International Journal of Mechanical, Industrial and Aerospace Sciences
ISSN: 2517-9950
Vol:10, No:5, 2016

Modeling of Steady State Creep in Thick-Walled Cylinders under Internal Pressure

Tejeet Singh, Ishavneet Singh

Abstract**—**The present study focused on carrying out the creep analysis in an isotropic thick-walled composite cylindrical pressure vessel composed of aluminum matrix reinforced with silicon-carbide in particulate form. The creep behavior of the composite material has been described by the threshold stress based creep law. The values of stress exponent appearing in the creep law were selected as 3, 5 and 8. The constitutive equations were developed using well known von-Mises yield criteria. Models were developed to find out the distributions of creep stress and strain rate in thick-walled composite cylindrical pressure vessels under internal pressure. In order to obtain the stress distributions in the cylinder, the equilibrium equation of the continuum mechanics and the constitutive equations are solved together. It was observed that the radial stress, tangential stress and axial stress increases along with the radial distance. The cross-over was also obtained almost at the middle region of cylindrical vessel for tangential and axial stress for different values of stress exponent. The strain rates were also decreasing in nature along the entire radius.

Keywords**—**Steady state creep, composite, cylinder, pressure.

I. Introduction

THICK cylinders are of critical importance to numerous industries such as nuclear power plants, chemical industries, armament and food processing industries. Failure of cylinders can be fatal to human being and its surroundings. Therefore, the reliability of the cylinder should be promised. Further, in many of these applications the material of the cylinders is subjected to high thermo-mechanical loadings [1] that lead to creep failure. Creep analysis of thick-walled cylinder made of an isotropic monolithic material, has been analyzed by many researchers [2]-[8].

Composites are one of the most widely used materials because of their adaptability to different situations and the relative ease of combination with other materials to serve purposes and exhibit desirable properties [9]. In many cases, the use of composites is more efficient owing to their unique and tailored properties such as low density, exceptional strength and stiffness, fatigue and corrosion resistance, high thermal conductivity and low coefficient of thermal expansion [10]. Singh and Gupta [11] analyzed the effect of material parameters on steady state creep in a composite cylinder subjected to internal pressure. In the recent years, the problem of composite cylinders made of Functionally Graded Materials (FGMs) operating at high pressures and temperatures are investigated by many researchers [12]-[15]. However, the studies related to creep in isotropic thick-walled cylinders made of Al-SiC$_p$ composite materials are few in number. Therefore, a research must be undertaken to study the creep behavior of a composite cylinder subjected to internal pressure. In the present work, a mathematical model has been developed to describe steady state creep in thick-walled cylinder made from Al-SiC$_p$ composite and the impact of stress exponent on steady state creep stresses and creep rates in thick-walled composite cylinder subjected to internal pressure has been estimated.

Tejeet Singh is with the Department of Mechanical Engineering, S.B.S. State Technical Campus, Ferozepur, 152004, Punjab, India (phone: +918288012011; fax: 01632-242138; e-mail: tejeetsingh@ rediffmail.com).

Ishavneet Singh, is with the Department of Mechanical Engineering, Baddi University, Solan, Himachal Pardesh, India (e-mail: ishavneetsingh86@gmail.com).

II. Creep Law and Creep Parameters

In aluminum based composites, undergoing steady state creep, the effective creep rate, $\dot{\varepsilon}_e$, is related to effective stress, σ_e, through threshold stress, σ_o, based creep law given by [16] and is given below,

$$\dot{\varepsilon}_e = [M(\sigma_e - \sigma_o)]^n \quad (1)$$

The stress exponent n is may be selected as 3, 5 and 8.

The creep parameters M and σ_o appearing in (1) are dependent on the type of composite material, operating temperature (T), size of reinforcement (P) and the content of reinforcement (V). In the present study, creep parameters are obtained from the experimental results [16]. The creep parameters are estimated by conducting the regression analysis on the experimental data for different values of stress exponent n i.e. $n = 3$, 5 and 8.

III. Mathematical Modeling and Analysis

Considering a thick-walled cylinder made of Al-SiC$_p$ composite having inner and outer radii of a and b respectively and subjected to internal pressure p. The radial ($\dot{\varepsilon}_r$) and tangential ($\dot{\varepsilon}_\theta$) strain rates in the cylinder are respectively given by:

$$\dot{\varepsilon}_r = \frac{d\dot{u}_r}{dr} \quad (2)$$

$$\dot{\varepsilon}_\theta = \frac{\dot{u}_r}{r} \quad (3)$$

where $\dot{u}_r = \frac{du}{dt}$ the radial displacement rate and u is the radial displacement.

Considering the equilibrium of forces on an element of the cylinder in the radial direction, we may write,

$$r\frac{d\sigma_r}{dr} = \sigma_\theta - \sigma_r \quad (4)$$

Material of the cylinder is assumed to be incompressible; therefore,

$$\dot{\varepsilon}_r + \dot{\varepsilon}_\theta + \dot{\varepsilon}_z = 0 \tag{5}$$

where $\dot{\varepsilon}_z$ is the strain rate in the axial (z) direction.

The generalized constructive equations for creep in an isotropic composite are given by,

$$\dot{\varepsilon}_r = \frac{\dot{\varepsilon}_e}{2\sigma_e}[2\sigma_r - \sigma_\theta - \sigma_z] \tag{6}$$

$$\dot{\varepsilon}_\theta = \frac{\dot{\varepsilon}_e}{2\sigma_e}[2\sigma_\theta - \sigma_z - \sigma_r] \tag{7}$$

$$\dot{\varepsilon}_z = \frac{\dot{\varepsilon}_e}{2\sigma_e}[2\sigma_z - \sigma_r - \sigma_\theta] \tag{8}$$

where $\sigma_r, \sigma_\theta, \sigma_z$ are respectively the radial, tangential and axial stresses.

The von-Mises [1] effective stress is given by,

$$\sigma_e = \frac{1}{\sqrt{2}}[(\sigma_\theta - \sigma_z)^2 + (\sigma_z - \sigma_r)^2 + (\sigma_r - \sigma_\theta)^2]^{1/2} \tag{9}$$

In a cylinder made of incompressible material with closed end, the plane strain condition exist *i.e.* the axial strain rate $\dot{\varepsilon}_z$ is zero. Therefore, (2), (3) and (5) on simplifying yields,

$$\dot{u}_r = \frac{C}{r} \tag{10}$$

where C is constant of integration,

Substituting (10) into (2) and (3);

$$\dot{\varepsilon}_r = -\frac{C}{r^2} \tag{11}$$

$$\dot{\varepsilon}_\theta = \frac{C}{r^2} \tag{12}$$

Under the assumption of plain strain case, (8) becomes,

$$\sigma_z = \frac{\sigma_r + \sigma_\theta}{2} \tag{13}$$

Using (13) into (9), we obtain;

$$\sigma_e = \frac{\sqrt{3}}{2}(\sigma_\theta - \sigma_r) \tag{14}$$

Substituting (11) and (13) into (6),

$$[\sigma_\theta - \sigma_r] = \frac{4}{3}\left(\frac{\sigma_e C}{\dot{\varepsilon}_e r^2}\right) \tag{15}$$

Putting $\dot{\varepsilon}_e$ and σ_e respectively from (1) and (14) into (15) and simplifying,

$$(\sigma_\theta - \sigma_r) = \frac{I_1}{r^{2/n}} + I_2 \tag{16}$$

where,

$$I_1 = \left(\frac{4}{3}\right)^{\frac{n+1}{2n}}\left(\frac{C^{1/n}}{M}\right)$$

and

$$I_2 = \frac{2}{\sqrt{3}}\sigma_o ,$$

On substituting (16) into (4) and integrating, we get,

$$\sigma_r = -\frac{n}{2}\frac{I_1}{r^{2/n}} + I_2 \ln r + C_1 \tag{17}$$

where C_1 is the constant of integration that can be evaluated by applying boundary conditions. The value of C_1, thus obtained are substituted in (17) to get the radial stress, σ_r,

$$\sigma_r = X\left(b^{-2/n} - r^{-2/n}\right) + I_2[\ln(r/b)] - p \tag{18}$$

where

$$X = \frac{[p + I_2(\ln a/b) - q]}{\left(a^{-2/n} - b^{-2/n}\right)},$$

Using (18) in (16), the tangential stress, σ_θ obtained,

$$\sigma_\theta = X\left[b^{-2/n} + r^{-2/n}\left(\frac{2}{n} - 1\right)\right] + I_2[1 + \ln(r/b)] - p \tag{19}$$

Substituting (18) and (19) in (13), we get the axial stress,

$$\sigma_z = X\left[b^{-2/n} - r^{-2/n} + \frac{r^{-2/n}}{n}\right] + I_2\left[\ln(r/b) + \frac{1}{2}\right] - q \tag{20}$$

On the basis of the analysis presented in the above section, a computer program has been generated to estimate the steady state behaviour of the composite cylinder subjected internal pressure for different value of stress exponent, as 3, 5 and 8. For the numerical evaluation, the inner and outer radius of the cylinder is taken 25.4 *mm* and 50.8 *mm* respectively. The size of SiC_p is 1.7 μm with volume fraction of 20%. The cylinder is assumed to operate at 673 K. The dimensions of cylinder selected in this study are similar to those used in earlier experimental work [17] on thick-walled cylinder. The radial, tangential and axial stresses at different radial locations of the cylinder are calculated respectively from (18), (19) and (20). The distributions of radial and tangential strain rates in the cylinder are computed from (6) and (7) respectively. The composite cylinder is subjected to internal pressure $p = 85.25$ *MPa* and zero external pressure.

IV. Results and Discussion

The creep stress and strain rates are estimated in a thick cylinder made of Al-SiCp and subjected to internal pressure.

A. Creep Stresses in Composite Cylinders

The radial stress in Fig. 1 remains compressive throughout the entire radial distance of composite cylinder. The value of radial stress is maximum at the inner radius and continuously decreasing minimum value to zero at outer radius of composite cylinder. The magnitude of the radial stress increases up to the middle region with the change of stress exponent from 3 to 8. Further, it can be seen from Fig. 1 that the maximum increase observed in the magnitude of the radial stress is at the middle region with change in stress exponent from 3 to 8.

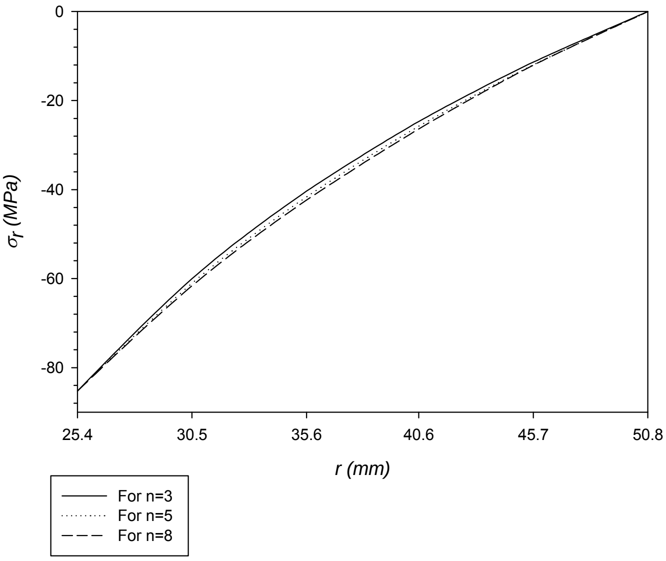

Fig. 1 Radial stress in composite cylinders

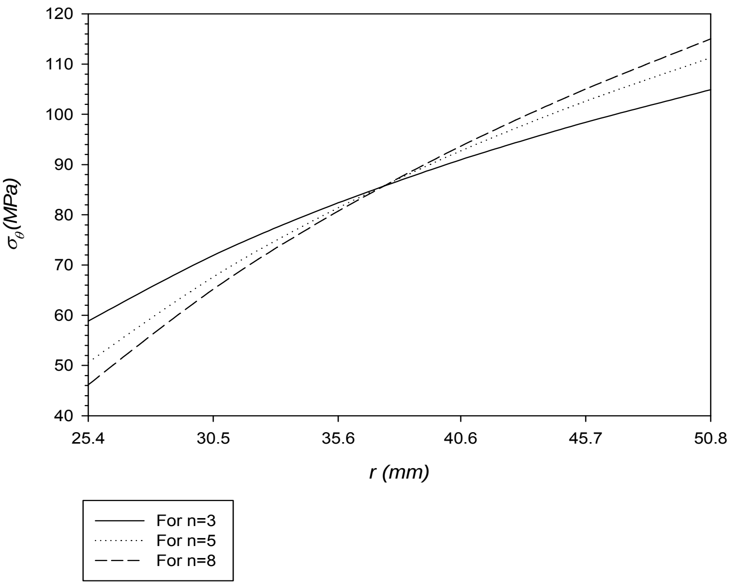

Fig. 2 Tangential stress in composite cylinders

The tangential stress as shown in Fig. 2 also remains tensile throughout the cylinder with minimum value at the inner radius and then continuously increasing to the maximum value at the outer radius. The change of stress exponent from 3 to 8 leads the tangential stress decreases at the inner radius but increases towards the outer radius of the cylinder. The tangential stress becomes maximum at the outer radius of the composite cylinder.

The variation of axial stress is shown in Fig. 3 and it is compressive at the inner region after that it remains tensile throughout the entire radius of the cylinder. The value of axial stress continuously decreases at the inner radius but increases towards the outer radius of the cylinder. The distribution of effective stress is shown in Fig. 4 for different stress exponents in the composite cylinders. The value of effective stress remains tensile throughout the entire radial distance of composite cylinder. The value of effective stress decreases throughout from inner radius to outer radius of composite cylinder, while changing the value of stress exponent from 3 to 8.

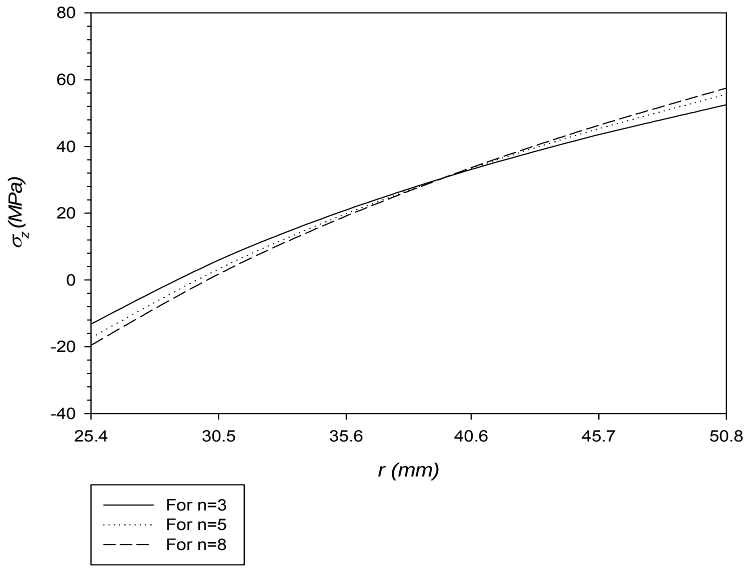

Fig. 3 Axial stress in composite cylinders

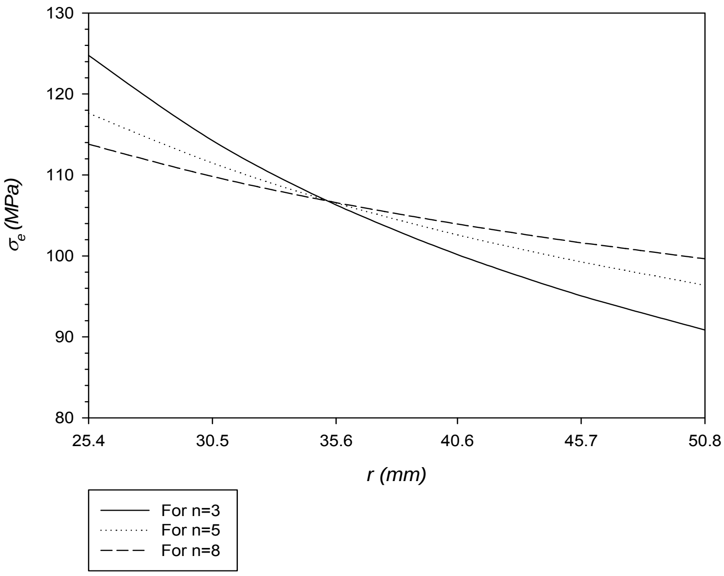

Fig. 4 Effective stress in composite cylinders

B.Strain Rates in Composite Cylinders

The effective strain rate ($\dot{\varepsilon}_e$) increases significantly by changing the stress exponent, as shown in Fig. 5. The effective strain rate reduces by about three orders of magnitude on changing the stress exponent from 3 to 8. The increase in effective strain rate, given by (1), may be because of increase in creep parameter *M* and decrease in threshold stress σ_o with the change of stress exponent from 3 to 8. Similarly, the radial and tangential strain rates also increase at the inner as well as outer surface of composite cylinder as shown in Fig. 6. Their magnitude remains same but nature is opposite due to plain strain condition. The increase observed in the value of $\dot{\varepsilon_\theta}/\dot{\epsilon_r}$ is almost similar at outer radius as well as to inner radius of composite cylinders. Similar to effective strain rate, the tangential/radial strain rates are also increases by about three orders of magnitude on changing stress exponent from 3 to 8. With changing stress exponent from 3 to 8 the threshold stress σ_o decreases and the creep parameter *M* increases as a result of which the strain rates in the composite cylinder increase to a significant extent.

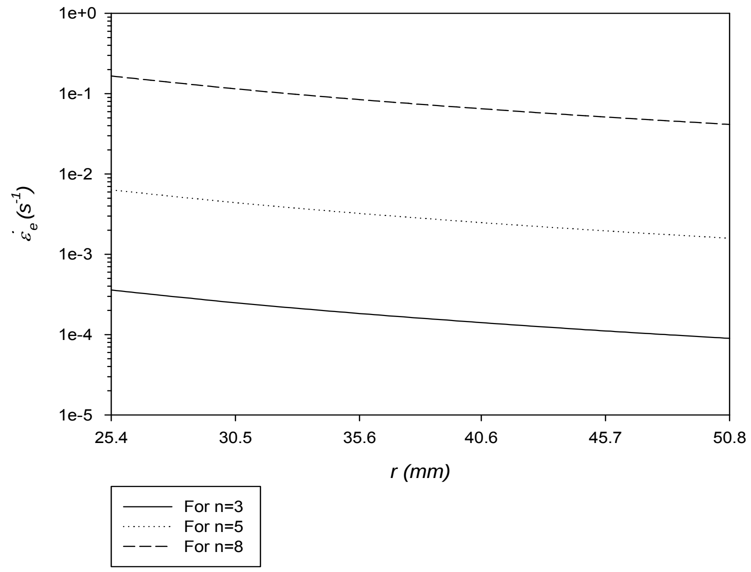

Fig. 5 Effective strain rates in composite cylinders

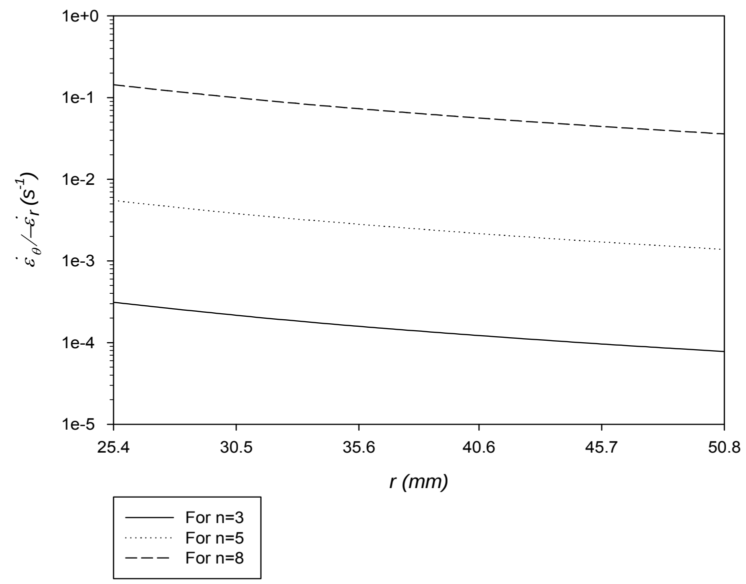

Fig. 6 Radial/Tangential strain rates in composite cylinders

V. Conclusions

The present study has led to following conclusions:

1) The radial stress in the cylinder remains maximum at the inner radius and continuously decreasing to minimum value of zero at outer radius. Further, a little variation in radial stress is seen in the middle region because of different value of stress exponent.
2) The tangential, axial stress and effective stresses are significantly affected by the value of stress exponent. A crossover is also seen at the middle region of the cylinder for different values of stress exponent i.e. 3, 5 and 8.
3) The steady state effective, tangential and radial strain rates in isotropic composite cylinder decreases everywhere in the composite cylinder with decrease in the value of stress exponent from 8 to 3.

References

[1] Dieter, G.E., (1988), Mechanical metallurgy, London: McGraw-Hill.
[2] Weir, C.D., (1957) The creep of thick-walled tube under internal pressure, J. Appl. Mech., 24: pp. 464–466.
[3] Davis, E.A., and Connelly, F.M., (1959), Stress distribution and plastic deformation in rotating cylinders of strain-hardening material, J. Appl. Mech., 26(1): pp. 25–30.
[4] Rimrott, F.P.J., (1959), Creep of thick-walled tubes under internal pressure considering large strains, Journal of Appl. Mech., 26: pp. 271–274.
[5] King, R.H., Mackie, W.W., (1967) Creep of thick-walled cylinders, ASME J. Basic Engng., 89(4): pp. 877–884.

[6] Robotnov, Y.N., (1969), Creep problems in structural members, Translated from Russion by Transcripta service Ltd., London, North-Holland Publishing Co. Amsterdam.
[7] Bhatnagar, N.S., Kulkarni, Pradnya, Arya, V.K., (1986), Analysis of an orthotropic thick-walled cylinder under primary creep conditions, International Journal of Pressure Vessels and Piping, 23(3): pp. 165-185.
[8] Bhatnagar, N.S., and Gupta, S.K., (1969), Analysis of thick-walled orthotropic cylinder in the theory of creep, J. of Physical Soc. of Japan, 27(6): pp. 1655–1662.
[9] Kaw, A. K., (2006), Mechanics of composite materials, CRC press, Taylor &Francis group, USA.
[10] Srivatson, T.S., Sudarshan, T.S., and Lavernia, E.J., (1995), Processing of discontinous-reinforced metal matrix composites by rapid solidification, Progress in Mater. Sci., 39: pp. 317–409.
[11] Singh, Tejeet, and Gupta, V.K., (2008), Effect of material parameters on steady state creep in a thick composite cylinder subjected to internal pressure, The Journal of Engng. Research Vol. 6, No. 2 (2009): pp. 20-32.
[12] You, L.H., Ou, H., and Zheng, Z.Y., (2005), Creep deformations and stresses in thick- walled cylindrical vessels of functionally graded materials subjected to internal pressure, Composites Structures, 78(2007): pp. 285-295.
[13] Singh, Tejeet, and Gupta, V.K., (2010), Modeling steady state creep in functionally graded thick cylinder subjected to internal pressure, Journal of Composite Mater. June 2010 vol. 44 no. 11: pp. 1317-1333.
[14] Singh, Tejeet, and Gupta, V.K., (2014), Analysis of steady state creep in whisker reinforced functionally graded thick cylinder subjected to internal pressure by considering residual stress, Mechanics of Advanced Materials and Structures, Taylor and Francis Ltd., Volume 21, Number 5, 28 May 2014,: pp. 384-392(9).
[15] Jamian, Saifulnizan., Sato, Hisashi., Hideaki, Tsukamoto., and Watanabe, Yoshimi., (2012), Creep analysis of functionally graded material thick-walled cylinder, International Conference on Mechanical and Manufacturing.
[16] Pandey, A.B., Mishra, R.S. and Mahajan, Y.R., 1992, "Steady State Creep Behavior of Silicon Carbide Particulate Reinforced Aluminium Composites, "Acta Metall Mater, Vol. 40(8), pp. 2045-2052.
[17] Johnson, A.E., Henderson, J., and Khan, B., (1961), Behavior of metallic thick-walled cylindrical vessels or tubes subjected to high internal or external pressures at elevated temperatures, Proc. Instn. Mech. Engrs., Vol. 175(25), pp. 1043-1069.