Performance Evaluation of One and Two Dimensional Prime Codes for Optical Code Division Multiple Access Systems

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***Abstract*—**In this paper, we have analyzed and compared the performance of various coding schemes. The basic ID prime sequence codes are unique in only dimension, i.e. time slots, whereas 2D coding techniques are not unique by their time slots but with their wavelengths also. In this research, we have evaluated and compared the performance of 1D and 2D coding techniques constructed using prime sequence coding pattern for Optical Code Division Multiple Access (OCDMA) system on a single platform. Analysis shows that 2D prime code supports lesser number of active users than 1D codes, but they are having large code family and are the most secure codes compared to other codes. The performance of all these codes is analyzed on basis of number of active users supported at a Bit Error Rate (BER) of 10-9.

***Keywords*—**CDMA, OCDMA, BER, OOC, PC, EPC, MPC, 2-D PC/PC, λc, λa.

I. Introduction

CODE Division Multiple Access (CDMA) communication system allows several users to send information over a single channel simultaneously. CDMA uses spread spectrum technique to multiplex users and each user is allotted distinct code. When coding is done in optical domain, then it is known as OCDMA. OCDMA has the advantage of using optical processing to perform certain network applications, like addressing and routing without resorting to complicated multiplexers or de-multiplexers. Thus, OCDMA has its utility in high speed LAN networks [1]-[4]. Prime codes are linear congruence codes which use congruence operations in finite field. They are a sequence of binary (0, 1) which has good auto and cross-correlation with help of Galois Field. The various coding parameters are code weight w, code length n, cross-correlation λc, and auto-correlation λa. Code weight and cross-correlation are two important parameters in assessing the performance of a code set. A small prime number denotes low code weight and hence less number of users. Larger code cardinality can be achieved by relaxing the cross correlation function λc at the expense of code performance.

From any coding technique, it is expected to obtain minimum BER, smaller weight of the code with minimum length of the sequence and it can support more number of the users at a time. In this paper we have compared the performance of various codes for the above mentioned coding parameters. This research would develop an understanding in terms of choosing amongst the various prime sequence coding techniques.

We begin with developing 1-D Extended Prime Code (EPC) in MATLAB and then proceed to Modified Prime Code (MPC) which is an improved version. 2-D prime codes (2-D PC) offer higher number of active users for the same BER. Lastly, we analyse multilevel PCs which offer a trade-off between code cardinality and cross-correlation and hence jeopardizing BER. In Section IV, the performance of these various codes are analyzed and compared.

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II. OCDMA Coding Techniques

Looking at the present scenario, where very high information transfer rate is required, it has been seen that there is a need of much larger size code family. In order to cope up with the increasing demand of higher information transfer rate and larger bandwidth, much work is going on towards increasing the family size of codes either by changing the length of sequence or weight of the code. But, as the weight of the code increases, more power is required for transmitting the data.

Kwong worked on 2n prime–sequence codes [1]. In this architecture, each datum bit 1 is encoded into a wave form (i.e., a binary code sequence) which represents the destination address of that bit, but data bits 0's are not encoded. Each bit period T is subdivided into small units, called chips, in order to accommodate all the elements. (i.e., 1's and 0's) of the binary code sequence. This construction results in the 2n prime sequence codes with fewer code words than the original prime-sequence codes because of the delay-distribution constraint. Though this architecture improves the performance of the OCDMA system, it reduces the cardinality [1]. Zhang and Sharma worked on extended prime sequence codes and designed a variable length prime sequence [2]. But, the limitation associated with these PCs is that their code weight w is always fixed to the number of code words (i.e. coder size) and must be a prime number P. To accommodate more users in an OCDMA system, a larger P is required, so is the code weight w. But, as code weight w is increased, there is a problem of more power consumption [2]-[4]. In 1-D PCs, the main shortcoming is the poor cross correlation between the active users in the network. 2-D PCs [4], [5] and multilevel codes aim to remove this shortcoming by relaxing λc.

III. Coding Algorithms

In OCDMA system, Optical Orthogonal Codes (OOC) and Prime Codes (PC) are the recent techniques used. In this section, we analyzed PC. At sender, end user data rate is XORed with pseudo-random sequence generated through these codes and sent over a common channel while at the receiver end reverse process is done to receive the original data. Light is transmitted at every '1s' with '0s' depicting no light.

A. 1-D PC

This type of code uses a technique which is spreading in time domain. Different users utilize different time slots for sending the desired information [6], [7]. PC generated is shown in Table II.

1-D PCs are developed by the following algorithm:

Step 1: Choose a prime number *p*.

Step 1-1: With this *p* and based on Galois field GF(*p*) construct a prime sequence, using (1).

$$A_i = \left(a_{i,0}, a_{i,1}, a_{i,2} \ldots\ldots\ldots\ldots\ldots .. a_{i,(p-1)}\right) \text{ where } \; i = 0,1,2,..p \qquad (1)$$

where element $A_{i,j} = \{i.j\}(\text{mod } p)$.

The prime sequence generated is shown in Table I.

Step 2: Obtain the following map for the code using (2)

$$D_{i,k} = \begin{cases} 1 & \text{if } k = a_{i,j} + j.p \text{ for } j = 0,1,2 \ldots\ldots p-1 \\ 0 & \text{elsewhere} \end{cases} \qquad (2)$$

Step 3: By using (1) and (2), we can obtain the following PC

$$D_i = \left(d_{i,0}, d_{i,1}, d_{i,2} \ldots .. d_{i,(n-1)}\right) \text{where } i = 0,1,2 \ldots (p-1) \qquad (3)$$

TABLE I
PRIME SEQUENCE AI, K

GF(5)	0	1	2	3	4
A0	0	0	0	0	0
A1	0	1	2	3	4
A2	0	2	4	1	3
A3	0	3	1	4	2
A4	0	4	3	2	1

TABLE II
PRIME CODE DI, K

Prime Sequence Ai	Extended Prime Code Di				
A0= (00000)	10000 0000	10000 0000	10000 0000	10000 0000	10000 0000
A1= (01234)	10000 0000	01000 0000	00100 0000	00010 0000	00001 0000
A2= (02413)	10000 0000	00100 0000	00001 0000	01000 0000	00010 0000
A3= (03142)	10000 0000	00010 0000	01000 0000	00001 0000	00100 0000
A4= (04321)	10000 0000	00001 0000	00010 0000	00100 0000	01000 0000

PC generated is shown in Table II. The code length and code weight of PC are n = p2 and w = p, respectively. Maximum autocorrelation side lobe and maximum cross-correlation function are λa=p-1 and λc= 2 respectively. The theoretical upper bound of cardinality (n, w, λa, λc) is as:

$$\phi(p^2, p, p-1, 2) = p \qquad (4)$$

Their main advantage is that they are very easy to construct and are the basic building block for constructing further codes.

B. Extended Prime Codes (EPC)

EPC is an extended version of basic 1-D PC. Here, in this EPC, the cross correlation is reduced but at the expense of larger code length. The construction algorithm is as follows:

Step 1: Construct 1-D PCs as shown in Table II.

Step 2: (p-1) "0s" are patched to each subsequence of each PC word.

Alternately EPCs can be obtained by using map of (5) after obtaining prime sequence from Table I.

$$D_{i,k} = \begin{cases} 1 & \text{if } k = a_{i,j} + j.(2p-1) \text{for } j = 0,1,2 \ldots\ldots p-1 \\ 0 & \text{elsewhere} \end{cases} \qquad (5)$$

The EPCs thus generated are shown in Table III.

TABLE III
EPC DI,K

Prime Sequence Ai	Prime code Di k=1 to 25				
D0=(00000)	10000	10000	10000	10000	10000
D1=(01234)	10000	01000	00100	00010	00001
D2=(02413)	10000	00100	00001	01000	00010
D3=(03142)	10000	00010	01000	00001	00100
D4=(04321)	10000	00001	00010	00100	01000

The code length and code weight of PC are n = p(2p-1) and w = p respectively. Maximum autocorrelation side lobe and maximum cross-correlation function are λa=p-1 and λc=, respectively. The theoretical upper bound of cardinality (n, w, λa, λc) is as:

$$\phi(p(2p-1), p, p-1, 1) = p \qquad (6)$$

EPCs have larger code length as compared to basic PC but cross-correlation property is considerably improved in this. The probability of error for M number of active users is given by:

$$p_e = \frac{\pi}{2}\left(1 + \text{erf}\left(\frac{-p}{\sqrt{2 \times 0.75 \times (M-1)}}\right)\right) \qquad (7)$$

C. Modified Prime Code (MPC)

MPC is generated by removing some pulses from the original PC known as redundant pulses. The construction is as follows [8].

Step 1: Choose a prime number p and a specific weight w is assigned.

Step 2: (p-w) "1s" are removed from this PC and remaining "1s" form a new code A` with constant weight w. The resulting A` has p distinct code-words

Step 3: Modified Prime Sequence is generated as:

$$A'_i = (a_{i,b_0}, a_{i,b_1}, a_{i,b_2} \dots\dots\dots a_{i,b_{w-1}}) \text{where } i \in GF(p)$$
$$A_{i,b_l} = \{i.b_l\}(\text{mod } p)$$
$$l = 0,1, \dots\dots.. w-1 \text{ and } b_l \in GF(p) \quad (8)$$

Step 4:Code word of MPC is generated above using (8) as:

$$D'_i = (d'_{i,a_0}, d'_{i,a_1}, d'_{i,a_2} \dots\dots\dots d'_{i,a_{(n-1)}}) \text{ for } i \in GF(p) \quad (9)$$

where

$$d'_{i,k} = \begin{cases} 1 & \text{if } k = a_{i,b_l} + b_l.p \quad \text{for } l = 0,1,2 \dots\dots w-1 \\ 0 & \text{elsewhere} \end{cases} \quad (10)$$

The code length and code weight of PC are n and w (w< p) respectively. Maximum autocorrelation side lobe and maximum cross-correlation function are λa= (w-1) and λc= 2 respectively. The theoretical upper bound of cardinality (n, w, λa, λc) is as

$$\phi(n, w, w-1, 2) = p \quad (11)$$

Thus, MPCs have an advantage of a shorter code length by removing redundant bits although they have a poorer cross correlation. The probability of error for M number of active users is given by

$$pe \le \sum_{i=0}^{w/2} (-1)^i \binom{w/2}{i} (1 - \frac{2ji}{w})^{M-1} \quad (12)$$

where;

$$j = \frac{w^2}{4p^2}.$$

D.Multilevel Prime Code

The 2-D code designed previously focused on keeping a low cross correlation. While it limits multiple access interference it significantly restricts code cardinality. Multilevel PCs offer a relaxation of cross-correlation and hence increasing code cardinality [9], [10].

Step 1:Choose a prime number (p) and specify a cross-correlation (λc) and wavelength (m).We construct the (m x n, w, λa=0, λc=r) multilevel PCs with Pr code matrices from Galois field GF (p) and a specified cross-correlation function λc=r.

Step 2:Each code matrix can be denoted as shown in (13)

$$D_{i_r,i_{r-1},i_{r-2}\dots i_j,} = (d_{i_r,i_{r-1},i_{r-2}\dots i_1,0}, d_{i_r,i_{r-1},i_{r-2}\dots i_1,1} \dots d_{i_r,i_{r-1},i_{r-2}\dots i_1,j} \dots d_{i_r,i_{r-1},i_{r-2}\dots i_1,w-1}) \quad (13)$$

in which the jth element determines the time slot position of j carrier wavelength.

Step 3:Each element is given by (14)

$$d_{i_r,i_{r-1},i_{r-2}\dots i_1,j} = (i_r j^r)(\text{mod } p)(i_{r-1} j^{r-1})(\text{mod } p) \dots (\text{mod } p)(i_1 j)$$
$$\text{where}\{i_r, \dots i_1\} \in GF(p) \quad (14)$$

This construction forms a new family of multilevel code where the number of wavelengths used is equal to code weight(w),code length(n) is equal to prime number p .The autocorrelation is 0 while maximum cross correlation is r . The theoretical upper bound for cardinality for (m x n, w, λa, λc) is given as:

$$\phi(w \times p, w, 0, r) = p^r \quad (15)$$

The probability of error for an arbitrary cross correlation n is given by

$$p_e = 0.5 \sum_{k=0}^{n} (-1)^k \binom{w}{k} \left[\sum_{j=0}^{n} q_{n,j} \frac{\binom{w-k}{j}}{\binom{w}{j}} \right]^{M-1} \quad (16)$$

where;

$$q_{n,j} = 0.5 \frac{h_{n,j}}{p(p^n-1)}.$$

The factor 0.5 is due to assumption that data bit ones and zeroes are transmitted with equal probability. The value of hn,j is given by

$$h_{n,0} = 2p(p^n-1) - h_{n,n} - h_{n,n-1} - \dots - h_{n,1}$$
$$h_{n,1} = w(p^n-1) - nh_{n,n} - \dots - 2h_{n,2}$$
$$h_{n,n} = \binom{W}{n}(p-1)$$
$$h_{n,j} = \left(\frac{h_{n-1,j-1} + h_{n-2,j-1} + \dots\dots\dots\dots + h_{j-1,j-1}}{h_{j-1,j-1}} \right) h_{j,j},$$

where;

$$j \varepsilon \, [2, n-1] \quad (17)$$

TABLE IV
MULTILEVEL PCS Di

	Level 2				
	Level 1 i2=0	Level 1 i2=1	Level 1 i2=2	Level 1 i2=3	Level 1 i2=4
i1=0	(0,0,0,0)	(0,1,4,4)	(0,2,3,3)	(0,3,2,2)	(0,4,1,1)
i1=1	(0,1,2,3)	(0,2,1,2)	(0,3,0,1)	(0,4,4,0)	(0,0,3,4)
i1=2	(0,2,4,1)	(0,3,3,0)	(0,4,2,4)	(0,0,1,3)	(0,1,0,2)
i1=3	(0,3,1,4)	(0,4,0,3)	(0,0,4,2)	(0,1,3,1)	(0,2,2,0)
i1=4	(0,4,3,2)	(0,0,2,1)	(0,1,1,0)	(0,2,0,4)	(0,3,4,3)

The frame of multilevel codes can be considered as tree where topmost level i.e. level r has Pr matrices with maximum cross-correlation of r. At the next level i.e. level (r-1) these Pr matrices are partitioned into P subsets of Pr-1 matrices in which cross correlation is λc =r-1. This division follows until lowest level which has Pr-1 subsets of P matrices each with λc = 1. It is noteworthy that one of the subset in this level is same as basic PCs.

IV. RESULTS AND ANALYSIS

The performance of all these above codes is analyzed to support various number of active users for the permissible BER of 10-9. The performance is evaluated by using MATLAB simulations.

Results shown in Fig. 1 compares BER performance of the (n, w, λa, λc) 1-D EPCs for prime number p= {73, 79, 83, 89, 97}. Here, code length n is taken as prime number p. In general larger value of p supports larger number of active users. Prime number 73 supports 168 users whereas prime numbers 79 and 97 can support 196 and 295 active users.

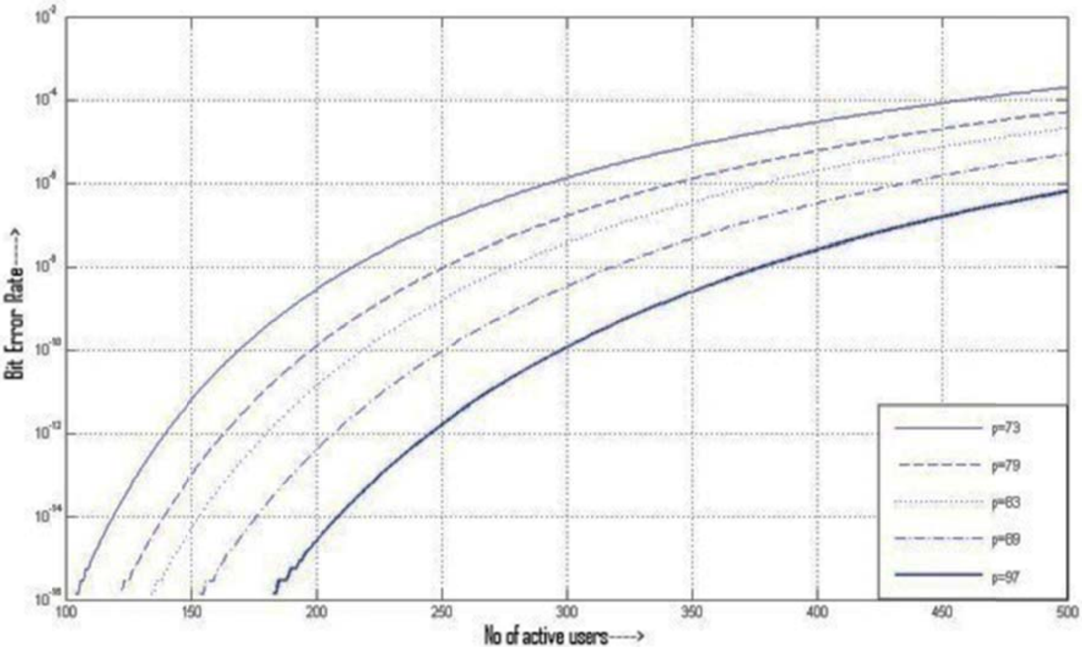

Fig. 1 BER versus number of users at various prime numbers for 1-D EPCs

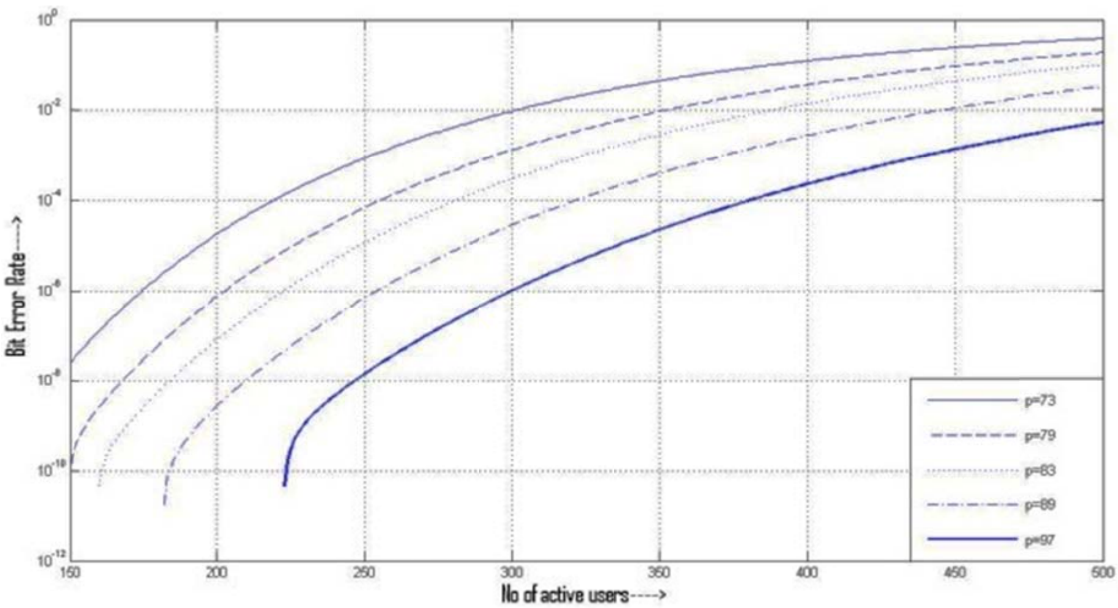

Fig. 2 BER versus number of users at various prime numbers for 1-D MPCs

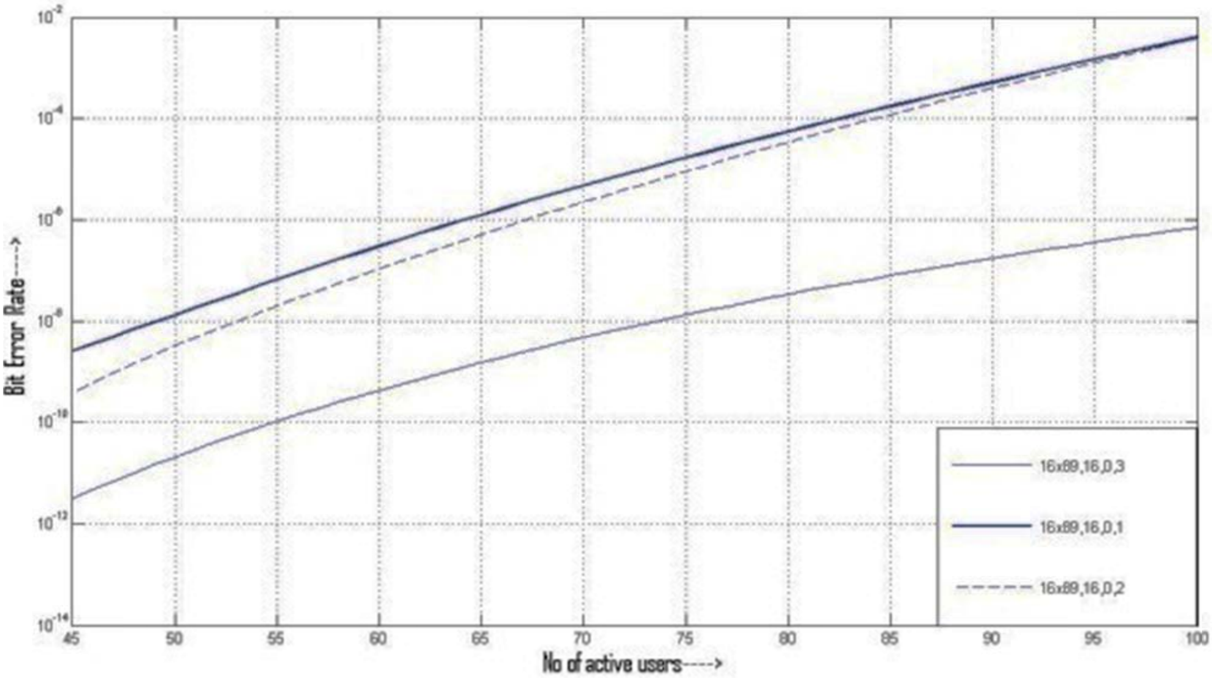

Fig. 3 BER vs. number of users at different cross correlation for 2-D multilevel PCs

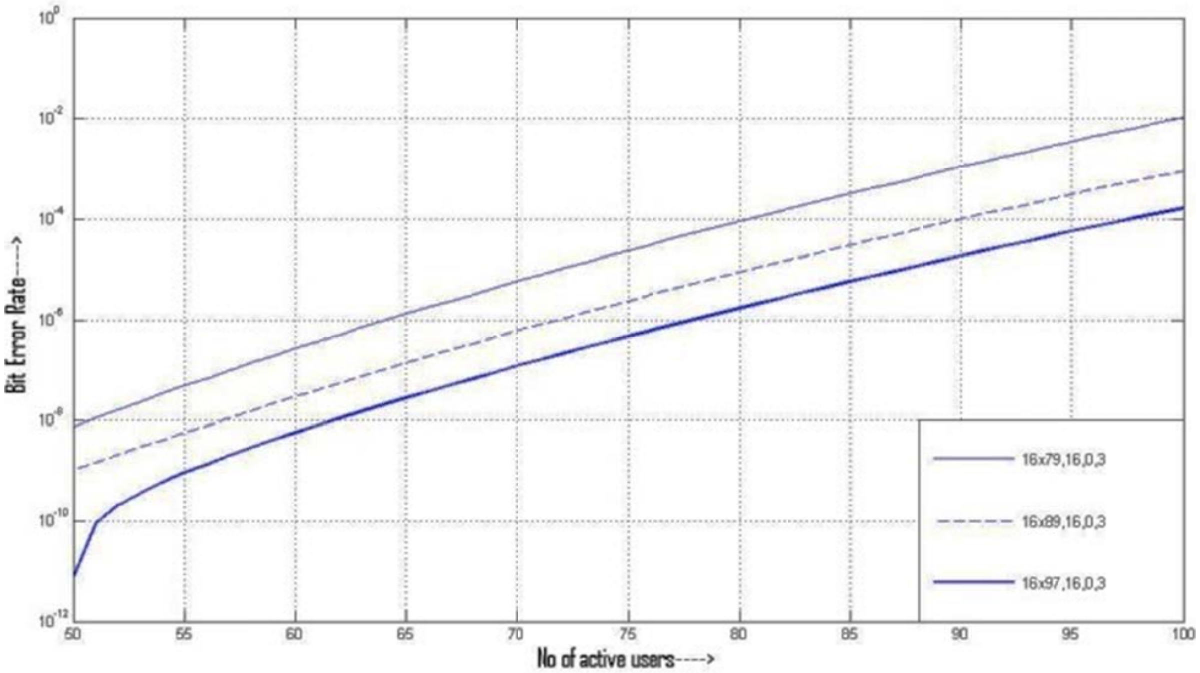

Fig. 4 BER vs. number of users at various prime numbers for 2-D multilevel PCs

In order to reduce the code length, 1-D MPCs were analyzed. Results shown in Fig. 2 compare BER performance of the (n, w, λa, λc) MPC sfor p= {73, 79, 83, 89, 97}. In this, weight is selected as w= {30} and code length n is taken as prime number p. In general, larger value of p supports larger number of active users. Prime number 73 supports 129 users, whereas prime numbers 79 and 97 can support 156 and 230 number of active users, respectively. MPC provides efficient design for OCDMA networks due to which resulting cost and optical power losses can be reduced.

The performance of 2-D multilevel PCs is evaluated here. Results shown in Fig. 3 compare BER performance of multilevel PCs for different values of cross correlation λc = {1,2,3}. The results have been taken for BER of 10-9 and for fixed prime number 89, number of active users supported are 48, 61, and 79 for λc=1, 2, and 3, respectively. Results shown in Fig. 4 compare BER performance of multilevel PCs for different values of prime numbers p = {79, 89, 97}. The results have been taken for BER of 10-9, the number of users supported are 71, 79, and 86 for p=79, 89, and 97 respectively.

By partitioning multilevel PCs into r levels, a trade-off between code cardinality and code performance can be achieved. If high performance is desired with less numbers of active users, lower levels of the tree structure of code matrices will be used while if higher code cardinality is to be achieved then higher levels of tree structure of code matrices should be used. Thus, a flexible system can be achieved which is the main advantage of multilevel PC.

The comparison between different types of 1-D PCs and 2-D multilevel PC is also done. Results shown in Fig. 5 compare BER performance of different PCs for prime number p={73,83}. It can be seen that for BER of 10-9, 1-D EPC can support larger number of active users as compared to those supported by MPCs. The numbers of users supported by EPC are 190 and by MPC, it is 129 for prime number 73 while for multilevel PCs it is 67 and 7, respectively.

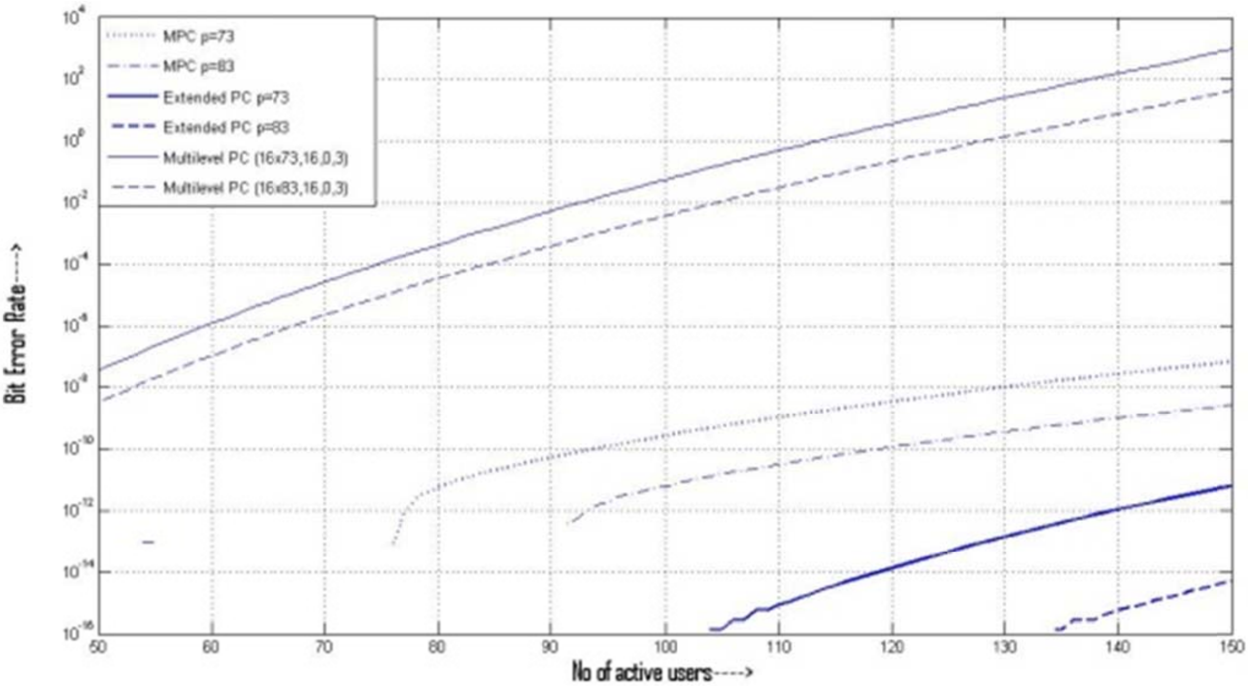

Fig. 5 BER comparison of 1-D EPC, MPC and 2-D Multilevel PCs

TABLE V
PERFORMANCE COMPARISON OF THE NUMBER OF ACTIVE USERS FOR DIFFERENT TYPES OF PC SAT BER OF 10^{-9}.

Prime Number	1-D extended PC	MPC	2-D Multilevel PC		
	λc=1	λc=2	λc=1	λc=2	λc=3
73	168	129	38	43	67
79	196	156	50	55	71
83	217	170	43	57	75
89	249	194	48	61	79

V. CONCLUSION

We had analyzed the performance of various 1-D and 2-D prime sequence coding techniques. 1-D EPC can support more number of active users compared to other codes but at the expense of larger code length which further increases the complexity of the code. MPC supports lesser number of active users at λc=2 but it has a lesser code length as compared to 1-D PC. MPC are comparatively less complex than 1-D EPC. Here although 2-D PC (multilevel PCs) supports lesser number of active users but they are the most secure codes compared to other codes as the users use both time and wavelength domain. Moreover, they add flexibility to the system and can be modified according to the system requirements. There is a trade-off between system capacity i.e. code cardinality and cross correlation and this property can be easily utilized in the case of multilevel PCs. In this paper, we have used mathematical modeling for evaluating the performance of the above codes. In future, the real or simulated systems could be designed and the performance of these codes can be verified. Further, the performance of these codes can be compared to some newly developed 3D codes.

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