Analysis of Scattering Behavior in the Cavity of Phononic Crystals with Archimedean Tilings

Yi-Hua Chen, Hsiang-Wen Tang, I-Ling Chang, Lien-Wen Chen

***Abstract*—**The defect mode of two-dimensional phononic crystals with Archimedean tilings was explored in the present study. Finite element method and supercell method were used to obtain dispersion relation of phononic crystals. The simulations of the acoustic wave propagation within phononic crystals are demonstrated. Around the cavity which is created by removing several cylinders in the perfect Archimedean tilings, whispering-gallery mode (WGM) can be observed. The effects of the cavity geometry on the WGM modes are investigated. The WGM modes with high Q-factor and high cavity pressure can be obtained by phononic crystals with Archimedean tilings.

***Keywords*—**Defect mode, Archimedean tilings, phononic crystals, whispering- gallery modes.

I. Introduction

PHONONIC crystals are composed of two or more different materials which are arranged periodically in space and have great ability to manipulate the propagation of acoustic wave. Due to Bragg's scattering, the incident acoustic wave within a particular range of frequency is forbidden from propagating in the phononic crystals along all the directions, which is called as the complete band gap. The wave propagation behavior and the range of complete band gap can be tuned by altering geometries, spatial arrangements and the constitutive materials.

Complete band gaps exist in perfect phononic crystals, however, when the periodicity is broken by creating defects inside the perfect phononic crystals, the new band gap will generate within the complete band gap which would cause highly localized resonance frequency or guide modes at the frequency [1], [2]. The acoustic stop bands in two-dimensional periodic arrays of liquid cylinders were first studied by [3]. Sigalas [4] created the defect in two-dimensional periodic square structures by changing the radius of one of cylinders in a unit cell and combined plane-wave expansion method (PWE) with supercell method to study the effect of geometry of defects on band gaps. Then [5] furthered to investigate the propagation of acoustic waves under both single and line defects in two-dimensional periodic composites which is made up of solid cylinders in air and suggested the applications as waveguide and acoustical filters. By analyzing the defects of circular and rectangular shapes, [6], [7] found that instead of geometry of defects and rotation angle, filling ratio plays an important role in affecting the band gap and defect modes. References [8]-[11] applied finite-difference time-domain (FDTD) method to probe into the acoustic wave propagation in acoustic waveguide realized by line defects in phononic crystals. Miyashita [12] also constructed the experiment of sharp bending sonic waveguide and a pair of coupled wave guides. Measured transmission ratio and coupling behavior of waves are presented. By creating defects in phononic crystals appropriately, the propagation of elastic waves can be confined in structures and many novel applications, such as filters, waveguides, mirrors, and sensors can be realized.

The whispering-gallery modes (WGMs) are a kind of waves moving around a concave surface which was first discovered in the dome of St. Paul's Cathedral in London, UK. WGMs can be observed around the edge of the cavity in photonic crystals due to total internal reflection. [13], [14] Xing et al. [15] investigated the WGMS in the photonic crystals by PWE and FDTD. The coupling efficiency of a waveguide and a localized cavity mode which was evaluated by observing the decay of WGMs was also studied [16].

Archimedean tilings are constituted of one or more types of convex polygon arranged periodically in two-dimensions. Archimedean tilings can break the confinement of rotational symmetry in traditional lattice arrangements such as square, triangular, or honeycomb matrix, and reach excellent rotational symmetry. Since Archimedean tilings are strictly periodic, the unit cell and Brillouin zone can be both well defined. Due to the complicated rules of arrangement within Archimedean tilings, some peculiar wave propagation behaviors can be induced. Wang [17] used photonic crystals with Archimedean-like tilings to construct coupled-resonator optical and more waveguide modes can be found. Then, [18] applied finite element method to evaluate the propagation of elastic waves in a phononic crystal slab with Archimedean-like tilings and advantages of Archimedean-like structures over the square lattice were presented. Xu et al. [19] also used finite element method to analyze the band structures of solid-solid and solid-air multi-atom Archimedean-like phononic crystals, more and wider band gaps than the traditional phononic can also be observed.

In this study, we apply the finite element method and supercell method to explore the wave propagation behavior, like band gaps and defect modes, inside phononic crystals with Archimedean tilings and the finite element software we use is *COMSOL Multiphysics®* [20].

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II. Computational Method

In this study, finite element method is employed to simulate the behavior of acoustic wave in phononic crystals. The acoustic wave equation is:

$$\frac{\omega^2 p}{\rho c_L^2} + \nabla \cdot \left(-\frac{1}{\rho} \nabla p + q \right) = 0 \quad (1)$$

where p is the pressure field, ρ is the density of fluid, c is the wave velocity of fluid and q is the dipole source.

Using the method of separation of variables, let p to be a time-harmonic field as:

$$p = p(x, y) e^{i\omega t} \quad (2)$$

where $p(x, y)$ is the amplitude of pressure and ω is the angular frequency. Substituting (2) into (1) and letting $\mathbf{q}$=0, then the Helmholtz equation can be obtained as

$$\nabla \cdot \left(-\frac{1}{\rho} \nabla p \right) - \frac{\omega^2 p}{\rho c_L^2} = 0 \quad (3)$$

Equation (3) is solved to yield the pressure field in the cavity of the phononic crystals.

Model of phononic crystals with the cavity is established in *COMSOL* software, the density and the wave velocity of materials are also set. Due to the limitation of computer memory, the space of structure cannot be infinite, therefore, impedance boundaries are set to avoid the reflection of the wave.

When the system is under the resonant frequency and the amplitude of signal is invariant with time, the quality factor Q is defined as the ratio of the storage energy in the system to the energy from outer environment per period which can be written as:

$$Q = \frac{\omega_0}{\Delta \omega} \quad (4)$$

where ω_0 is the resonant frequency and $\Delta\omega$ is the full width at half maximum (FWHM). The higher Q represents the better performance of the resonator [21]. Generally, Q factor ranges from 10^3 to 10^6 is considered to be high and great performance of the resonator is expected.

III. Results and Discussions

In this study, the two-dimensional phononic crystal with (3, 4, 6, 4) Archimedean tilings which is composed of PMMA cylinders in the air background is considered. The mass density ρ and wave speed C of PMMA and air are $\rho_{PMMA} = 1690\ kg$, $\rho_{air} = 1.25\ kg$, $C_{PMMA} = 2694$ m/s and $C_{air} = 343$ m/s respectively.

The (3, 4, 6, 4) Archimedean tilings are composed of triangles, squares, and hexagons as shown in Fig. 1. and its corresponding Brillouin zone is also presented. In Fig. 1 (a), the dash portion is the primitive unit cell, the lattice constant a_0=0.109m, the radius of the cylinder r=0.016m, and the distance between two adjacent cylinders a=0.04m. The filling ratio is 0.47.

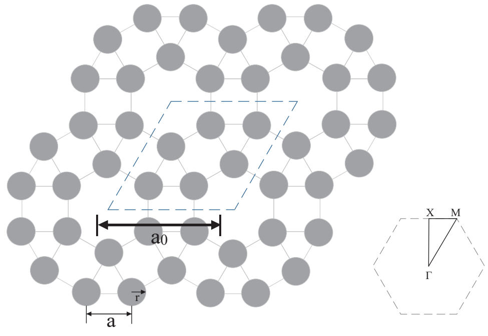


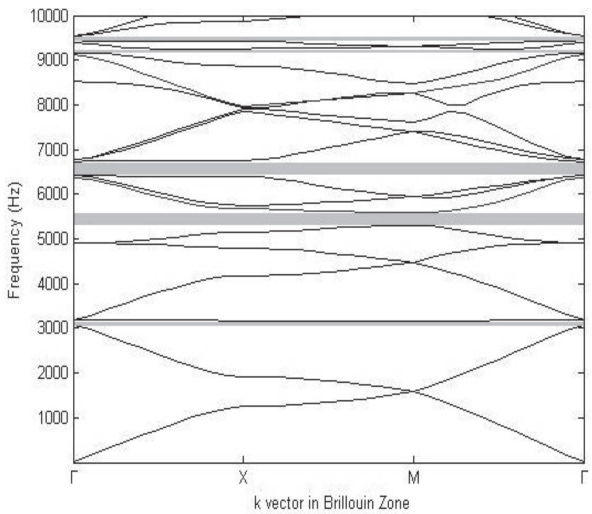
Fig. 1 (a) Phononic crystal with Archimedean tilings (b) Corresponding Brillouin zone

Fig. 2 demonstrates the band structures of phononic crystals with two different kinds of spatial arrangements, triangular lattice and Archimedean tilings. The same material and lattice constant are used in the simulation. For the triangular lattice, the radius of cylinders is set to be 0.039m and the filling ratio is 0.46 which is very close to the value of the Archimedean tilings. Comparing Figs. 2 (a) and (b), we can find that more band gaps exist in the arrangement of Archimedean tilings than that of triangular lattice within the frequency range of 0-10000Hz which shows that the phononic crystals with Archimedean tilings have advantages in audible applications.

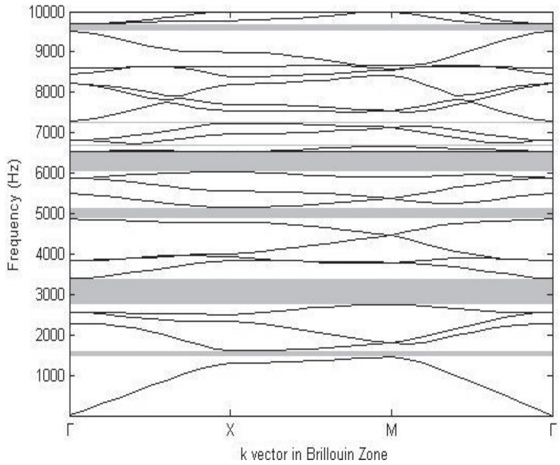
By moving out some cylinders from the perfect Archimedean tilings, we can create a circular-shape defect in phononic crystal as shown in Fig. 3 (b) and the band structure of this structure is presented in Fig. 3 (a). The defect mode shown in Fig. 3 (b) is the 2nd band gap with the frequency 3427 Hz. At the left boundary, an incident wave with the pressure of 1 Pa is given and at the right side, the impedance match boundary is set to prevent from the reflective wave. Due to the circular-shape cavity, the WGM is favorable to be excited and it can be seen in Fig. 3 (b). Fig. 4 shows the intensity in the cavity of phononic crystals as the function of frequency. The highest peak occurs at 3427 Hz which indicates the WGM resonant frequency and the quality factor Q at 3427 Hz is calculated to be at a high value of 1898.

Different arrangements of the cavity which means the different defects will change the resonant frequency. We put several cylinders with the radius 12.5 mm around the cavity and the simulation result of pressure field is shown in Fig. 5. After adding the cylinders, the resonant frequency will shift to 3057Hz which is lower than the former structure. If one more layer of cylinders are added as shown in Fig. 6, the resonant frequency will shift to a lower value of 2999 Hz. Furthermore, the relationship between radius and resonant frequency is investigated. Fig. 7 (a) shows the shift of resonant frequency as only one layer of cylinders are added in the cavity and the radius of cylinders is increased from 10 mm to 12.5 mm with the increment 0.5 mm. Similarly, Fig. 7 (b) shows the shift of resonant frequency when the radius of first layer of cylinders is fixed at 12.5mm and the radius of the second layer of cylinders' changes from 5mm to 7.5 mm with the increment 0.5 mm. It can be seen from Fig. 7 that the resonant frequencies will shift to lower values as the radius of cylinders gets bigger, the resonant frequency can be tuned by changing the radius of cylinders.

Next, the case which a hollow cylinder is put inside the center of the cavity as presented in Fig. 8 is investigated. In Fig. 8, the simulation results of pressure field under different outer radius of hollow cylinder are presented. The thickness of the hollow cylinder is fixed at 10 mm. After adding the hollow cylinder, it can be found that the resonant frequency will get lower and Q-factor will be much higher than those in the original cavity as in Fig. 3 (a). Adding the hollow cylinder will compress the space of resonance, so the pressure field can be strengthened. From Figs. 8 (a)-(c), it is obvious that the intensity of pressure field increases as the outer radius of the hollow cylinder increases. Fig. 9 shows the effect of outer radius of hollow cylinder on Q-factor and maximum pressure value. Opposite trends of Q-factor and maximum pressure when radius gets bigger are found. Bigger radius will lead to higher Q-factor but lower maximum pressure. Therefore, if good performances of both Q-factor and pressure are required, the hollow cylinder with outer radius 40 mm will be a good choice.

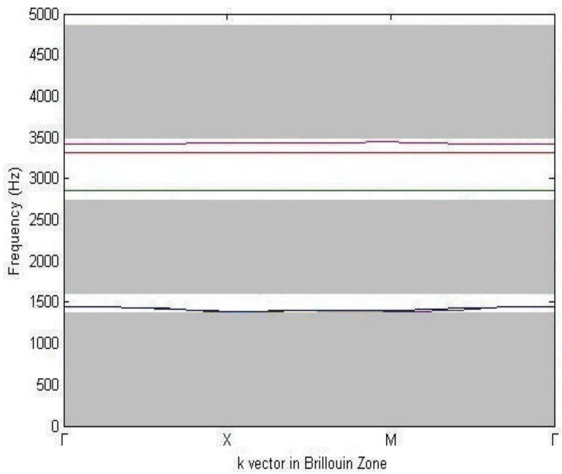


(a)

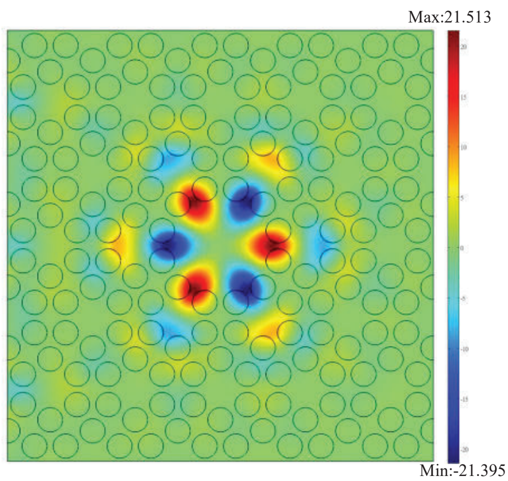


(b)

Fig. 2 Band structures of two-dimensional phononic crystals with (a) triangular lattice and (b) Archimedean tilings



(a)



(b)

Fig. 3 (a) Band structure and (b) pressure field at the resonant frequency 3427 Hz of phononic crystals with Archimedean tilings with the defect

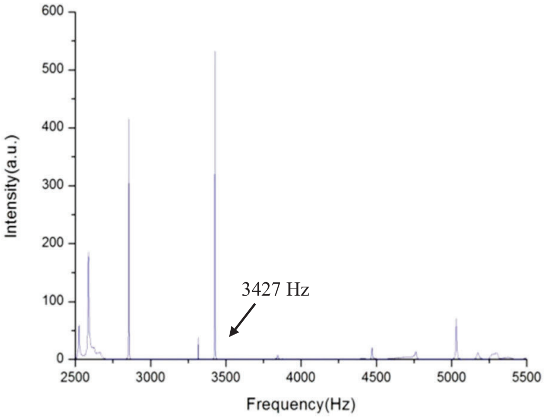


Fig. 4 Intensity in the cavity of phononic crystals as the function of frequency

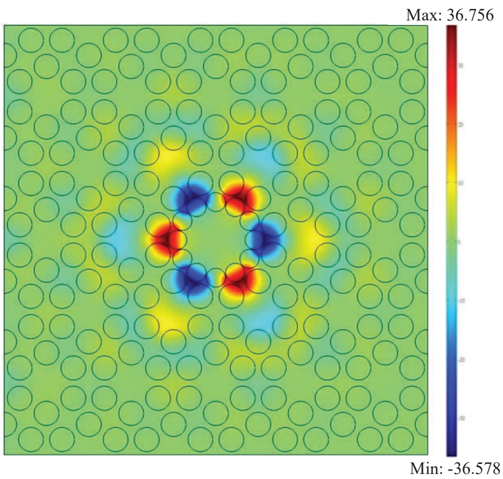


Fig. 5 Pressure field of one-layer-cylinders cavity at the resonant frequency 3057 Hz

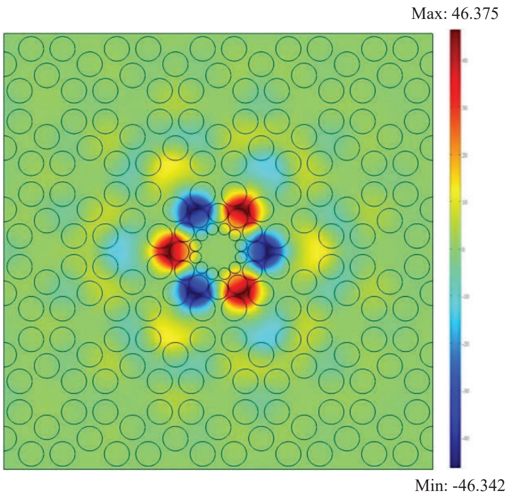
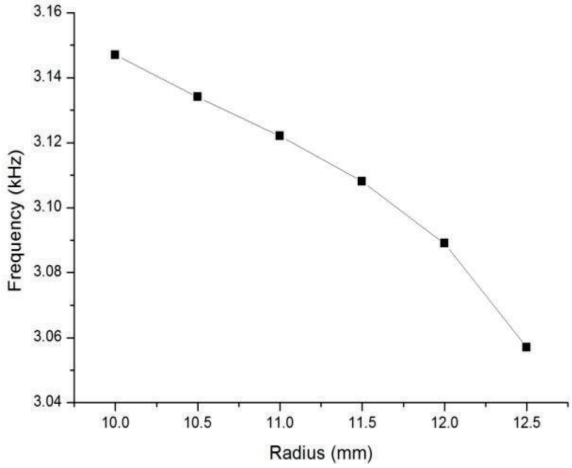
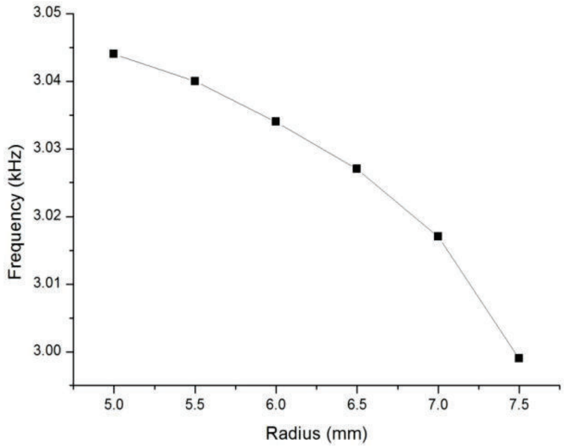


Fig. 6 Pressure field of two-layer-cylinders cavity at the resonant frequency 2999 Hz

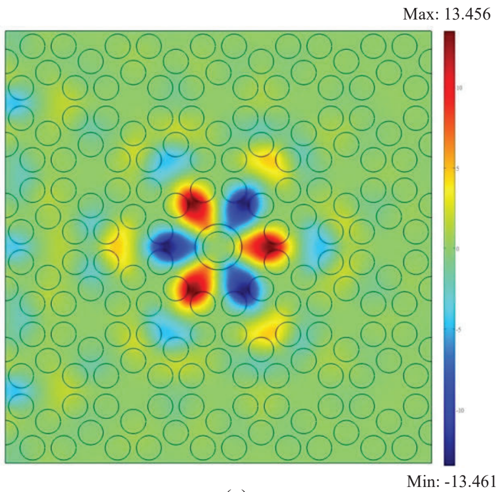


(a)

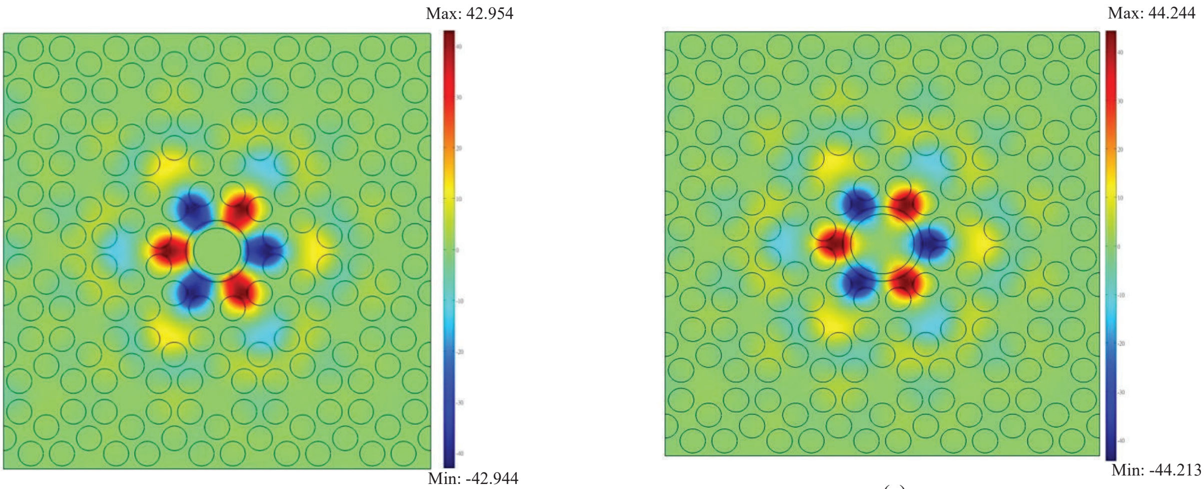


(b)

Fig. 7 The resonance frequency shift toward the radius of cylinders (a) one-layer-cylinders (b) two-layer-cylinders



(a)



(b) (c)

Fig. 8 Pressure field of hollow-cylinder cavity. The outer radius of the hollow cylinder is (a) 30 mm (b) 40 mm (c) 50 mm. The resonant frequency is (a) 3336 Hz (b) 3156 Hz (c) 2979 Hz

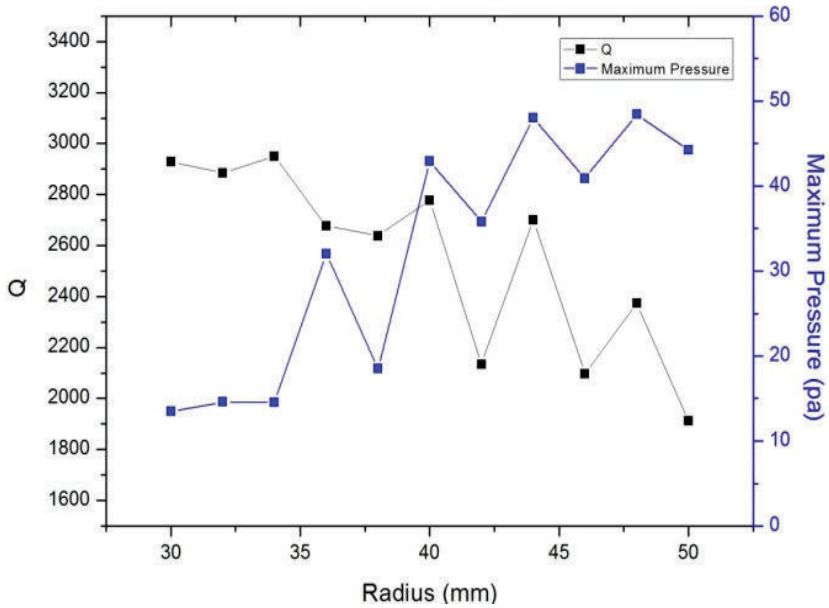


Fig. 9 Q-factor and maximum pressure as the function of the radius of cylinders

IV. Conclusions

In this study, the whispering-gallery modes in a phononic crystal with Archimedean tilings (3, 4, 6, 4) which is composed of rigid cylinders in air background is investigated. The circular shape cavity can be easily generated in the Archimedean tilings (3, 4, 6, 4) arrangement which is favorable to excite WGM. Different designs in the cavity are discussed. By adding cylinders in a ring shape or putting in the hollow cylinder can increase the values of both pressure and Q-factor. Q-factors are all within the range of high Q-factor which shows the good performance of cavity design. This study provides the clear view of the cavity design of phononic crystal with Archimedean tilings.

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