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Topology Optimization of Structures with Web-Openings

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***Abstract*—**Topology optimization technique utilizes constant element densities as design parameters. Finally, optimal distribution contours of the material densities between voids (0) and solids (1) in design domain represent the determination of topology. It means that regions with element density values become occupied by solids in design domain, while there are only void phases in regions where no density values exist. Therefore the void regions of topology optimization results provide design information to decide appropriate depositions of web-opening in structure. Contrary to the basic objective of the topology optimization technique which is to obtain optimal topology of structures, this present study proposes a new idea that topology optimization results can be also utilized for decision of proper web-opening's position. Numerical examples of linear elastostatic structures demonstrate efficiency of methodological design processes using topology optimization in order to determinate the proper deposition of web-openings.

***Keywords*—**Topology optimization, web-opening, structure, element density, material.

I. Introduction

In general, when web-openings [1], [2] with varied shapes and sizes are placed in members of structure, structural stability of the members can be investigated through strain energy, i.e. stiffness of structure. Since these web-openings which have an influence on stiffness of structure can be defined as varied models with respect to shapes, number, size and so on, optimization techniques which yield maximal stiffness of structure under defined design conditions can be employed for appropriate web-opening's deposition in structure. Although shapes, numbers and sizes of web-openings have to be also considered in the structural design, optimal deposition of web-openings into members is the most serious interest in this study because shapes, numbers and sizes must be treated secondly after the determination of depositions of web-openings.

In this study, a topology optimization [3], [4] is utilized in order to decide optimal depositions of web-openings. Topology optimization of structures yields optimal topology as well as optimal shape in views of global structural systems. In discretization of continuous design domain, a density is defined as a material property of each element, i.e. an optimization design parameter. Therefore optimal shape and topology are represented by optimal density distribution contours which have maximal stiffness of structure.

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In this study, proper web-opening's depositions of linear elastostatic structures are investigated using density distribution method, i.e. SIMP [4]-[6] of topology optimization methods and efficiency of the proposed method is demonstrated.

II. Topology Optimization Problem

A. Optimization Problem

In design domain $\Omega_x \subseteq \Re^n$ (n =2) which dominates linear elastostatic structures, topology optimization problems are defined as follows.

$$\text{Minimize: } f = \left[-\frac{1}{2} \int_{\Omega_x} \varepsilon^T(\boldsymbol{u}) \boldsymbol{C}(\boldsymbol{E}) \varepsilon(\boldsymbol{u}) \, d\Omega_x \right] \tag{1}$$

$$\text{Subject to: } \int_{\Omega_x} \varepsilon^T(\boldsymbol{u}) \boldsymbol{C}(\boldsymbol{E}) \varepsilon(\delta \boldsymbol{u}) \, d\Omega_x = \int_{\Omega_x} \boldsymbol{b}^T \delta \boldsymbol{u} \, d\Omega_x + \int_{\Gamma_t} \boldsymbol{t}^T \delta \boldsymbol{u} \, d\Gamma_t \tag{2}$$

$$\int_{\Omega_x} d\Omega \le \boldsymbol{V}_0 \tag{3}$$

where, (1) denotes an objective function f, i.e. minimal strain energy or maximal stiffness. ε, $\boldsymbol{C}$ and $\boldsymbol{u}$ are respectively strains, material tensors and displacements. Equations (2) and (3) are optimization constraints. Equation (2) is equilibrium. $\delta\boldsymbol{u}$, $\boldsymbol{b}$ and $\boldsymbol{t}$ are virtual displacements, body forces and traction forces. Equation (3) is a volume constraint and $\boldsymbol{V}_0$ is the limit of feasible volumes in design domain.

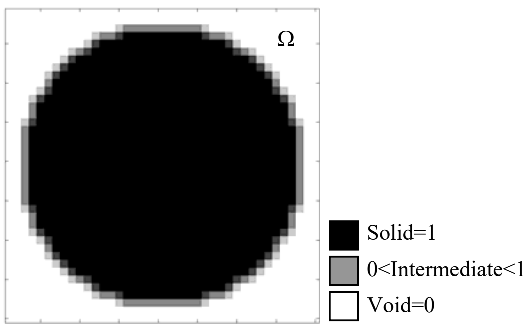

Fig. 1 Material density distributions

According to topology optimization problems defined as (1) and (2), optimal solutions are material density distribution contours with maximal stiffness. The material density values consist of void phases (0, white), solid phases (1, black) and intermediate phases (0<value<1, gray) in domain Ω as shown in Fig. 1.

B. Sensitivity Analysis

Since displacement fields depend on optimization design parameter s, a total sensitivity of objective function f in terms of s is written as a partial derivative [7] as follows:

$$\nabla_s \boldsymbol{f} = \nabla_s^{ex} \boldsymbol{f} + \overline{\nabla}_u \boldsymbol{f}^T \nabla_s \boldsymbol{u} \tag{4}$$

where, $\nabla_s^{ex} \boldsymbol{f}$ and $\overline{\nabla}_u \boldsymbol{f}^T \nabla_s \boldsymbol{u}$ denote an explicit and implicit partial derivative terms, respectively.

Suppose that in discrete processes, body forces $\boldsymbol{b}$, traction forces $\boldsymbol{t}$, differential tensor $\boldsymbol{L}$ and Jacobi matrix are independent of design parameter s, finally the total sensitivity formulation of objective function $\boldsymbol{f}$ is simply rewritten as follows:

$$\nabla_s \boldsymbol{f} = -\frac{1}{2}\boldsymbol{u}_i \int_{\Omega_i} \boldsymbol{B}_i^T \nabla_s \boldsymbol{C}_i(\Phi)\boldsymbol{B}_i \, d\Omega_i \; \boldsymbol{u}_i \tag{5}$$

where, $\boldsymbol{u}_i$, $\boldsymbol{B}_i$ and $\boldsymbol{C}_i$ are nodal displacement vector, operator matrix and material tensor of element $\boldsymbol{i}$.

III. Numerical Algorithm for Determination of Optimal Deposition of Web-Openings

The numerical algorithm for optimal depositions of web-openings using topology optimization results is shown in Fig. 2 and the procedures of deposition design of web-openings are as follows:

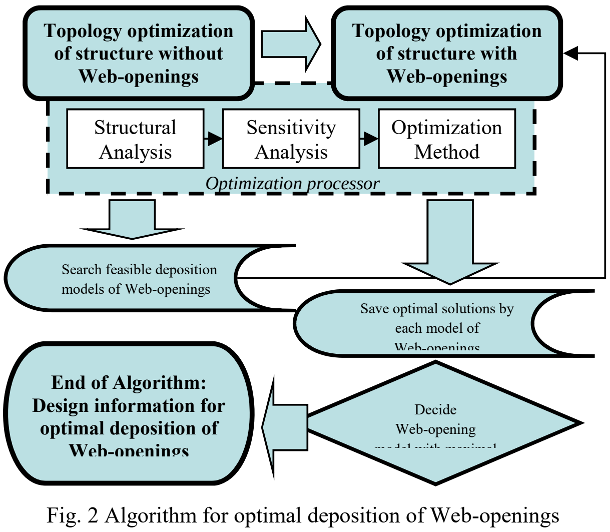

Fig. 2 Algorithm for optimal deposition of Web-openings

1. It is assumed that web-opening's shapes, numbers and sizes are not considered as variables of structural design. They might be treated in future works. Here, one square of 8×8 finite elements is only fixed as web-opening's geometry and number.

At first, topology optimization of nominal structure without web-openings is carried out. Through yielded material density distributions, void phases are searched and web-opening's deposition models are decided into the parts.

2. Topology optimization is executed according to each model of web-opening's depositions which is decided in the first step 1.
3. Seek a web-opening's model with the smallest strain energy of all optimal solutions obtained in previous step 2. It means that the structure with this model includes maximal stiffness and gets the greatest structural safety.
4. The optimal deposition result of web-openings is used as profitable information for web-opening's design.

The topology optimization processes are composed of structural analysis, sensitivity analysis and optimization method in turn. When the repetitive solution is converged during optimization procedures, all iterations are finished and optimal results are obtained. For structural analysis, finite element method is utilized and variational sensitivity using adjoint method [7] is applied. An OC method [8] of gradient-based concepts is used as the optimization method in order to reduce computational consumption of many design variables.

IV. Numerical Example

As a numerical test, a linear elastostatic structure with beam-to-column connection is considered. The geometry, boundary and loading conditions of the analytical model are shown in Fig. 3.

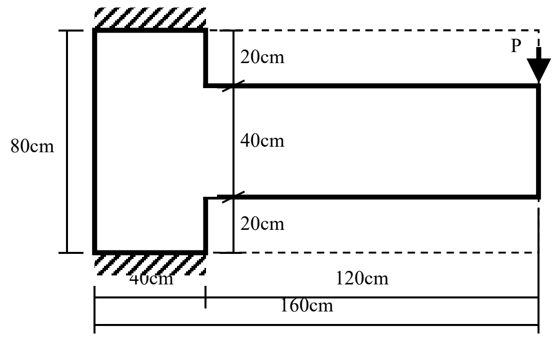

Fig. 3 Analytical model: a structure with beam-to-column connection

The design domain (160cm×80cm) of dash lines is discretized as finite elements of 80×40. As topology optimization problems, an objective function is minimal strain energy (N·m) and a volume constraint is restricted by 27% of total volumes. Nominal values of Young's modulus, Poisson's ratio and an external force are $\boldsymbol{E}_0 = 2.1\times10^6$ kg/cm², $\nu_0 = 0.3$ and $\boldsymbol{P}_0 = 360$ N, respectively. The penalty parameter for SIMP is $\boldsymbol{k} = 5.0$. In the structure with beam-to-column connection, one square of 16cm×16cm (finite elements of 8×8) is considered as design conditions of web-opening.

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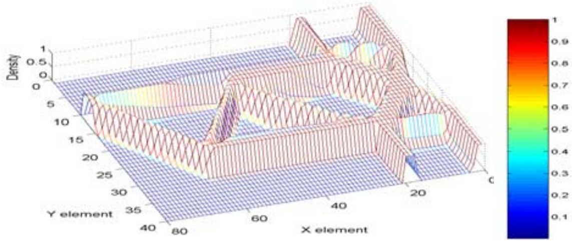
Fig. 4 shows results of topology optimization without considering web-opening. From Fig. 5, feasible models of web-opening's deposition can be investigated and they are (a)-(g) as shown in Fig. 5. The model (f) is infeasible but here it is considered for comparison with feasible models.

(a) Optimal material density distribution contours



(b) 3D optimal density function

Fig. 4 Optimal solutions of SIMP of a structure with beam-to-column connection without web-opening

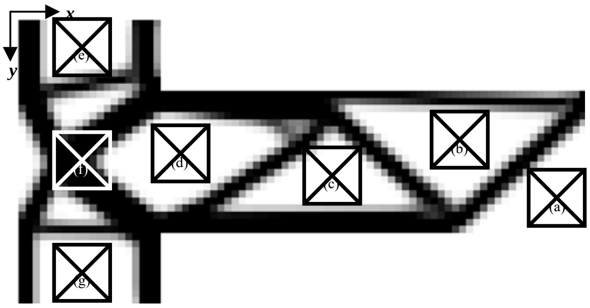


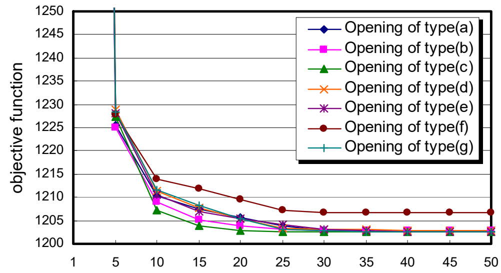
Fig. 5 Models (Types) of (a)~(g) of depositions of web-opening

Table I shows deposition coordinates of models of web-opening in design domain. Here, elx and ely denote x and y coordinates of elements.

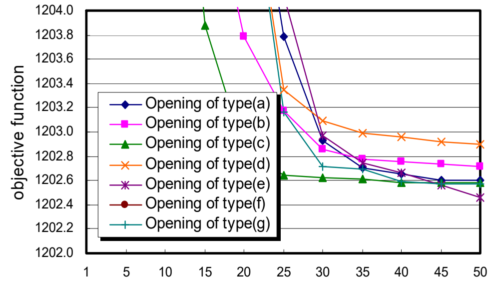
TABLE I
MODELS OF DEPOSITION OF WEB-OPENINGS

Model (Type)	elx	ely
(a)	73 ~ 80	22 ~ 29
(b)	58 ~ 65	14 ~ 21
(c)	41 ~ 48	19 ~ 26
(d)	20 ~ 27	16 ~ 23
(e)	6 ~ 13	1 ~ 8
(f)	6 ~ 13	17 ~ 24
(g)	6 ~ 13	33 ~ 40

The convergence histories of objective function in topology optimization of the structure with beam-to-column connection with each model of Fig. 5 and Table I are shown in Fig. 6. Figs. 6 (a) and (b) illustrate global and local convergence histories of objective function, respectively.



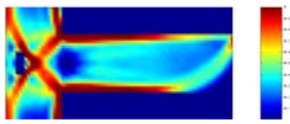
(a) iteration (global scale)



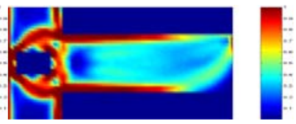
(b) iteration (local scale)

Fig. 6 Convergence histories of objective function in case of model (a) ~ (g) of web-opening

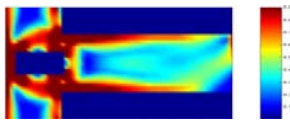
Fig. 7 shows changes of intermediate density distributions in beam-to-column connection with respect to size effects of web-opening.



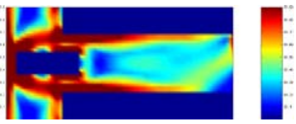
(a) web-opening of 16*14



(b) web-opening of 16*24



(c) web-opening of 16*34



(d) web-opening of 16*44

Fig. 7 Intermediate density distributions by size effects of web-opening

V. CONCLUSIONS

Structural optimization is a sequential and mathematical technique to archive the optimum of objective that is satisfied with defined constraints in structural problems. This method can optimize the design variables such as topologies, shapes, and sizes for optimal solutions. Especially topology optimization method that yields optimal topologies for the solution utilizes design variables of constant densities into finite elements and its solution is represented as optimal distributions of material densities between 0 and 1. It means that the positions with densities in design domain have to be occupied by materials for structural stiffness and there is no requirement of materials in regions where no densities exists. Therefore the void regions of topology optimization results can become design information for appropriate deposition of web-opening into which it has no material.

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