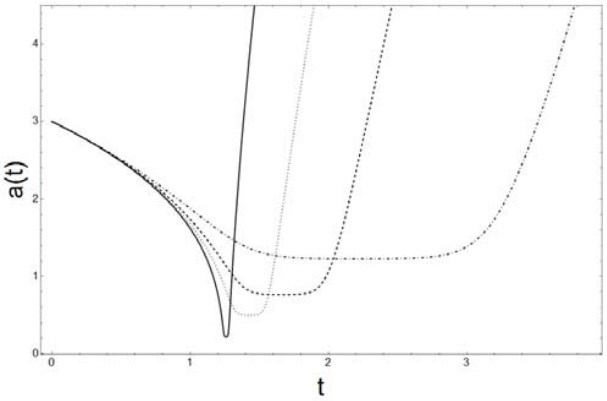

Fig. 2 The time behavior of the scale factor for different values of deformation parameter, $\ell = -0.07385$ (solid curve), $\ell = -0.1266$ (dotted curve), $\ell = -0.19412$ (dashed curve) and $\ell = -0.38402$ (dotted-dashed curve)

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