International Journal of Architectural, Civil and Construction Sciences
ISSN: 2415-1734
Vol:10, No:3, 2016

Formulating the Stochastic Finite Elements for Free Vibration Analysis of Plates with Variable Elastic Modulus

Mojtaba Aghamiri Esfahani, Mohammad Karkon, Seyed Majid Hosseini Nezhad, Reza Hosseini-Ara

***Abstract*—**In this study, the effect of uncertainty in elastic modulus of a plate on free vibration response is investigated. For this purpose, the elastic modulus of the plate is modeled as stochastic variable with normal distribution. Moreover, the distance autocorrelation function is used for stochastic field. Then, by applying the finite element method and Monte Carlo simulation, stochastic finite element relations are extracted. Finally, with a numerical test, the effect of uncertainty in the elastic modulus on free vibration response of a plate is studied. The results show that the effect of uncertainty in elastic modulus of the plate cannot play an important role on the free vibration response.

***Keywords*—**Stochastic finite elements, plate bending, free vibration, Monte Carlo, Neumann expansion method.

I. Introduction

STRUCTURAL systems always design under the uncertainty conditions. The existence of inherent uncertainty in the behavior of these systems has made the scientists realize the importance of analysis with stochastic nature in reviewing engineering systems. Several factors lead to the emergence of uncertainty in the structure, among which not constant characteristic of forming substance such as elastic modulus can be pointed out [1].

Using the mean values and probability theories has long been a common approach in dealing with these uncertainties. This procedure, which has been utilized in most of the engineering works, is based on extreme values and mean of factors with uncertainty. In fact, in this approach, it is implicitly assumed that the results of the analysis undoubtedly have a nature of uncertainty. This approach is not appropriate in most cases and cannot result in an optimal design. Stochastic analysis approach provides this possibility that even in complex systems with uncertainty, the optimal design can be achieved. One of the most powerful tools in stochastic analysis is the stochastic finite element method (SFEM). This method which is extended from the conventional finite element method can be used in analysis of variety of static and dynamic problems with stochastic nature. One of the common methods used to calculate stochastic response of a system is Monte Carlo simulation (MCS) [2]. Furthermore, by benefiting from stochastic theories and functions, data are quantified with uncertainty. Due to the simplicity and lack of experimental data, estimation of Gaussian stochastic fields is used [1].

So far, a few researches have been conducted on the stochastic effect of material characteristics on the free vibration response of plate bending. Sepahvand et al. studied the effect of uncertainty of elastic modulus on the free vibration of orthotropic plates using generalized polynomial chaos expansion [3]. Also, Dey et al. analyzed free vibration response of laminated composite plates with the assumption of stochastic elastic modulus, mass density and orientation angle of the layers [4]. In addition, free vibration analysis of FGM plates with stochastic variables was performed by Talha and Singh [5]. Moreover, Chakraborty et al. proposed a novel method for modeling stochastic field based on polynomial correlated function expansion (PCFE) for free vibration analysis of laminated composite plates [6].

In this study, the effect of uncertainty in the elastic modulus on angular frequencies of thin plate bending is investigated. For this end, it is assumed that the stochastic field contains Gaussian distribution. Furthermore, the distance autocorrelation function is used for stochastic field. In this approach, the field variations in a limited component are approximated with the mean of distance. Then, by using finite element method (FEM) and MCS as well as Neumann expansion method, the stochastic finite element relations are extracted. It is noteworthy that, four-node and four-side component (MZC) with 12 degrees of freedom can be used for finite element analysis.

II. Simulation of Stochastic Field

In this study, it is assumed that the elastic modulus changes stochastically and proportional to the distance on the surface of plate. Therefore, we can write:

$$A(x) = A_0\left[1+\alpha(x)\right] \tag{1}$$

In the above equation A_0 is the mean of elastic modulus and $\alpha(x)$ is the level of volatility and changes.

$$E\left[\alpha(x)\right]=0 \tag{2}$$

Here, the symbol $E[\cdot]$ represents the average value. Furthermore, the autocorrelation function is defined as [1]:

Mojtaba Aghamiri Esfahani is with the Payame Noor University, Iran, Islamic Republic Of (e-mail: mojtaba.aghamiri@gmail.com).

$$R_{\alpha\alpha} = E\left[\alpha(r)\alpha(r+\xi)\right] \tag{3}$$

In this equation, the coordinates of the desired location (the center of the component) and vector distance is between r and $r+\xi$ points (the distance between centers of the two components). By considering the homogeneous and isotropic stochastic field, the autocorrelation function only depends on the amount of distance $|\xi|$ and the amount of variance σ_α^2. Thus, the distance autocorrelation function can be assumed as:

$$R_{\alpha\alpha}(\xi) = \sigma_\alpha^2 \exp\left[-\left(\frac{|\xi|}{d}\right)\right] \tag{4}$$

In (4), d is the length of correlation; therefore, as d increases, the level of correlation increases, and when it decreases, the level of correlation decreases. Moreover, σ_σ is the standard deviation of $\alpha(r)$. This quantity represents the coefficient of variation of stochastic field $A(x)$.

In FEM, the structure is divided into n elements. In addition, it is assumed that the amount of stochastic field within the element is fixed. Typically, as the number of elements increases, the response accuracy also increases. Therefore, if n components are used for the plate, we will have n random value of $\alpha_i = \alpha(r_i)\ i = 1,2,\cdots,n$ which are dependent on these elements. The correlation of these values is determined by the covariance matrix. This matrix can be obtained as:

$$\operatorname{cov}\left[\alpha_i\ \alpha_j\right] = R_{\alpha\alpha}\left(\xi_{ij}\right) \tag{5}$$

If the structure is divided into n finite elements, vector of correlated stochastic field (with mean of zero) $\{\alpha\} = \{\alpha_1, \alpha_2, \ldots, \alpha_N\}$ can be obtained as:

$$\{\alpha\} = [L]\{Z\} \tag{6}$$

Vector $\{Z\} = \{Z_1, Z_2, \ldots, Z_N\}$ is independent Gaussian stochastic vector with mean of zero and standard deviation of one. Lower triangular matrix $[L]$ is obtained from the Cholesky analysis of the covariance matrix. After finding the matrix $[L]$, correlated stochastic vector can be easily calculated for each simulation. In general, the degree of accuracy in the mean and standard deviation of response depend on the number of samples.

III. Formulating Finite Elements

Equation of free vibration of plate bending can be written in matrix form as:

$$\left([K] - \omega^2[M]\right)\{w\} = 0 \tag{7}$$

where $\{w\}$ is the nodal displacement vector, ω is the angular frequency, $[K]$ is the stiffness matrix and $[M]$ is the mass matrix of the whole structure. Stiffness matrix $[k]$ and mass matrix $[m]$ for elements (MZC) can be calculated as:

$$[k] = \int_v [B]^T [D]_e [B]\, dv \tag{8}$$

$$[m] = \rho\int_v [N]^T [N]\, dv \tag{9}$$

where $[N]$, $[D]_e$, $[B]$, and ρ factors in (8) and (9) are interpolation functions, matrix of materials, strain matrix and density of plate, respectively. Interpolation functions and strain matrix $[B]$ of element (MZC) can be found in reference [7]. Moreover, matrix of materials for thin bending element is as:

$$[D]_e = \frac{Et^3}{12\left(1-\nu^2\right)}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix} \tag{10}$$

where, E, v, and t factors are elastic modulus, Poisson's ratio and plate thickness, respectively.

If the material matrix $[D]_e$ has an uncertain, stochastic nature, this matrix can be written as follows in terms of stochastic factor h_α:

$$[D]_e = h_\alpha [C]_e \tag{11}$$

which, h_α represents stochastic of material matrix on the surface of the element. In addition, $[C]_e$ is the certain matrix. In order to enter stochastic nature into the finite element relationships, stochastic parameter, h_α, can be written as the sum of mean and standard deviation $h_\alpha = h_0 + \Delta h$.

$$E = E_0\left(1+\alpha_e\right) \tag{12}$$

If the material matrix has a stochastic nature, the element stiffness matrix and the total structure stiffness matrix will have an uncertain, stochastic nature. As a result, the structural response will also have an uncertain, stochastic nature. Therefore, stiffness matrix structure will also have two parts of mean and deviation:

$$\left[k(h)\right] = \left[k(h_0)\right] + \left[k(\Delta h)\right] \tag{13}$$

The parts of mean and deviation of stiffness matrix can be obtained as:

$$\left[k\left(h_0\right)\right]=E_0\int_v\left[B\right]^T\left[C\right]_0\left[B\right]dv \quad (14)$$

$$\left[k\left(\Delta h\right)\right]=E_0\alpha_e\int_v\left[B\right]^T\left[C\right]_0\left[B\right]dv \quad (15)$$

It is noteworthy that the mass matrix, $[m]$, is a certain matrix. By overlaying mass and stiffness matrices of elements, stiffness matrix and total structure mass matrix can be obtained as

$$\left[K\right]=\sum_{e=1}^{n}\left[k\left(h\right)\right]_e \quad (16)$$

$$\left[M\right]=\sum_{e=1}^{n}\left[m\right]_e \quad (17)$$

In Monte Carlo method, initially, stochastic variables are numerically simulated. Then, for each of the simulated variables, structural response will be calculated. Vector of stochastic variable $\{\alpha\}=\{\alpha_1,\alpha_2,\ldots,\alpha_N\}$ can represent each of the stochastic factor such as materials, geometry, load, etc. By calculating the structural response for all simulated variables, reaction of structure relative to these stochastic factors can be achieved.

IV. Numerical Test

The word “data” is plural, not singular. In this section, to investigate the effect of stochasticity of elastic modulus on free vibration of bending plate, a numerical test is conducted. To this purpose, a square plate with simple supports is analysed. This plate consists of length and thickness. Also, the elastic modulus, Poisson's ratio and density of the plate are, respectively $E=10.92\times10^9\,N/m^2$, $\nu=0.3$, and $\rho=1000\,kg/m^3$. Using analysis, the operator without the first mode frequency dimension, is calculated as

$$\lambda_1^2=\omega_1 a^2\sqrt{\frac{\rho t}{D}} \quad (18)$$

For Finite element analysis, a 10×10 network is used. Also, using the Monte Carlo method, stochastic simulation was conducted 5,000 times. In Fig. 1, effect of the coefficients of variation of stochastic fields without correlation (d=2) is shown. Fig. 2 also shows the effect of the coefficients of variation of stochastic fields with perfect correlation (d=200) on the free vibration response. According to the graph, it is observed that the stochasticity of the elastic modulus fields, has little effect on free vibration response. It is also observed that although the increase on Cross-correlation of stochastic fields increases the coefficient of variation of response, but the effect is low. Also in Fig. 3, the sensitivity graph of vibration response to the coefficient of variation and correlation of stochastic fields has been shown.

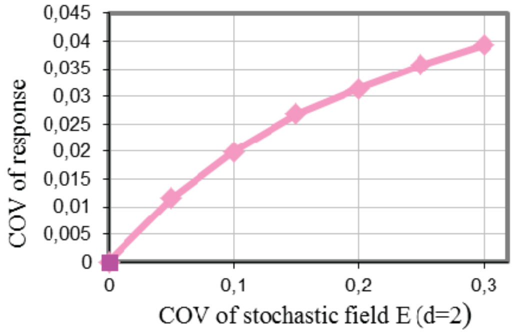

Fig. 1 Effect of the coefficients of variation of stochastic fields without correlation (d=2)

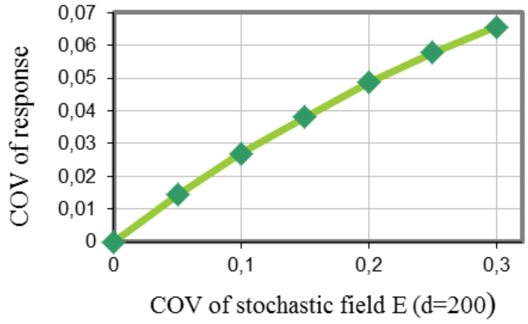

Fig. 2 Effect of the coefficients of variation of stochastic fields with perfect correlation (d=200)

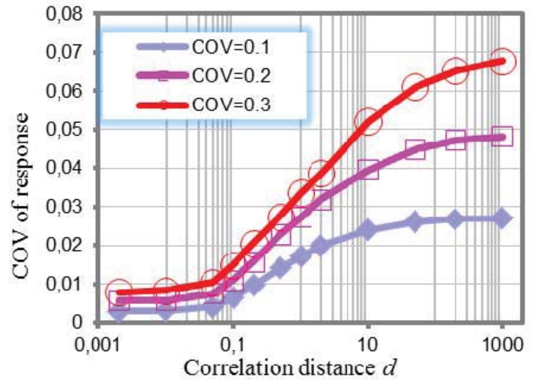

Fig. 3 Sensitivity of vibration response to the coefficient of variation and correlation of stochastic fields

V. Conclusion

Submission of a manuscript is not required for participation in a conference. In this article, the effect of stochasticity of elastic modulus on free vibration of bending plate was investigated. For this purpose, the FEM and MCS were used. Also, a normally distributed stochastic field was modeled with using spatial autocorrelation function. With stochastic vibration analysis of plate, it was observed that the variability of the elastic modulus does not have much effect on free vibration response of plate; so if the coefficient of variation of stochastic field is equal to 30%, the coefficient of variation of responses, depending on the amount of stochastic correlation, will be between 4 and 5.6 percent.

REFERENCES

[1] B. S. de Lima and N. F. Ebecken, "A comparison of models for uncertainty analysis by the finite element method," Finite Elements in Analysis and Design, vol. 34, pp. 211-232, 2000.
[2] H.-C. Noh and T. Park, "Monte Carlo simulation-compatible stochastic field for application to expansion-based stochastic finite element method," Computers & structures, vol. 84, pp. 2363-2372, 2006.
[3] K. Sepahvand, S. Marburg, and H.-J. Hardtke, "Stochastic free vibration of orthotropic plates using generalized polynomial chaos expansion," Journal of Sound and Vibration, vol. 331, pp. 167-179, 2012.
[4] S. Dey, T. Mukhopadhyay, and S. Adhikari, "Stochastic free vibration analysis of angle-ply composite plates–a RS-HDMR approach," Composite Structures, vol. 122, pp. 526-536, 2015.
[5] M. Talha and B. Singh, "Stochastic vibration characteristics of finite element modelled functionally gradient plates," Composite Structures, vol. 130, pp. 95-106, 2015.
[6] S. Chakraborty, B. Mandal, R. Chowdhury, and A. Chakrabarti, "Stochastic free vibration analysis of laminated composite plates using polynomial correlated function expansion," Composite Structures, vol. 135, pp. 236-249, 2016.M. Young, The Techincal Writers Handbook. Mill Valley, CA: University Science, 1989.
[7] I. Ramu and S. Mohanty, "Modal Analysis of Functionally Graded Material Plates Using Finite Element Method," Procedia Materials Science, vol. 6, pp. 460-467, 2014.

Mojtaba Aghamiri Esfahani is member of Department of Mechanical Engineering, Payame Noor University, Tehran, Iran. He has bachelor of mechanical engineering from Malekeashtar University of Technology in Esfahan Iran and master of science in space structures from Ferdowsi University of Mashhad from Mashhad, Iran.

He, works in reliability and optimization of structures and composites and has some article in this field.

e.g. Mojtaba Aghamiri Esfahani is member of Iran Construction Engineering Organization and Mechanical Engineering Organization of Iran.