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Lyapunov-Based Tracking Control for Nonholonomic Wheeled Mobile Robot

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***Abstract*—**This paper presents a tracking control strategy based on Lyapunov approach for nonholonomic wheeled mobile robot. This control strategy consists of two levels. First, a kinematic controller is developed to adjust the right and left wheel velocities. Using this velocity control law, the stability of the tracking error is guaranteed using Lyapunov approach. This kinematic controller cannot be generated directly by the motors. To overcome this problem, the second level of the controllers, dynamic control, is designed. This dynamic control law is developed based on Lyapunov theory in order to track the desired trajectories of the mobile robot. The stability of the tracking error is proved using Lupunov and Barbalat approaches. Simulation results on a nonholonomic wheeled mobile robot are given to demonstrate the feasibility and effectiveness of the presented approach.

***Keywords*—**Mobile robot, trajectory tracking, Lyapunov, stability.

I. Introduction

IN recent years there has been enormous activity in the study of a class of mechanical control systems called nonholonomic mobile robot. In addition to their practical applications, the theoretical challenges of both nonholonomic characteristic and nonlinearity modeling have attracted the attention of many researchers. However, the issues associated with nonlinearity modeling are unable to be solved by conventional linear control theory, and therefore other possibilities have therefore been explored by various researchers. Controlling such nonholonomic systems turns out to be a nontrivial problem for many reasons. Even in the simplest case, which we shall study here, the kinematic model of a two-wheel mobile robot, the stabilization problem at a given position requires a nontrivial controller [1]-[5].

Many efforts have been devoted to research the tracking problems of wheeled mobile robots in recent years. A robust fuzzy logic controller is presented in [6] for the trajectory tracking of a mobile robot based on controlling the robot at a higher level. The controller is highly robust, flexible, and can automatically follow a sequence of discrete way points. Moreover, no interpolation of the waypoints is needed to generate a continuous reference trajectory. A robust sliding mode tracking control for a nonholonomic mobile robot has been presented in [7]. A feedback linearized by the computed-torque method for the dynamic equation of the robot is used and the position is calculated by polar coordinates. Other control schemes such as adaptive control and based on neural networks are proposed in recent years [8]-[10]. Robust trajectory tracking controllers for a two-wheeled mobile robot was developed by [11] using its kinematic and dynamic model in the presence of slip. The authors in [12] presented an integrated motion planning and control framework for the control of a wheeled mobile robot based on the differential flatness property. A centralized feedback linearizing control strategy is used in [13] with an extended Kalman Filter to achieve a desired formation. Backstepping approach was used in [14] for the dynamic model of a mobile robot.

Many control strategies based on Lyapunov approach have been proposed for mobile robot. For unicycle-like vehicles, [15] used Lyapunov approach to develop the control law. The author proved that, with a special choice of the system state variables, global stability properties can be guaranteed by smooth feedback control law. Since that, researchers have been working to improve Lyapunov based controller performances in terms of convergence of tracking errors and time response. For corridor navigation and wall following, the authors in [16], proposed a Lyapunov based control law using sonar and odometric sensorial information. In [17], Lyapunov based controller is employed for mobile robot for tracking a moving target with limited velocities.

This paper presents a tracking control strategy for the mobile robots. This control strategy is divided into 2 levels to achieve a smooth tracking movement while the robot moving forward on a predefined trajectory. In the first step, a kinematic controller is used to generate a velocity control law in order to adjust the right and left wheel velocities. This control law uses the desired and the real position/orientation of the platform to generate a desired velocity that will be used as input for the next level. Since this desired velocity cannot be generated directly by the motors, control torques are designed for the mobile platform based on the dynamic model. In the second step, the dynamic controller is designed based on Lyapunov approach to track the desired trajectory. Simulation results show the effectiveness of the proposed control strategy for controlling the mobile robot.

The rest of this paper is organized as follows. In Section II, the system description is presented. Section III presents kinematic and dynamic modeling of the mobile. The control law for the mobile robot is presented in Section IV. The simulation results are discussed in Section V, and the conclusion is presented in Section VI.

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II. System Description

The system used in this paper consists of two driving wheels mounted on the same axis at the front while the two back wheels can freely rotate as shown in Fig. 1. r is the radius of each driving wheel, L is the distance between driving wheel and the axes of symmetry, C_c is the center of mass of the mobile robot, $C_0 - XY$ is the coordinate system fixed to the mobile robot, C_0 is the origin of the coordinate system $C_0 - XY$ and is the center point between the right and left driving wheels, and d is the distance from C_0 to C_c. The generalized coordinate $q = [x, y, \emptyset]^T$ denotes the position and orientation vector of the robot, and $q_d = [x_d, y_d, \emptyset_d]^T$ represents the desired trajectory.

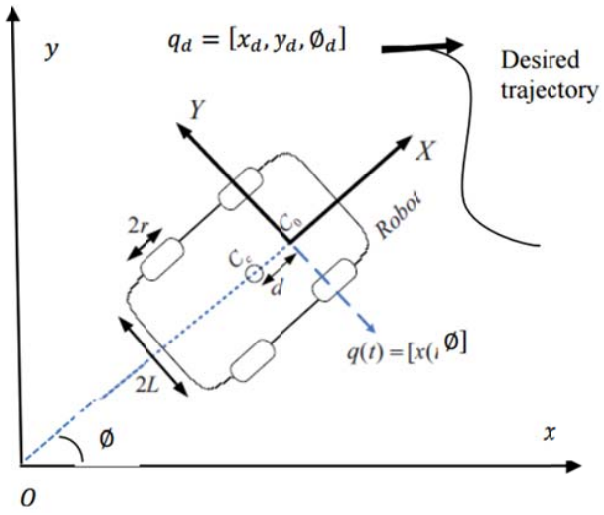

Fig. 1 Mobile robot

The objective is to develop a control law in order to track the desired trajectory. Fig. 2 illustrates the block diagram of the control strategy. There are two levels of the controller, being kinematic and dynamic controller respectively. The kinematic controller provides the desired velocity which will be used by the dynamic controller in order to calculate the suitable torque to be applied to the right and left motors of the wheels.

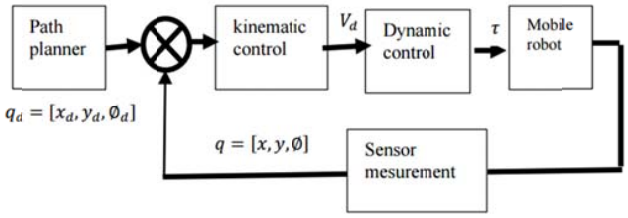

Fig. 2 Control design

III. Dynamic and Kinematic Modeling

The mobile manipulator considered in this paper consists of a wheeled mobile robot shown in Fig. 1. The general dynamic equation is described by [18] as:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = B(q)\tau - A^T(q)\lambda \quad (1)$$

where $\tau = [\tau_r, \tau_l]^T \in R^r$ is the input vector and consists of motors' torques τ_r and τ_l which act on the right and left wheels, $\lambda \in R^m$ is the vector of constraint forces, $M(q) \in R^{n\times n}$ is a symmetric and positive-definite inertia matrix, $C(q,\dot{q}) \in R^{n\times n}$ is the centripetal and Coriolis vector, $G(q) \in R^n$ is the gravitational vector, $B(q) \in R^{n\times r}$; is the input transformation matrix, and $A(q) \in R^{m\times n}$ is the matrix associated with the constraints. We consider that the robot is moving on a flat terrain and thus $G(q) = 0$. M, C, G, q in (1) can be expressed as:

$$M(q) = \begin{bmatrix} m & 0 & -2m_w d sin\emptyset \\ 0 & m & 2m_w d cos\emptyset \\ -2m_w d sin\emptyset & 2m_w d cos\emptyset & I \end{bmatrix}$$

$$C(q,\dot{q})\dot{q} = \begin{bmatrix} -2m_w\dot{\emptyset}^2 d cos\emptyset \\ -2m_w\dot{\emptyset}^2 d sin\emptyset \\ 0 \end{bmatrix}; G = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix};$$

$$A^T = \begin{bmatrix} sin\emptyset \\ -cos\emptyset \\ -d \end{bmatrix}; B(q) = \frac{1}{r}\begin{bmatrix} cos\emptyset & cos\emptyset \\ sin\emptyset & sin\emptyset \\ L & -L \end{bmatrix}; q = \begin{bmatrix} x \\ y \\ \emptyset \end{bmatrix}$$

$$m = m_c + 2m_w;\ I = I_c + 2m_w(d^2 + L^2) + 2I_m$$

where m_c is the mass of the robot without the driving wheels, m_w is the mass of each driving wheel plus the motor rotor, I_c is the moment of inertia of the platform without the driving wheels and I_m is the moment of inertia of each wheel and the motor rotor about a wheel diameter; the kinematic constraints can be denoted as:

$$A(q)\dot{q} = 0 \quad (2)$$

$$\dot{x}sin\emptyset - \dot{y}cos\emptyset = 0 \quad (3)$$

When selecting a full rank matrix $S(q)$ to be a basis of null space A(q), the constraint equation will be:

$$A(q)S(q) = 0 \quad (4)$$

where

$$S = \begin{bmatrix} \frac{r}{2L}(Lcos\emptyset + dsin\emptyset) & \frac{r}{2L}(Lcos\emptyset - dsin\emptyset) \\ \frac{r}{2L}(Lsin\emptyset - dcos\emptyset) & \frac{r}{2L}(Lsin\emptyset + dcos\emptyset) \\ \frac{r}{2L} & -\frac{r}{2L} \end{bmatrix}$$

Therefore, we have:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\emptyset} \end{bmatrix} = S(q)\begin{bmatrix} \omega_l \\ \omega_r \end{bmatrix} = S(q)W \quad (5)$$

where $\omega_r; \omega_l$ represent the angular velocities of the right and left wheels and $W = [\omega_l \quad \omega_r]^T$. If we consider $v; \omega$ as the linear and angular velocities of the mobile robot, the relation between $v; \omega$ and $\omega_r; \omega_l$ can be explained as:

$$\begin{bmatrix}\omega_l\\ \omega_r\end{bmatrix}=\begin{bmatrix}1/r & L/r\\ 1/r & -L/r\end{bmatrix}\begin{bmatrix}v\\ \omega\end{bmatrix}\rightarrow W=HV \tag{6}$$

where $V=[v \quad \omega]^T$. From (5) and (6) it is clear that:

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\emptyset}\end{bmatrix}=\begin{bmatrix}cos\emptyset & dsin\emptyset\\ sin\emptyset & -dcos\emptyset\\ 0 & 1\end{bmatrix}\begin{bmatrix}v\\ \omega\end{bmatrix}=SV \tag{7}$$

The derivative of (7) gives:

$$\dot{q}=SV\Rightarrow\ddot{q}=\dot{S}V+S\dot{V} \tag{8}$$

IV. Control Law Design

In this section, the control torque based on dynamic model is developed for nonholonomic wheeled mobile robot. First, the velocity control based on kinematic model is designed to develop the desired velocity. Next, the torques for mobile platform are developed using such a desired velocity.

When multiplying (1) by S^T, the constraint force term $A^T(q)\lambda$ can be eliminated using (4). So, we have:

$$S^TM\ddot{q}+S^TC\dot{q}=S^TB\tau \tag{9}$$

Introducing (8), the dynamic equation (9) becomes:

$$\bar{M}\dot{V}+\bar{C}V=\bar{\tau} \tag{10}$$

where $\bar{M}=S^TMS$; $\bar{C}=S^TM\dot{S}+S^TCS$ and $\bar{\tau}=S^TB\tau$

From the modified model, we consider the following properties that will be used in the stability analysis:

P1. The inertia-mass matrices M(q) and $\bar{M}(q)$ are symmetric positive definite**.**

P2. The inertia-mass matrix $\bar{M}(q)$ and the Coriolis matrix $\bar{C}(q,\dot{q})$ satisfy the following skew-symmetric property:

$$X^T(\bar{M}-2\bar{C})X=0\quad\forall X\in\mathcal{R}^n \tag{11}$$

Note that the objective is to track a reference trajectory by the mobile platform. Then, the desired position is $q_d=[x_d \quad y_d \quad \phi_d]$ and the desired velocity is $V_d=[v_d \quad \omega_d]^T$. Therefore, the tracking errors is obtained using the Kanayama transformation [19] as:

$$\begin{bmatrix}\tilde{x}\\ \tilde{y}\\ \tilde{\phi}\end{bmatrix}=\begin{bmatrix}cos\phi & sin\phi & 0\\ -sin\phi & cos\phi & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x_d-x\\ y_d-y\\ \phi_d-\phi\end{bmatrix} \tag{12}$$

Proposition 1: Using (2), (3) and (12), the error dynamics can be given as:

$$\begin{aligned}\dot{\tilde{x}}&=\omega\tilde{y}-v+v_dcos\emptyset\\ \dot{\tilde{y}}&=-\omega\tilde{x}+v_dsin\emptyset\\ \dot{\tilde{\phi}}&=\omega_d-\omega\end{aligned} \tag{13}$$

Proof: See Appendix.

Proposition 2: The error dynamics (13) are asymptotically stable when using the following velocity control law:

$$V(t)=\begin{bmatrix}v\\ \omega\end{bmatrix}=\begin{bmatrix}k_x\tilde{x}+v_dcos\tilde{\phi}\\ \omega_d+k_1v_d\tilde{y}\end{bmatrix} \tag{14}$$

where $k_1>0$; $k_x>0$ are the controller gains.

To prove proposition 2, we consider the following positive Lyapunov function:

$$W=\frac{1}{2}\tilde{x}^2+\frac{1}{2}\tilde{y}^2+\frac{1-cos\tilde{\phi}}{k_1} \tag{15}$$

The time derivative of W is given as:

$$\begin{aligned}\dot{W}(t)=\tilde{x}\dot{\tilde{x}}+\tilde{y}\dot{\tilde{y}}+\dot{\tilde{\phi}}\frac{sin\tilde{\phi}}{k_1}&\\ =\tilde{x}(\omega\tilde{y}-v+v_dcos\emptyset)+\tilde{y}(-\omega\tilde{x}+v_dsin\emptyset)&\\ +(\omega_d-\omega)\frac{sin\tilde{\phi}}{k_1}&\\ =-v\tilde{x}-\omega\frac{sin\tilde{\phi}}{k_1}+\tilde{x}v_dcos\emptyset+\tilde{y}v_dsin\emptyset+\omega_d\frac{sin\tilde{\phi}}{k_1}&\end{aligned}$$

Using the control law (14), the time derivative of the Lyapunov function becomes:

$$\dot{W}(t)=-k_x\tilde{x}^2 \tag{16}$$

The time derivative of $W(t)$ is negative because k_x is a positive gain. Then, using Barbalat's theorem [20], the error dynamics are asymptotically stable.

The above velocity control law, (14), is based on the kinematic model. However, the motors generate control torques, and cannot directly generate the velocity controls. Therefore, it is necessary to design the torques for the mobile platform based on the dynamic model, and then the control torque will result in an actual velocity.

Using the actual and desired velocities, the velocity tracking error can be expressed as:

$$z=V-V_d \tag{17}$$

where V and V_d are the actual and desired velocity of the mobile robot, respectively.

Let us propose the following control law:

$$\bar{\tau}=\bar{M}\dot{V}_d+\bar{C}V_d-Kz \tag{18}$$

where K is a positive gain.

Inserting the controller equation (18) in the dynamic equation (10), the error dynamics can be expressed as:

$$\bar{M}\dot{Z}+\bar{C}Z+Kz=0 \tag{19}$$

Proposition 3: The closed loop system is asymptotically stable when using the proposed controllers (14) and (18).

To prove the stability of the closed loop system, we propose the following positive Lyapunov function:

$$V=\frac{1}{2}z^T\bar{M}z \tag{20}$$

The time derivative of V is given as follows:

$$\dot{V} = z^T \bar{M}_{11} \dot{z} + \frac{1}{2} z^T \dot{\bar{M}}_{11} z \tag{21}$$

Using the error dynamics (19), (21) becomes:

$$\dot{V} = z^T [-\bar{C}Z - Kz] + \frac{1}{2} z^T \dot{\bar{M}}_{11} z \tag{22}$$

Using the property (11), the time derivation becomes:

$$\dot{V} = -z^T K z \tag{23}$$

Since K is a positive definite matrix, the time derivative $\dot{V}$ is negative. Using Barbalat's theorem [20], the error dynamics are globally asymptotically stable.

V. Simulation Results

The mobile robot which is shown in Fig. 1 is utilized to demonstrate the effectiveness of the proposed control strategy. The dynamic parameters for this mobile robot are illustrated in Table I.

TABLE I
Simulation Parameters

Parameters	Values
m_c (kg)	2.3
m_ω (kg)	.28
L (m)	.1
r (m)	0.04
d(m)	0.02
I_c (kgm^2)	.01
I_m (kgm^2)	.0021

The reference trajectory is chosen as $q_{rd} = [x_r \quad y_r \quad \emptyset_r]^T$ then $\dot{x}_r = V_r cos\emptyset_r$; $\dot{y}_r = V_r sin\emptyset_r$, $\dot{\emptyset}_r = \omega_r$. where the linear velocity and the reference angular velocity are chosen as $V_r = 1m/s$ and $\omega_r = 0.3\, rad/s$, respectively. The reference initial position of the mobile platform is $q_d(0) = [2 \quad 2 \quad 45^o]^T$, while the actual initial position is $q(0) = [0 \quad 0 \quad 0^o]^T$. The kinematic controller is defined in (14), where the gains are set to be $k_x = 8$; $k_y = 10$; $and\ k_\phi = 12$. The dynamic controller gains are $k_d = diag[25,25]$. The simulation results obtained are shown in Figs. 3-9.

Regarding the simulation results, a good tracking in x-position is shown in Fig. 3. Fig. 6, which shows the related tracking errors, confirms such a good tracking. In the y-position, Fig. 4 shows a good tracking of the desired trajectory. According to the related tracking error presented in Fig. 7, a good tracking in y-position is obtained. For the $\emptyset$-direction, the tracking of the desired trajectory is presented in Fig. 5. It is clear from Fig. 8 that the related tracking error converges to zero, which again confirms a good tracking. Finally, from these simulation results, despite the different starting point of the desired and the real values, the steady state errors are very small and converge to zero, which demonstrate an effective control performance on nonholonomic wheeled mobile robot.

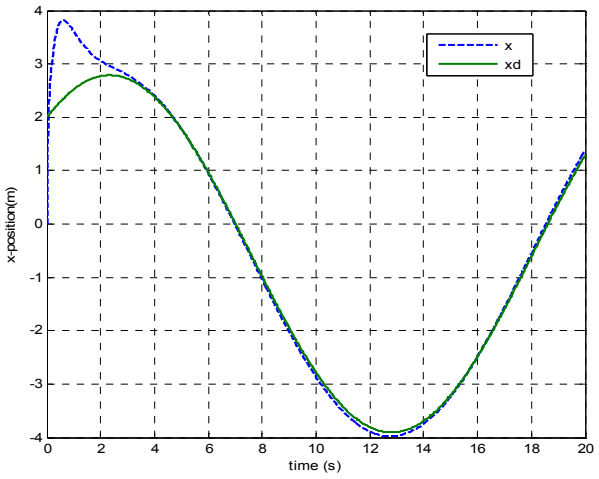

Fig. 3 Tracking trajectory of x-position

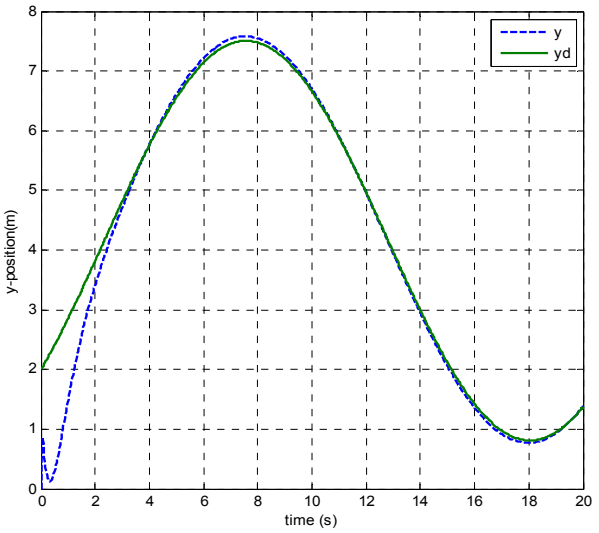

Fig. 4 Tracking trajectory of y-position

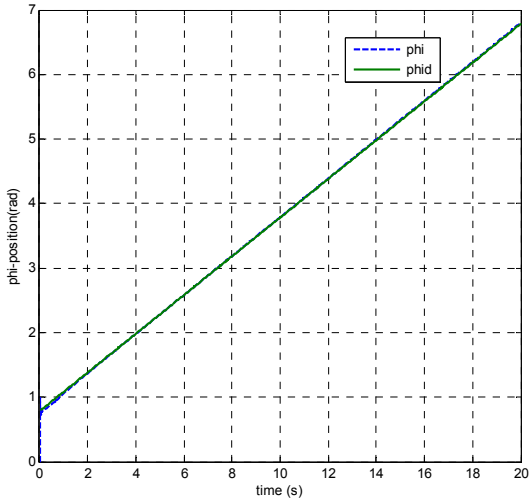

Fig. 5 Tracking trajectory of ϕ-direction

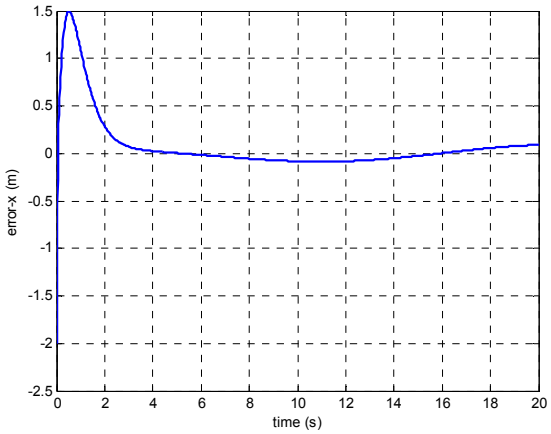

Fig. 6 Tracking error of x-position

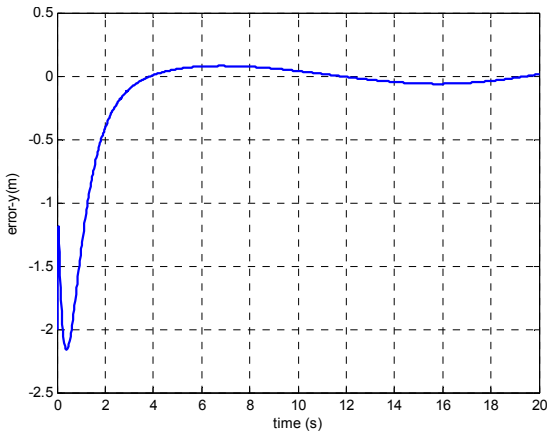

Fig. 7 Tracking error of y-position

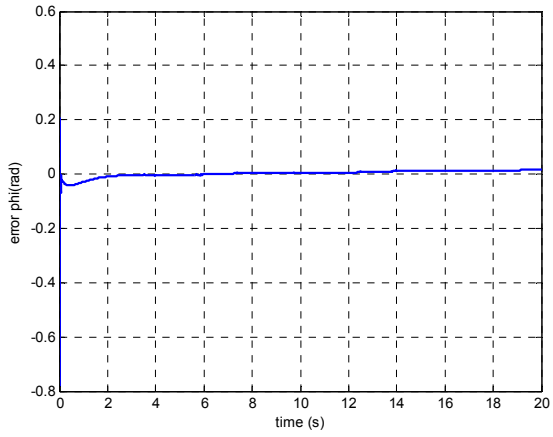

Fig. 8 Tracking error of ϕ-direction

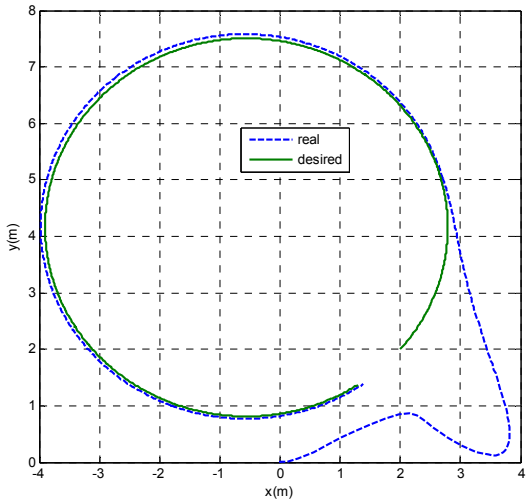

Fig. 9 Tracking trajectory in (X, Y) plan

VI. Conclusion

This paper presents a control strategy based on Lyapunov approach for nonholonomic wheeled mobile robot. First, a kinematic controller was designed to generate a desired velocity for left and right wheels. By considering the kinematic controller of mobile platform, the dynamic controller is developed to ensure a good tracking of the desired trajectory. Simulation results obtained have demonstrated the efficacy of the proposed control method in controlling mobile robots. As a future work, the proposed control strategy will be validated experimentally and will also be applied to other kinds of electro- mechanical systems.

Appendix

A. Proof of Proposition 1

In this section, we attempt to prove the error dynamics given in (13). From the nonholonomic constraint given by (2) and $A_v = [-sin(\phi) \quad cos(\phi) \quad 0]$, we can write:

$$\dot{x}sin\phi - \dot{y}cos\phi = 0 \Rightarrow \dot{x}_d sin\phi_d - \dot{y}_d cos\phi_d = 0 \qquad (24)$$

From the kinematic model given in (7) with d=0, we can write:

$$\begin{bmatrix} cos\phi & 0 \\ sin\phi & 0 \\ 0 & 1 \end{bmatrix}^T \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} cos\phi & 0 \\ sin\phi & 0 \\ 0 & 1 \end{bmatrix}^T \begin{bmatrix} cos\phi & 0 \\ sin\phi & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \qquad (25)$$

Thus

$$\begin{bmatrix} v \\ \omega \end{bmatrix} = \begin{bmatrix} cos\phi & 0 \\ sin\phi & 0 \\ 0 & 1 \end{bmatrix}^T \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\phi} \end{bmatrix} \qquad (26)$$

We can now write:

$$\begin{aligned} v &= \dot{x}\, cos\phi + \dot{y}\, sin\phi \\ v_d &= \dot{x}_d\, cos\phi_d + \dot{y}_d\, sin\phi_d \end{aligned} \qquad (27)$$

Taking the first time derivative of (12), we get:

$$\begin{aligned}\dot{\tilde{x}} &= (\dot{x}_d - \dot{x})cos\phi + (\dot{y}_d - \dot{y})sin\phi - \dot{\phi}(x_d - x)sin\phi \\ &\quad + \dot{\phi}(y_d - y)cos\phi \\ &= \tilde{y}\omega - v + \dot{x}_d cos\phi + \dot{y}_d sin\phi \\ &= \tilde{y}\omega - v + \dot{x}_d cos(\phi_d - \tilde{\phi}) + \dot{y}_d sin(\phi_d - \tilde{\phi}) \\ &= \tilde{y}\omega - v + \dot{x}_d \left(cos(\phi_d)cos(\tilde{\phi}) + sin(\phi_d)sin(\tilde{\phi})\right) \\ &\quad + \dot{y}_d \left(sin(\phi_d)cos(\tilde{\phi}) - cos(\phi_d)sin(\tilde{\phi})\right) \\ &= \tilde{y}\omega - v + \left(\dot{x}_d cos(\phi_d) + \dot{y}_d sin(\phi_d)\right)cos(\tilde{\phi}) \\ &\quad + (\dot{x}_d sin\phi_d - \dot{y}_d cos\phi_d)sin(\tilde{\phi})\end{aligned}$$

Then:

$$\dot{\tilde{x}} = \tilde{y}\omega - v + v_d cos(\tilde{\phi}) \tag{28}$$

$$\begin{aligned}\dot{\tilde{y}} &= -(\dot{x}_d - \dot{x})sin\phi + (\dot{y}_d - \dot{y})cos\phi - \dot{\phi}(x_d - x)cos\phi \\ &\quad - \dot{\phi}(y_d - y)sin\phi \\ &= -\tilde{x}\omega + \dot{x}\, sin\phi - \dot{y}\, cos\phi - \dot{x}_d sin\phi + \dot{y}_d\, cos\phi \\ &= -\tilde{x}\omega - \dot{x}_d sin(\phi_d - \tilde{\phi}) + \dot{y}_d\, cos(\phi_d - \tilde{\phi}) \\ &= -\tilde{x}\omega - \dot{x}_d \left(sin(\phi_d)cos(\tilde{\phi}) - cos(\phi_d)sin(\tilde{\phi})\right) + \\ &\dot{y}_d \left(cos(\phi_d)cos(\tilde{\phi}) + sin(\phi_d)sin(\tilde{\phi})\right) \\ &= -\tilde{x}\omega - \left(\dot{x}_d sin(\phi_d) - \dot{y}_d cos(\phi_d)\right)cos(\tilde{\phi}) \\ &\quad + \left(\dot{x}_d cos(\phi_d) + \dot{y}_d sin(\phi_d)\right)sin(\tilde{\phi})\end{aligned}$$

After simplification, we obtain:

$$\dot{\tilde{y}} = -\tilde{x}\omega + v_d sin(\tilde{\phi}) \tag{29}$$

and the third term is given as follows:

$$\dot{\tilde{\phi}} = \dot{\phi}_d - \dot{\phi} = \omega_d - \omega \tag{30}$$

REFERENCES

[1] C. Samson and K. Ait-Abderrahim, "Feedback control of a nonholonomic wheeled cart in cartesian space," in *Robotics and Automation, 1991. Proceedings., 1991 IEEE International Conference on*, 1991, pp. 1136-1141.

[2] J.-B. Pomet, "Explicit design of time-varying stabilizing control laws for a class of controllable systems without drift," *Systems & control letters,* vol. 18, pp. 147-158, 1992.

[3] R. Murray, G. Walsh, and S. Sastry, "Stabilization and tracking for nonholonomic control systems using time-varying state feedback," *IFAC Nonlinear control systems design,* pp. 109-114, 2014.

[4] A. Bloch and S. Drakunov, "Stabilization of a nonholonomic system via sliding modes," in *Decision and Control, 1994., Proceedings of the 33rd IEEE Conference on*, 1994, pp. 2961-2963.

[5] C. C. De Wit, H. Berghuis, and H. Nijmeijer, "Practical stabilization of nonlinear systems in chained form," in *Decision and Control, 1994., Proceedings of the 33rd IEEE Conference on*, 1994, pp. 3475-3480.

[6] E. Maalouf, M. Saad, and H. Saliah, "A higher level path tracking controller for a four-wheel differentially steered mobile robot," *Robotics and Autonomous Systems,* vol. 54, pp. 23-33, 2006.

[7] J.-M. Yang and J.-H. Kim, "Sliding mode control for trajectory tracking of nonholonomic wheeled mobile robots," *Robotics and Automation, IEEE Transactions on,* vol. 15, pp. 578-587, 1999.

[8] R. Fierro and F. L. Lewis, "Control of a nonholonomic mobile robot using neural networks," *Neural Networks, IEEE Transactions on,* vol. 9, pp. 589-600, 1998.

[9] O. Mohareri, R. Dhaouadi, and A. B. Rad, "Indirect adaptive tracking control of a nonholonomic mobile robot via neural networks," *Neurocomputing,* vol. 88, pp. 54-66, 2012.

[10] B. S. Park, S. J. Yoo, J. B. Park, and Y. H. Choi, "Adaptive neural sliding mode control of nonholonomic wheeled mobile robots with model uncertainty," *Control Systems Technology, IEEE Transactions on,* vol. 17, pp. 207-214, 2009.

[11] J.-C. Ryu and S. K. Agrawal, "Differential flatness-based robust control of a two-wheeled mobile robot in the presence of slip," in *ASME 2008 Dynamic Systems and Control Conference*, 2008, pp. 915-921.

[12] C. P. Tang, "Differential flatness-based kinematic and dynamic control of a differentially driven wheeled mobile robot," in *Robotics and Biomimetics (ROBIO), 2009 IEEE International Conference on*, 2009, pp. 2267-2272.

[13] G. L. Mariottini, G. Pappas, D. Prattichizzo, and K. Daniilidis, "Vision-based localization of leader-follower formations," in *Decision and Control, 2005 and 2005 European Control Conference. CDC-ECC'05. 44th IEEE Conference on*, 2005, pp. 635-640.

[14] R. Fierro and F. L. Lewis, "Control of a nonholonomic mobile robot: backstepping kinematics into dynamics," in *Decision and Control, 1995., Proceedings of the 34th IEEE Conference on*, 1995, pp. 3805-3810.

[15] M. Aicardi, G. Casalino, A. Bicchi, and A. Balestrino, "Closed loop steering of unicycle like vehicles via Lyapunov techniques," *Robotics & Automation Magazine, IEEE,* vol. 2, pp. 27-35, 1995.

[16] R. Carelli and E. O. Freire, "Corridor navigation and wall-following stable control for sonar-based mobile robots," *Robotics and Autonomous Systems,* vol. 45, pp. 235-247, 2003.

[17] L. Huang and L. Tang, "Dynamic Target Tracking Control for a Wheeled Mobile Robots Constrained by Limited Inputs," in *Proceedings of 17th World Congress, IFAC, Seoul, Korea*, 2008, pp. 3087-3091.

[18] T. Fukao, H. Nakagawa, and N. Adachi, "Adaptive tracking control of a nonholonomic mobile robot," *Robotics and Automation, IEEE Transactions on,* vol. 16, pp. 609-615, 2000.

[19] Y. Kanayama, Y. Kimura, F. Miyazaki, and T. Noguchi, "A stable tracking control method for an autonomous mobile robot," in *Robotics and Automation, 1990. Proceedings., 1990 IEEE International Conference on*, 1990, pp. 384-389.

[20] M. W. Spong, S. Hutchinson, and M. Vidyasagar, *Robot modeling and control* vol. 3: Wiley New York, 2006.