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Conjugate Free Convection in a Square Cavity Filled with Nanofluid and Heated from Below by Spatial Wall Temperature

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Abstract—The problem of conjugate free convection in a square cavity filled with nanofluid and heated from below by spatial wall temperature is studied numerically using the finite difference method. Water-based nanofluid with copper nanoparticles are chosen for the investigation. Governing equations are solved over a wide range of nanoparticle volume fraction ($0 \leq \phi \leq 0.2$), wave number (($0 \leq \lambda \leq 4$) and thermal conductivity ratio ($0.44 \leq K_r \leq 6$). The results presented for values of the governing parameters in terms of streamlines, isotherms and average Nusselt number. It is found that the flow behavior and the heat distribution are clearly enhanced with the increment of the non-uniform heating.

Keywords—Conjugate free convection, nanofluid, spatial temperature.

I. Introduction

FREE convective heat transfer is a significant phenomenon in engineering systems due to its wide applications in operations of solar collectors, heat exchangers, storage tanks, double pane windows, etc. Ostrach [1] introduced a number of these applications. Many researchers have considered nanofluids as working mediums that enhance thermal conductivity with the presence of nanoparticles, thus making nanofluids appear as a decent candidate for heat removal devices in workable, fluid-based, thermal applications. Shu and Wee [2] numerically studied the free convection in a square cavity by simple-generalized differential quadrature method. Turan et al. [3] numerically investigated the laminar free convection of Bingham fluids in a square cavity with differentially heated side walls. Recently, Alsabery et al. [4] considered the free convection heat transfer in a square cavity partially filled with porous media numerically by using finite element method. A very first comprehensive work on free convection in partially occupying nanofluids in cavities was done by Khanafer et al. [5] The work of Jou and Tzeng [6] considered the free convective heat transfer in nanofluids occupying a rectangular cavity. Nasrin and Parvin [7] used the finite element method to study the buoyancy-driven flow and heat transfer in a trapezoidal cavity filled with water-Cu nanofluid.

Conjugate free convection heat transfer in cavities has received much attention because of its importance to many engineering systems, such as solar energy collectors, material processing, heat preservation of thermal transport circuits, building energy components, and the cooling of electrical units. House et al. [8] investigated the effect of a centred heat-conducting body on the free convection heat transfer in a square cavity. Zhao et al. [9] studied the effect of a centred heat-conducting body on the conjugate free convection heat transfer in a square cavity.

The problem of free convection in closed cavities with boundary walls, including spatial wall temperatures, has been considered in a few studies. Saeid and Yaacob [10] considered numerically the free convection in a square cavity filled with pure fluid in addition to a non-uniform hot-wall temperature and a uniform cold-wall temperature. However, conjugate free convection in a square cavity filled with nanofluid and heated from below by spatial wall temperature has not yet been undertaken. The aim of this study is to investigate the effect of spatial wall temperature variation on conjugate free convection in a square cavity filled with nanofluid.

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II. Mathematical Formulation

Consider two-dimensional steady free convection in a square cavity with length L, as illustrated in Fig. 1. The bottom horizontal wall of the cavity is heated to spatial temperature $\overline{T}_h$ and the top horizontal wall is maintained at a constant cold temperature T_c, while the vertical walls are kept adiabatic. The boundaries of the cavity are assumed to be impermeable, the fluid within the cavity is a waterbased nanofluids having Cu nanoparticles. The Boussinesq approximation is applicable, the nanofluid physical properties are constant except for the density. By considering these assumptions, the continuity, momentum and energy equations for the laminar free convection can be written as:

$$\frac{\partial u_{nf}}{\partial x} + \frac{\partial v_{nf}}{\partial y} = 0 \tag{1}$$

$$u_{nf}\frac{\partial u_{nf}}{\partial x} + v_{nf}\frac{\partial u_{nf}}{\partial y} = -\frac{1}{\rho_{nf}}\frac{\partial p_{nf}}{\partial x} + \nu_{nf}\left(\frac{\partial^2 u_{nf}}{\partial x^2} + \frac{\partial^2 u_{nf}}{\partial y^2}\right) \tag{2}$$

$$u_{nf}\frac{\partial v_{nf}}{\partial x} + v_{nf}\frac{\partial v_{nf}}{\partial y} = -\frac{1}{\rho_{nf}}\frac{\partial p_{nf}}{\partial x} + \nu_{nf}\left(\frac{\partial^2 v_{nf}}{\partial x^2} + \frac{\partial^2 v_{nf}}{\partial y^2}\right) + \beta_{nf} g\left(T_{nf} - T_c\right) \tag{3}$$

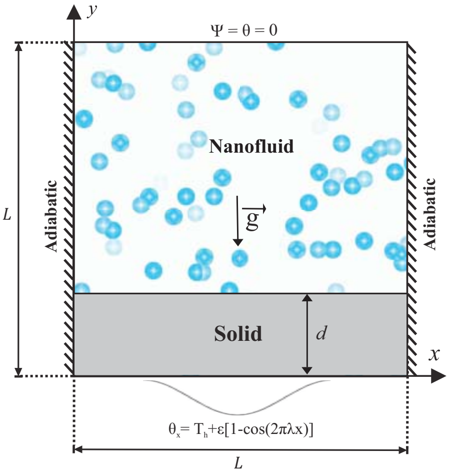

Fig. 1 Physical model of convection in a square cavity together the coordinate

$$u_{nf}\frac{\partial T_{nf}}{\partial x}+v_{nf}\frac{\partial T_{nf}}{\partial y}=\alpha_{nf}\left(\frac{\partial^2 T_{nf}}{\partial x^2}+\frac{\partial^2 T_{nf}}{\partial y^2}\right) \quad (4)$$

and the energy equation for the impermeable wall is:

$$\frac{\partial^2 T_w}{\partial x^2}+\frac{\partial^2 T_w}{\partial y^2}=0. \quad (5)$$

where x and y are the Cartesian coordinates measured in the horizontal and vertical directions respectively, g is the acceleration due to gravity. We assumed that the temperature of the hot vertical wall has a sinusoidal variation almost a minimum value of T_h in the form

$$T_h(y)=\bar{T}_h+\varepsilon\left(\bar{T}_h-T_c\right)\left[1-\cos\left(\frac{2\pi\lambda x}{L}\right)\right] \quad (6)$$

α_{nf} is the effective thermal diffusivity of the nanofluids, ρ_{nf} is the effective density of the nanofluids and μ_{nf} is the effective dynamic viscosity of the nanofluids, which are defined as

$$\alpha_{nf}=\frac{k_{nf}}{(\rho Cp)_{nf}},\quad \rho_{nf}=(1-\phi)\rho_{bf}+\phi\rho_{sp},$$
$$\frac{\mu_{nf}}{\mu_{bf}}=\frac{1}{(1-\phi)^{2.5}} \quad (7)$$

where, ϕ is the solid volume fraction of nanoparticles, the heat capacitance of the nanofluids given is

$$(\rho Cp)_{nf}=(1-\phi)(\rho Cp)_{bf}+\phi(\rho Cp)_{sp} \quad (8)$$

The thermal expansion coefficient of the nanofluids can be determined by

$$\beta_{nf}=(1-\phi)(\beta)_{bf}+\phi\beta_{sp} \quad (9)$$

$$(\rho\beta)_{nf}=(1-\phi)(\rho\beta)_{bf}+\phi(\rho\beta)_{sp} \quad (10)$$

The thermal conductivity based on Maxwell-Garnett's (MG) model is given below:

$$\frac{k_{nf}}{k_{bf}}=\frac{k_{sp}+2k_{bf}-2\phi(k_{bf}-k_{sp})}{k_{sp}+2k_{bf}+\phi(k_{bf}-k_{sp})} \quad (11)$$

In terms of the stream function ψ and the vorticity ω, which are defined in the usual way as:

$$u=\frac{\partial\psi}{\partial y},\quad v=-\frac{\partial\psi}{\partial x} \quad (12)$$

$$\omega=\frac{\partial v}{\partial x}-\frac{u\psi}{\partial y} \quad (13)$$

Now, we introduce the following non-dimensional variables:

$$X=\frac{x}{L},\quad Y=\frac{y}{L},\quad \Omega=\frac{\omega L^2}{\alpha_{bf}}$$
$$\Psi=\frac{\psi}{\alpha_{bf}},\quad \theta=\frac{T-T_c}{T_h-T_c} \quad (14)$$

This then yields the dimensionless governing equations:

$$\frac{\partial^2\Psi_{nf}}{\partial X^2}+\frac{\partial^2\Psi_{nf}}{\partial Y^2}=-\Omega_{nf} \quad (15)$$

$$\frac{\partial\Psi_{nf}}{\partial Y}\frac{\partial\Omega_{nf}}{\partial X}-\frac{\partial\Psi_{nf}}{\partial X}\frac{\partial\Omega_{nf}}{\partial Y}=\frac{\mu_{nf}}{\mu_{bf}}\left[\frac{Pr}{(1-\phi)+\phi\frac{\rho_{sp}}{\rho_{bf}}}\right]$$
$$\times\left(\frac{\partial^2\Omega_{nf}}{\partial X^2}+\frac{\partial^2\Omega_{nf}}{\partial Y^2}\right)+\frac{\beta_{nf}}{\beta_{bf}}RaPr\left(\frac{\partial\theta_{nf}}{\partial X}\right) \quad (16)$$

$$\frac{\partial\Psi_{nf}}{\partial Y}\frac{\partial\theta_{nf}}{\partial X}-\frac{\partial\Psi_{nf}}{\partial X}\frac{\partial\theta_{nf}}{\partial Y}=\frac{(\rho Cp)_{bf}}{(\rho Cp)_{nf}}\frac{(k)_{nf}}{(k)_{bf}}$$
$$\times\left(\frac{\partial^2\theta_{nf}}{\partial X^2}+\frac{\partial^2\theta_{nf}}{\partial Y^2}\right) \quad (17)$$

$$\frac{\partial^2\theta_w}{\partial X^2}+\frac{\partial^2\theta_w}{\partial Y^2}=0. \quad (18)$$

where $Ra_{bf}=g\rho_{bf}\beta_{bf}(\bar{T}_h-T_c)L^3/(\mu_{bf}\alpha_{bf})$ is the Rayleigh number, and $Pr=\nu_{bf}/\alpha_{bf}$ is the Prandtl number for the base fluid.

The dimensionless boundary conditions of (15)–(18) are:

$$\theta_w(X,0)=0.5+\varepsilon\left[1-\cos(2\pi\lambda X)\right];\theta_{nf}(X,1)=-0.5$$

$$\partial\theta_w(0,Y)/\partial X=0;\partial\theta_{nf}(0,Y)/\partial X=0$$

$$\partial\theta_w(1,Y)/\partial X=0;\partial\theta_{nf}(1,Y)/\partial X=0$$

$$\theta_{nf}(X,D)=\theta_{nf}(X,D);$$
$$\partial\theta_{nf}(X,D)/\partial Y=K_r\partial\theta_w(X,D)/\partial Y \quad (19)$$

where $K_r=K_w/K_{bf}$ is the thermal conductivity ratio. The local Nusselt numbers along the hot and the cold horizontal walls can be defined as:

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$$Nu_h = hL\frac{k_w}{k_{bf}} = \left(-\frac{\partial \theta_w}{\partial Y}\right)_{Y=0} \tag{20}$$

$$Nu_c = hL\frac{k_{nf}}{k_{bf}} = \left(-\frac{\partial \theta_{nf}}{\partial Y}\right)_{Y=1} \tag{21}$$

Finally, the average Nusselt numbers can be defined as:

$$\overline{Nu}_w = \int_0^1 \left[-\left(\frac{k_w}{k_{bf}}\right)\frac{\partial \theta_w}{\partial Y}\right] dY \tag{22}$$

$$\overline{Nu}_{nf} = \int_0^1 \left[-\left(\frac{k_{nf}}{k_{bf}}\right)\frac{\partial \theta_{nf}}{\partial Y}\right] dY \tag{23}$$

III. Results and Discussion

In this section, we present numerical results for the streamlines and isotherms with various values of nanoparticle volume fraction ($0 \leq \phi \leq 0.2$), wave number ($0 \leq \lambda \leq 4$), thermal conductivity ratio ($0.44 \leq K_r \leq 6$), and other parameters fix at Rayleigh number ($Ra = 10^5$), nondimensional temperature ($\varepsilon = 0.4$), ratio of wall thickness ($D = 0.3$) and Prandtl number $Pr = 6.2$. The values of the average Nusselt number have been calculated for various values of λ and ϕ.

Fig. 2 shows the effects of the wave number on the streamlines (left) and isotherms (right) of waterCu at $Ra = 10^5$, $\varepsilon = 0.4$, $K_r = 1$ and $D = 0.3$. Fig. 2 (a) presents the effect of low wave number ($\lambda = 0.2$). on the streamlines and the isotherm patterns. The pure fluid flow structure appears with singular streamlines cell in the clockwise direction, while two streamlines cells occur for the nanofluid within the cavity. When the streamlines circulated as vortices in the clockwise direction (negative sign of Ψ), the strength of the flow circulation is denoted as $\Psi_{\min}$. By the nanofluid addition ($\phi = 0.05$), the thermal conductivity increases which leads to increase the strength of the flow circulation (see $\Psi_{\min}$ values). The convection heat transfer enhanced by the non-uniform heating; as such the isotherm patterns of the pure fluid tend to take almost a horizontal lines, while the nanofluid isotherm patterns appear with almost a curved lines. Increasing wave number up to 0.9 leads to increase the strength of the flow circulation. Significant change appears on the flow motion especially for the pure fluid, the pure fluid streamline cell tends to break into two cells, one in the clockwise direction close to the right wall and one in anti-clockwise direction near to the left wall. Clearly, the isotherm patterns are influenced by the increase of λ value, next to the horizontal, the isotherm patterns arise with irregular lines. At a higher λ value ($\lambda = 3$), the streamlines are significantly affected, the streamlines for the pure fluid are almost similar to that for the nanofluid. The strength of the flow circulation decreases due to the strong effect of the non-uniform heating. The isotherm patterns near to the bottom wall are transferred from irregular-shape to the spots-shape due to the high non-uniform heating, as displayed in Fig. 2 (c).

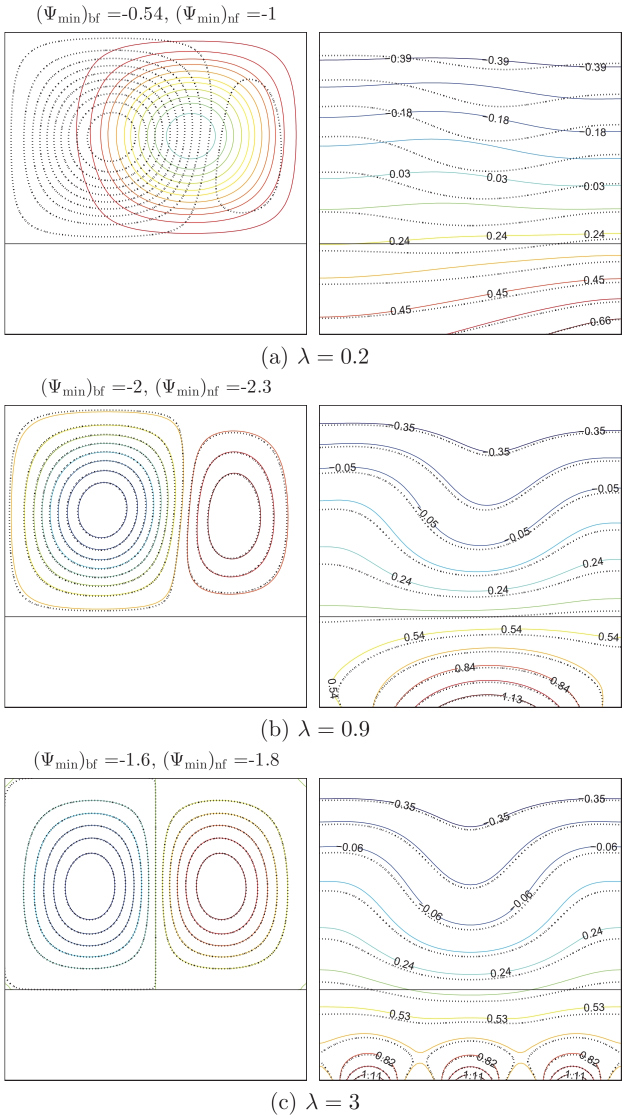

Fig. 2 Streamlines (left) and isotherms (right) evolution by wave number for $Ra = 10^5$, $\varepsilon = 0.4$, $K_r = 1$ and $D = 0.3$

Fig. 3 (a) illustrates the effect of nanoparticle volume fractions on the average Nusselt number with wave number for waterCu at $Ra = 10^5$, $\varepsilon = 0.4$, $K_r = 1$ and $D = 0.3$. We observe that the convection heat transfer is enhanced by the raise of wave number, the significant increasing appears for lower λ values ($\lambda \geq 2$). Increasing λ values leads to decrease and then increase the average Nusselt number which depicts the different enhancement of the effects of the spatial heating on the convection heat transfer from the uniform heating. Maximum values of average Nusselt number appear together with higher value of nanoparticle volume fractions due to the higher thermal conductivity of the nanoparticles. Fig. 3 (b) presents the effect of thermal conductivity ratio on the average Nusselt numbers with nanoparticle volume fractions at $Ra = 10^5$, $\varepsilon = 0.4$, $K_r = 1$ and $D = 0.3$. Due to the increase in convection by the addition of nanoparticles, the

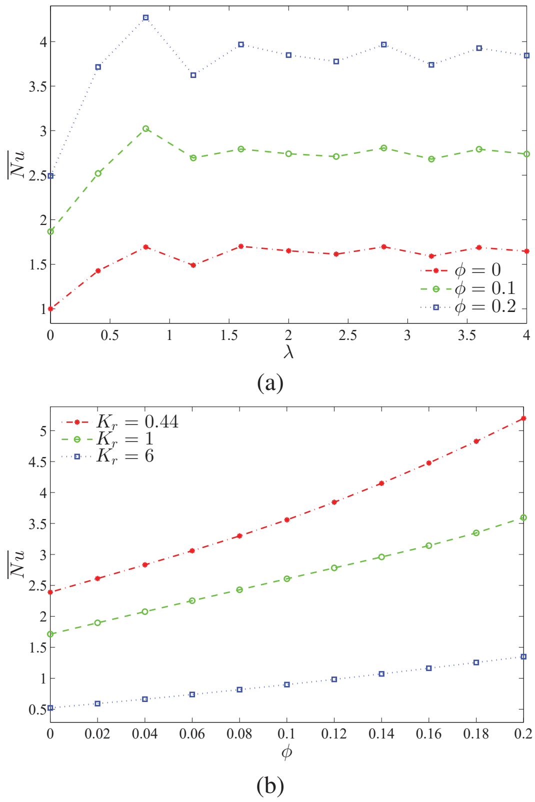

Fig. 3 (a) Variation of average Nusselt numbers interfaces with wave number for different nanoparticle volume fraction; (b) Variation of average Nusselt numbers interfaces with nanoparticle volume fraction for different thermal conductivity ratio

average Nusselt number increases. A significant enhancement in the convection heat transfer is obtained by lower thermal conductivity ratio $K_r = 0.44$, which leads to the maximum value of the average Nusselt number.

IV. Conclusion

The present numerical study considers the effect of spatial wall temperature variation on conjugate free convection in a square cavity filled with nanofluid and heated from below. The Finite Difference Method (FDM) has been used numerically to solve the dimensionless governing equations (15)-(18) subject to the boundary conditions (19). The streamlines are affected by applying the non-uniform heating, the pure fluid flow structure appears with singular streamlines cell in the clockwise direction, while two streamlines cells occur for the nanofluid within the cavity. A significant enhancement in the convection heat transfer is obtained by lower thermal conductivity ratio, which leads to the maximum value of the average Nusselt number.

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