

# Robust Control of a Parallel 3-RRR Robotic Manipulator via $\mu$ -Synthesis Method

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**Abstract**—Control of some mechanisms is hard because of their complex dynamic equations. If part of the complexity is resulting from uncertainties, an efficient way for solving that is robust control. By this way, the control procedure could be simple and fast and finally, a simple controller can be designed. One kind of these mechanisms is 3-RRR which is a parallel mechanism and has three revolute joints. This paper aims to robust control a 3-RRR planner mechanism and it presents that this could be used for other mechanisms. So, a significant problem in mechanisms control could be solved. The relevant diagrams are drawn and they show the correctness of control process.

**Keywords**—3-RRR, dynamic equations, mechanisms control, structural uncertainty.

## I. INTRODUCTION

A robot is said to be a serial robot or open-loop manipulator if its kinematic structure takes the form of an open-loop chain, a parallel manipulator if it is made up of a closed-loop chain, and a hybrid manipulator if it consists of both open- and closed-loop chains [1]. Parallel robots work at high velocities due to their closed kinematic loops (chains) with relatively very low exertion [2]-[4]. Therefore, an output speed is important for parallel manipulators to be used in parallel kinematic robots and machines. In comparison with the serial complements, some of the prominent possible benefits of parallel designs are more stable structure, higher accuracy, greater stiffness with less weight, dynamic stability, higher load bearing capacity and appropriate positional configuration of actuators. Furthermore, there are some problems in these useful manipulators; for example, several complicated singular configurations and restricted workspace. Some works have been done on such mechanisms in the static areas [5] and dynamic area [6].

In order to enhance the trajectory tracking performance of parallel mechanisms, considerable researches have been done for achieving a better controller [7]. Robust and adaptive control systems were designed to decrease the tracking errors.  $\mu$ -synthesis is an efficient method for robust control against uncertainties and other changes.  $\mu$ -synthesis can supply appropriate robustness, tracking performances to overcome various uncertainties and external disturbances, and as a defensive technique from system parameter variations such as

it used in aircrafts [8]. Before this, it has been used for serial and parallel robots different uncertainties and disturbances [9], [10], [11]. When the uncertainties are large, especially in parallel robots, larger gain values should be incorporated and the amount of gain increases.

The main purpose of this research is to present an efficient control system which has robustness against uncertainties in the model and has good performance for different conditions and its inverse kinematics. Parallel robotic end effectors have attracted a lot of considerations in developing control systems for usual Multi-input Multi-output (MIMO) systems [12]. The review of literature on control of robots and end effectors discloses that a lot of mechanisms are restricted to positioning capabilities. Therefore, a parallel manipulator, to supply planar movements with essential tools for a trajectory planning, is crucial and vital.

In the present research, a 3-DOF planar parallel robot has been considered, including rigid bodies whose kinematics had been depicted by means of the Newton method. A control system has been designed to increase the performance from the aspect of positioning and trajectory tracking. Consequently, it is expected that the suggested plan for the mechanism will be robust enough to achieve trajectory tracking tasks other than positioning tasks with required correctness. The research mainly relates to the design and investigations of the robust control system for a typical 3-RRR parallel robot.

## II. OVERALL ROBUST CONTROL STEPS

In robust control, the first step is extracting dynamic equations. After that, the state matrices should be found and transfer function could be calculated by the help of them. One of the most popular ways which helps to compute weighting functions is drawing block diagrams. Furthermore, to cover bad influences of uncertainties, uncertainty families are defined. Uncertainty families help to complete the system block diagrams and ratio of input to output of delta function could be calculated. Finally, an appropriate controller could be designed by equaling it smaller than one.

## III. STATE SPACE EQUATIONS

First of all, dynamic equations should be determined. It could be done by Newton's law, so its overall dynamic equations would be written as [6]:

$$\begin{bmatrix} m_p & 0 & 0 \\ 0 & m_p & 0 \\ 0 & 0 & I_p \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{\theta} \end{Bmatrix} = \begin{Bmatrix} \sum F_x \\ \sum F_y \\ \sum M \end{Bmatrix} = U \quad (1)$$

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where  $z$ : State function

$$Z = \{x \ y \ \theta \ \dot{x} \ \dot{y} \ \dot{\theta}\}^T \tag{2}$$

$$\dot{Z}_{6 \times 1} = A_{6 \times 6} Z_{6 \times 1} + B_{6 \times 3} U_{3 \times 1} \tag{3}$$

With the help of above form, state matrices can be calculated as:

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \tag{4}$$

$$D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{5}$$

$$B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{6}$$

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -K & 0 & 0 & -C & 0 & 0 \\ 0 & -K & 0 & 0 & -C & 0 \\ 0 & 0 & -K & 0 & 0 & -C \end{bmatrix} \tag{7}$$

The system parameters and uncertainties values are [13], [14]:

- Stiffness coefficient:  $K = 0.15 \text{ (kgm}^2\text{s}^{-2}\text{)}$
- Damping coefficient:  $C = 0.1 \pm 15\% \text{ (kgm}^2\text{s}^{-1}\text{)}$
- Mass:  $m_p = 2.245 \text{ kg}$
- First moment of area:  $I_p = 0.006 \pm 15\% \text{ m}^3$

After finding the state space matrices, transfer function could be computed. The first, second and third element of transfer function is as:

$$G_{11} = G_{22} = \frac{4000}{(4000 \cdot s^2 + 1796 \cdot s + 1347)} \tag{8}$$

$$G_{33} = \frac{10000}{(10000 \cdot s^2 + 12 \cdot s + 9)} \tag{9}$$

The other elements are zero. In this system, the input forces are the whole inputs and the outputs are every direction displacement. Cad model of a 3-RRR is shown in Fig. 1.

There is an uncertainty in inertia and damping matrices. So, it is necessary to define an uncertainty family for each of them. The inverse of inertia function family and Damping family are:

$$\frac{1}{\tilde{I}} = \frac{1}{I + \Delta\omega_1 I} = \frac{1/I}{1 + \Delta\omega_1} \tag{10}$$

$$\tilde{C} = \frac{c}{1 + \Delta\omega_2 c} \tag{11}$$

Regards to these families, the system diagram becomes as like as Fig. 2. To find the robust stability, the  $x'_1/x'_2$  and  $x_1/x_2$  should be calculated.

$$\frac{x_1}{x_2} = -\omega_1 \frac{1}{1 + \frac{(c+K/I)^{-1}}{s}} \Rightarrow \left| \frac{-\omega_1 s^2}{s^2 + csI^{-1} + kI^{-1}} \right| < 1 \tag{12}$$

$$\frac{x'_1}{x'_2} = -\omega_2 \frac{c}{1 + c \frac{(\frac{K}{s})I^{-1}}{1 + (\frac{K}{s})I^{-1}}} \Rightarrow \left| \frac{-\omega_2 s^2 c - \omega_2 c k I^{-1}}{s^2 + csI^{-1} + kI^{-1}} \right| < 1 \tag{13}$$

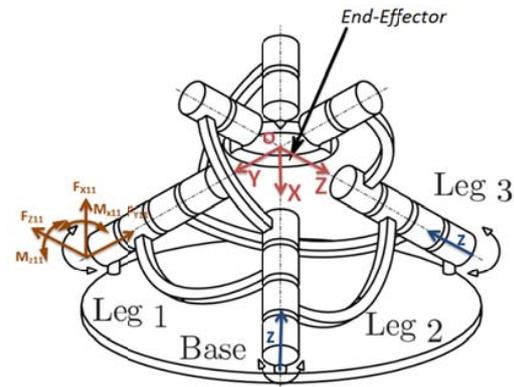


Fig. 1 CAD model of 3-RRR [15]

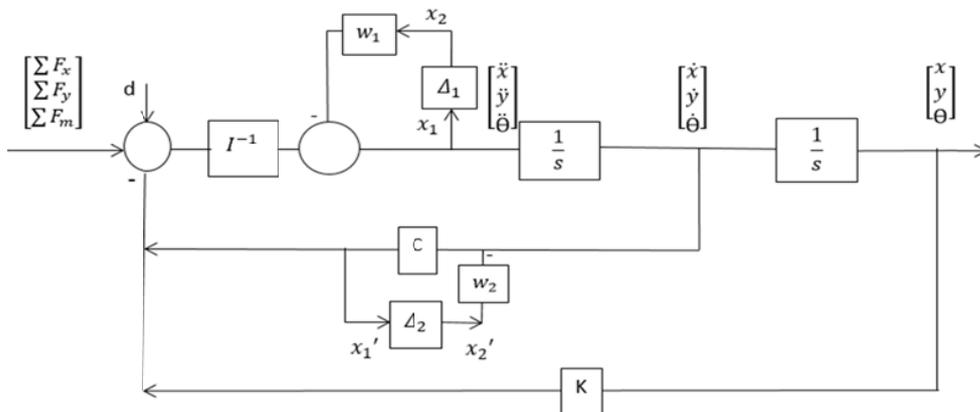


Fig. 2 Block diagram of system regards to weighting functions

To reach the stability, the system should be stable for each element of transfer function. This system is stable and its stability is confirmed by several ways as like as Routh–Hurwitz stability criterion or the step response diagram of the system. Finding two weighting functions and a controller in which  $\mu$ -value is smaller than one is the next step. In nominal performance,  $W_1S$  should be smaller than one, so  $W_1$  is the function which is less than  $1/S(s)$ , and can be calculated by the  $x_1/x_2$  ratio. In robust stability,  $W_2T$  should be smaller than one, so  $W_2$  can be found in the same way by the  $x_1/x_2$  ratio. Final estimated weighting functions are calculated as:

$$w_1 = \frac{(0.3s+0.06)}{0.9s+10} \quad (14)$$

$$w_2 = \frac{(0.9s+0.6)}{s+1} \quad (15)$$

Now  $W_1$ ,  $W_2$  and transfer functions are calculated. So, the next step is finding the controller with the  $\mu$ -value smaller than one. In the previous section, the first and second elements of principal diameter are equal, so the weighting functions and controller are the same. But the third element is different, so its controller should be different. The consequence diagrams of some sufficient iterations in MATLAB are presented in Figs. 3 and 4.

As it could be seen in Figs. 3 and 4 after second graph, maximum values of  $\mu$  are smaller than one. So, these weighting functions could provide robust stability of system and control it. The outcome controller, which provides robust stability is  $K$ .

$$K = \begin{bmatrix} K_1 & 0 & 0 \\ 0 & K_2 & 0 \\ 0 & 0 & K_3 \end{bmatrix} \quad (16)$$

in which:

$$K_1 = K_2 = [(13.146(s+11.11)(s^2+0.5337s+0.2614)(s^2+0.45s+0.3383)(s^2+6.124s+90.03)] / [(s+4.135)(s+1)(s^2+0.5386s+0.2639)(s^2+4.578s+86.34)(s^2+10.15s+124.5)] \quad (10)$$

$$K_3 = [5.2577(s+11.11)(s+0.02254)(s+0.0121)(s^2+0.001038s+0.0008995)(s^2+0.006778s+0.001038)(s^2+15.17s+77.38)] / [(s+2.565)(s+1)(s+0.02246)(s+0.01222)(s^2+0.006745s+0.001036)(s^2+15.06s+77.15)(s^2+16.49s+91.5)] \quad (11)$$

#### IV. RESULTS AND DISCUSSION

First of all, it should be shown that this controller could make the step response of the system better. To confirm that, the initial step responses of the system for each element of the matrix should be extracted, then it should be compared with their response with the help of the controller. The first and second elements are the same, so their responses are alike but the third element response is different. The first and second

responses before and after adding controller could be seen in Figs. 5 and 6. Such systems need to reduce unwanted vibrations and make the system faster. As it could be seen, the robust controller eliminates all of undesirable vibrations and it decreases the settling time, which makes the system faster. It could be seen in the diagrams for the third element, too (Figs. 7 and 8).

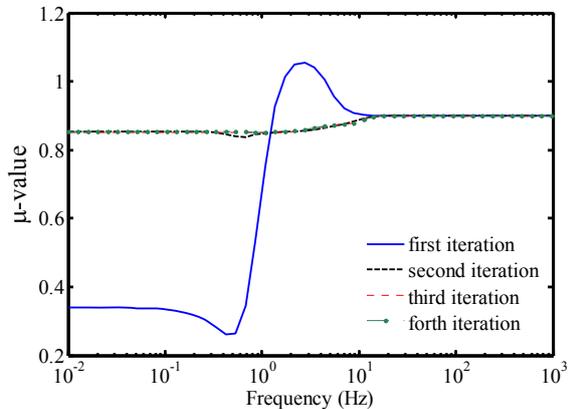


Fig. 3 Maximum  $\mu$ -value for iteration of the first and second elements of principal diameter

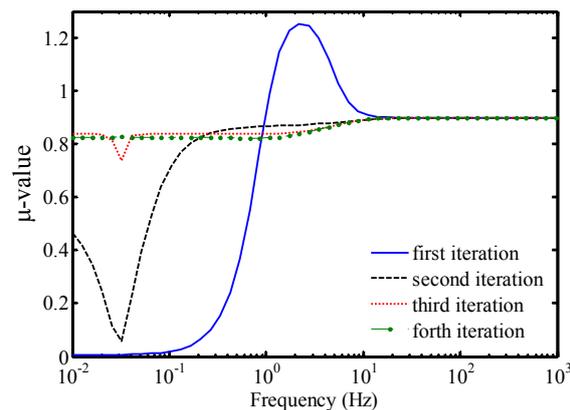


Fig. 4 Maximum  $\mu$ -value for iteration of the Third element of principal diameter

#### V. CONCLUSION

In this paper, a robust controller for 3-RRR mechanism is designed. The dynamic of this system has several complexities, which make the dynamic analysis so difficult. Therefore, its control is challenging. One of the innovations in this paper is the idea of putting any lateral items as uncertainty and designing a robust controller for it. There are several steps in the presented design described with details. The obtained results are satisfying both robust stability and performance. As it can be seen in the diagrams, the designed controller eliminates all the unwanted vibrations in the system, and it decreases the settling time, therefore, makes the system faster and more accurate.

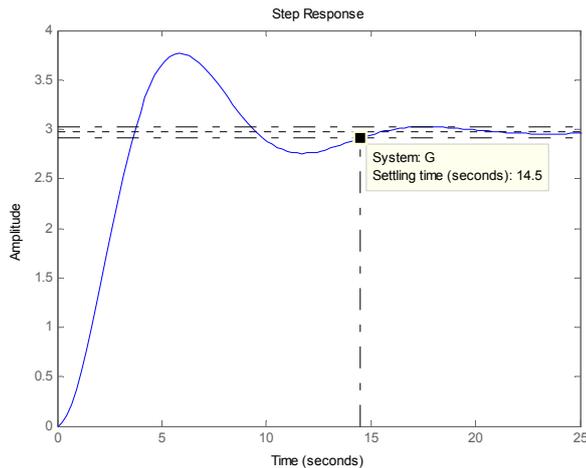


Fig. 5 First and Second element initial step response

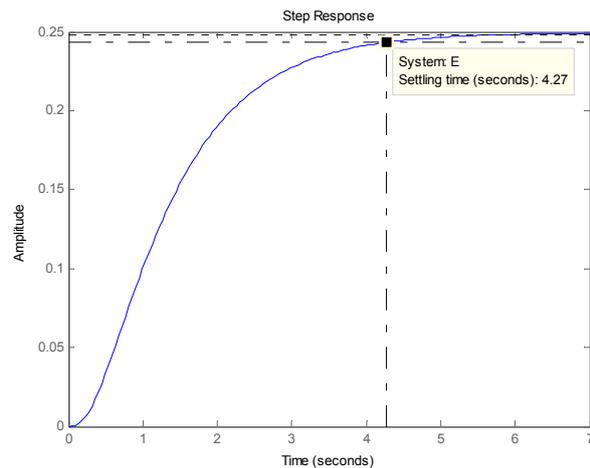


Fig. 8 Third element step response with the help of controller

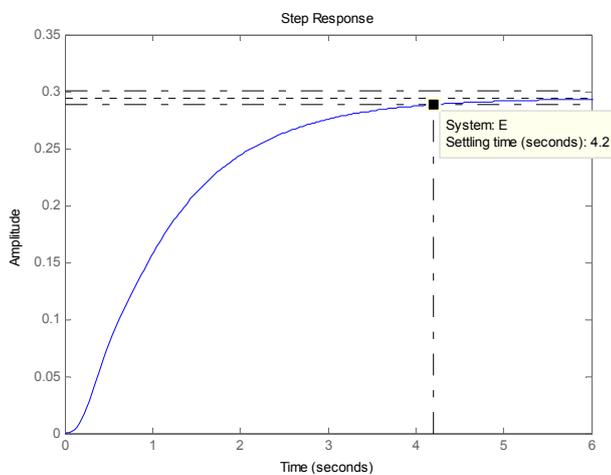


Fig. 6 First and Second element step response with the help of controller

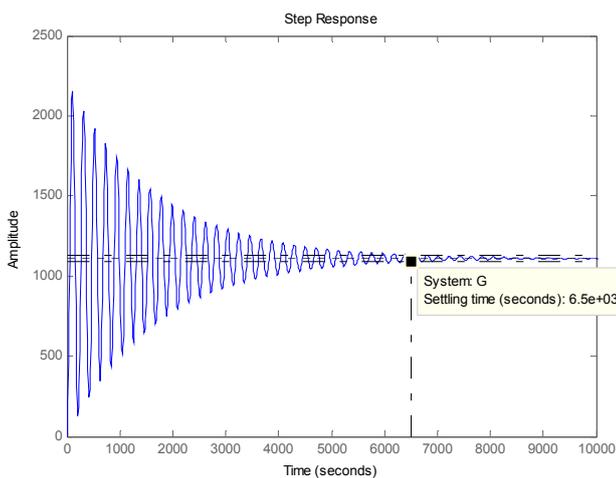


Fig. 7 Third element initial step response

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