

# Numerical Simulation of Punching Shear of Flat Plates with Low Reinforcement

Fatema-Tuz-Zahura, Raquib Ahsan

**Abstract**—Punching shear failure is usually the governing failure mode of flat plate structures. Punching failure is brittle in nature which induces more vulnerability to this type of structure. In the present study, a 3D finite element model of a flat plate with low reinforcement ratio and without any transverse reinforcement has been developed. Punching shear stress and the deflection data were obtained on the surface of the flat plate as well as through the thickness of the model from numerical simulations. The obtained data were compared with the experimental results. Variation of punching stress with respect to deflection as obtained from numerical results is found to be in good agreement with the experimental results; the range of variation of punching stress is within 5%. The numerical simulation shows an early and gradual onset of nonlinearity, whereas the same is late and abrupt as observed in the experimental results. The range of variation of punching stress for different slab thicknesses between experimental and numerical results is less than 15%. The developed numerical model is useful to complement available punching test series performed in the past. The results obtained from the numerical model will be helpful for designing retrofitting schemes of flat plates.

**Keywords**—Flat plate, finite element model, punching shear, reinforcement ratio.

## I. INTRODUCTION

THE greatest disadvantage of flat plate systems is the risk of brittle punching failure at the slab-column connection due to transfer of shear and unbalanced moment. Vertical loads acting on the floor system and moments transferred from the columns may create excessive shear stresses around the slab-column connection. Unbalanced moments naturally occur at corner and edge slab-column connections. They may also occur at interior connections with unequal vertical loads on adjacent spans, or at any connection due to combined vertical and lateral forces as a result of wind effects or earthquake excitations.

In 1960, Kinnunen and Nylander [1] defined in their model the punching strength as a function of the slab deformation. This approach was later adopted by other researchers and further developed. Similar to the model of Kinnunen and Nylander [1], Muttoni [2] developed the Critical Shear Crack Theory (CSCT), which served as a basis for the fib Model Code 2010 and describes the punching strength as a function of the slab rotation whereby the slab response can be calculated with a Quadrilinear moment curvature relationship approach. However, it has to be noted that most of the models

using such an approach are based on the theory of an axisymmetric slab. Nevertheless, most punching tests were performed with specimens that were not axisymmetric and thus the validation of the model cannot directly be performed. In these cases several adjustments have to be made such as the consideration of an orthogonal flexural reinforcement layout, the shape of the column and the shape of the slab specimen.

Lips and Muttoni [3] examined influence of punching shear reinforcement on the flexural response of flat slabs. They performed an investigation on the flexural response of 6 full-scale flat slab specimens with the aim to investigate the punching strength and the rotation capacity of flat slabs with and without shear reinforcement. Fernández and Muttoni [4] performed analysis on applications of Critical Shear Crack Theory to punching of reinforced concrete slabs with transverse reinforcement. Mirzaei and Sasani [5]-[7] performed extensive experimental and analytical study on post-punching behavior of reinforced concrete slabs.

Fariborz [8] proposed a formula to calculate the punching shear strength of flat plates with good accuracy for a wide range of slab thicknesses, tensile reinforcement ratios, and concrete compressive strengths. In this method, it is assumed that punching shear failure occurs due to the crushing of the critical concrete strut adjacent to the column. A large number of experimental results of slab test specimen, reported in the literature were gathered to evaluate the accuracy of the proposed formula, as well as the punching shear formulae in some of the internationally recognized standards such as AS 3600-2009 [9], ACI 318-05 [10], CSA A23.3-04 [11], DIN 1045-1:2001 [12], Eurocode2 [13], and NZS 3101:2006 [14].

Punching shear is usually the governing failure mode for flat slabs supported on columns, with or without capitals. This subject has been thoroughly investigated in the past by various researchers dealing with the theoretical and/or experimental aspects of the phenomenon. Current ACI design code provisions for checking punching shear follow a format similar to that of ACI 318-08 [15], which relates the punching shear strength to the effective flexural depth of the slab  $d$  and the control perimeter  $b_0$  of a critical section (at a distance  $d/2$  from the face of the column for ACI 318-08 [15]). Guandalini et al. [16] conducted experimental study on the punching behavior of flat plate with low reinforcement ratio. There is a lack of study on the effects of clear cover, reinforcement ratio, column reinforcement, thickness of the slab, column size, reinforcement size, integrity reinforcement, material property, and nonlinearity on punching shear behavior. A 3D finite element (FE) model has been developed in the present study to explore effects of the parameters mentioned above. The model

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is first to be validated before it can be used in designing retrofitting schemes of flat plate column connection.

## II. OBJECTIVE AND SCOPE

Current methods for the design of punching shear reinforcement rely on empirical formulation to estimate the contributions of concrete and shear reinforcement at failure. One aim of this study is to develop a numerical model for the prediction of the punching capacity and post punching behavior of flat plate. A parametric study is conducted to develop better understanding of retrofitting of flat plate column connection.

## III. MODEL DEVELOPMENT

Using ABAQUS, a 3D FE model is developed in this study to investigate the punching behavior of a reinforced flat plate. The main focus of interest of the study is the slab column connection. Due to symmetry and simplicity an interior column along with half of a slab panel in all direction is modeled in this study. The geometry of a model is described by elements and their nodes. Slab is modeled by using a three dimensional eight noded continuum solid element and the reinforcement is modeled using truss element (Fig. 1).

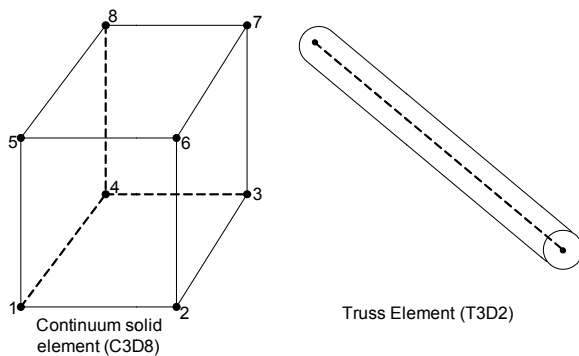


Fig. 1 Elements used in model development [17]

The final shape of the model is shown in Fig. 2. The length of the panel and the dimensions of the column varied according to the purpose. Typical layout of flexure reinforcement is shown in Fig. 3.

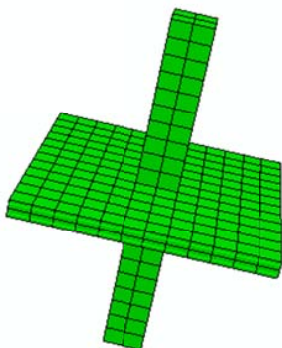


Fig. 2 Finite element model of flat plate

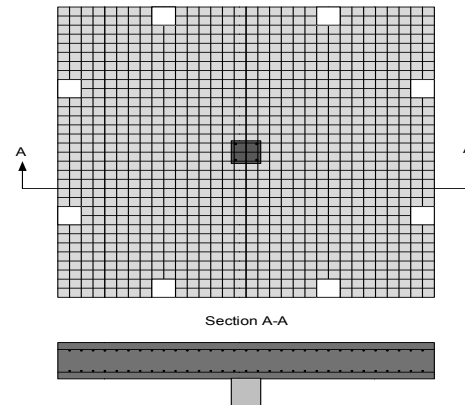


Fig. 3 Typical Layout of flexure reinforcement (verification model)

## IV. MATERIAL PROPERTY

Steel and concrete are used in the FE model for numerical investigation. Inelastic properties are considered for steel. The damage plasticity model of concrete is adopted for the numerical investigation.

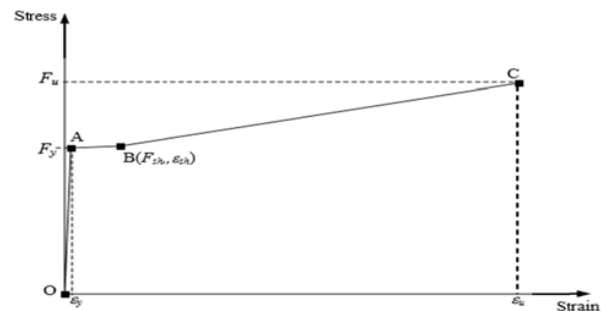


Fig. 4 Stress-Strain Curve for Steel used in the Numerical Analysis

The steel material properties are simulated with a trilinear elasto-plastic model that includes strain hardening. A tri-linear curve shown in Fig. 4, where point A is the stress-strain yield point, point B refers to the onset of strain hardening and point C is the ultimate stress point. The subscripts y, sh and u in the table represent the yield, onset of strain hardening and ultimate strain respectively.

The value of the Poisson's ratio for steel used in the numerical analysis is 0.3.

The damage plasticity model in ABAQUS was used to simulate the concrete material behavior. Complete stress-strain relationships for concrete under uniaxial compression and tension are necessary to predict the structural response of punching of flat plate by nonlinear finite element analysis. There have been many attempts to developed analytical formulations to represent stress-strain relationships for normal and high strength concrete in uniaxial compression. Among these formulations the models proposed by Carreira and Chu [18] has been used to generate the compression and tension stress-strain curve for concrete material. Carreira and Chu [19] proposed the following equation to calculate the compressive

stress-strain diagram of concrete.

$$f_c = f_{cu} \left[ \frac{\beta \left( \frac{\epsilon}{\epsilon_{cu}} \right)}{\beta - 1 + \left( \frac{\epsilon}{\epsilon_{cu}} \right) \beta} \right] \quad (1)$$

where  $\beta$  is a material parameter that depends on the shape of the stress-strain diagram. A simplified expression for  $\beta$  as a function of  $f'_c$  is used in this research:

$$\beta = \left[ \frac{f'_c}{32.4} \right]^3 + 1.55 \quad (f'_c \text{ in MPa}) \quad (2)$$

The uniaxial tensile strength of concrete,  $f'_t$  was set at 10% of the  $f'_c$  concrete strength. To generate the tension stress-strain diagrams the following equation is proposed by Carreira and Chu [18] which has been implemented here.

$$f_c = f'_t \left[ \frac{\beta \left( \frac{\epsilon}{\epsilon'_t} \right)}{\beta - 1 + \left( \frac{\epsilon}{\epsilon'_t} \right) \beta} \right] \quad (3)$$

The compressive and tensile stress-strain (developed by using Carreira and Chu equations) used in the finite element simulation are shown in Figs. 5 and 6.

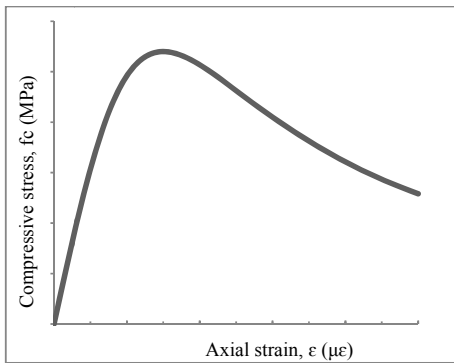


Fig. 5 Uniaxial Compressive Stresses for 27 MPa Concrete

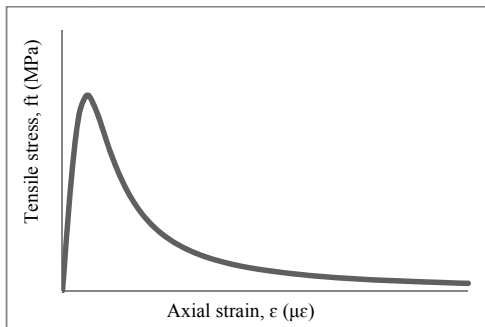


Fig. 6 Uniaxial tensile Stresses for 27 MPa Concrete

V. LOADING CONDITION AND SOLUTION STRATEGY

The load is applied on eight locations on the top surface of the flat plate. The sides of the flat plate and the mid height of the column are fixed in all directions (Fig. 7).

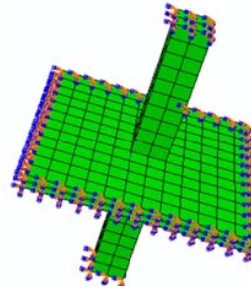


Fig. 7 Boundary conditions of FE model

Riks solution strategy has been implemented to trace stable post peak behavior of the composite column upto failure. This method is generally used to predict unstable geometrically nonlinear collapse of a structure.

VI. VERIFICATION WITH EXPERIMENTAL DATA

In order to validate the proposed model the results obtained from experimental data are compared with the numerical data. Guandalini et al. [16] performed several experiments on punching tests of slabs with low reinforcement ratios. For verification purpose numerical model similar to full size model PG1 of Guandalini et al. [16] has been simulated by using ABAQUS (Figs. 8 and 9).

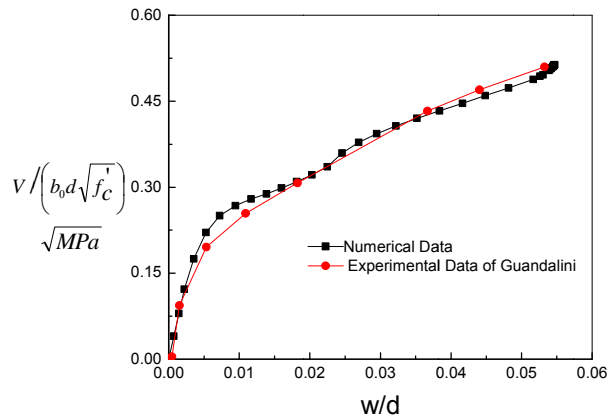


Fig. 8 Normalized load-deflection curve (deflection w was measured between center of column and reaction points at perimeter)

VII. CONCLUSION

Verification of the numerical data with the experimental data shows that the model development is in good agreement with the experimental setup. So this model can be successfully used for predicting the behavior of punching of flat plate.

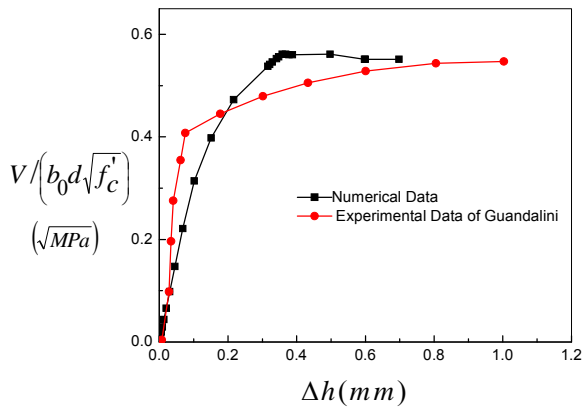


Fig. 9 Change in thickness of slab

## REFERENCES

- [1] Kinnunen, S. and Nylander, H., Punching of Concrete Slabs Without Shear Reinforcement, Transactions of the Royal Institute of Technology, No. 158, 112 pp., Stockholm, Sweden, 1960.
- [2] Muttoni, A., Punching shear strength of reinforced concrete slabs without transverse reinforcement, ACI Structural Journal, Vol. 105, No. 4, pp. 440-450, USA, 2008.
- [3] Lips S. and Muttoni A., Influence of punching shear reinforcement on the flexural response of flat slabs, fib Symposium Tel-Aviv 2013, Israel, 2013, 4.
- [4] Fernández, R. M. and Muttoni, A., Applications of the critical shear crack theory to punching of R/C slabs with transverse reinforcement, ACI Structural Journal, Vol. 106 No. 4, pp. 485-494, USA, 2009.
- [5] Mirzaei, Y., Post punching behavior of reinforced slab column connection, 7th fib PhD Symposium in Stuttgart, 2008.
- [6] Mirzaei, Y. and Sasani, M., Punching shear failure in progressive collapse analysis, Proc., Structures Congress, 2941-2950.
- [7] Mirzaei, Y. and Sasani, M., Progressive collapse resistance of flat slab-Modeling post punching behavior, Computers and Concrete 12 (3), 351-375.
- [8] Fariborz M, Concentric Punching Shear Strength of Reinforced Concrete Flat Plates, Master of Engineering thesis, Swinburne University of Technology, Melbourne, Australia, June 2012.
- [9] AS 3600 Supplement 1:2014 Concrete structures—Commentary (Supplement to AS 3600—2009).
- [10] ACI, Building Code Requirements for Structural Concrete, ACI American Concrete Institute, ACI 318-05, 430 . USA, 2005
- [11] CSA STANDARD A23.3-04, Canadian Standard Association, 232, 2004.
- [12] DIN 1045-1 Tragwerke aus Beton und Stahlbeton , DIN 1045-1, Deutsches Institut für Normung, 2nd Edition, 148 p., Berlin, Germany, German, 2005.
- [13] Eurocode 2: Design of concrete structures - Part 1-1: General rules and rules for buildings, European Committee for Standardization (CEN), Brussels, 2004.
- [14] NZS 3101-1 (2006) (English): Concrete structures standard - The design of concrete structures.
- [15] ACI Committee 318, "Building Code Requirements for Structural Concrete (ACI 318-08) and Commentary," American Concrete Institute, Farmington Hills, MI, 2005, 465 pp.
- [16] Guandalini S., Burdet O., Muttoni A., Punching tests of slabs with low reinforcement ratios, ACI Structural Journal, V. 106, N°1, USA, 2009, pp. 87-95.
- [17] Abaqus Analysis User's Manual, Article 23.1.1 Element library, Figure 23.1.1-1.
- [18] Carreira, D.J., Chu K.H.(1984)."Stress-Strain relationship for plain concrete in compression". ACI Journal, Technical Paper, Title no. 82-72.
- [19] Carreira, D.J., Chu, K.H. (1984)."Stress-Strain relationship for Reinforced concrete in Tension". ACI Journal, Technical Paper, Title no. 83-3.