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Investigation of Stoneley Waves in Multilayered Plates

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Abstract**—**Stoneley waves are interface waves that propagate at the interface between two solid media. In this study, the dispersion characteristics and wave structures of Stoneley waves in elastic multilayered plates are displayed and investigated. With a perspective of bulk wave, a reasonable assumption of the potential function forms of the expansion wave and shear wave in *n*th layer medium is adopted, and the characteristic equation of Stoneley waves in a three-layered plate is given in a determinant form. The dispersion curves and wave structures are solved and presented in both numerical and simulation results. It is observed that two Stoneley wave modes exist in a three-layered plate, that conspicuous dispersion occurs on low frequency band, that the velocity of each Stoneley wave mode approaches the corresponding Stoneley wave velocity at interface between two half infinite spaces. The wave structures reveal that the in-plane displacement of Stoneley waves are relatively high at interfaces, which shows great potential for interface defects detection.

Keywords**—**Characteristic equation, interface waves, dispersion curves, potential function, Stoneley waves, wave structures.

I. Introduction

STONELEY waves, one kind of elastic interface waves existing at solid-solid interfaces, were first found by R. Stoneley in 1924, and the characteristic equation were further derived in two half space [1]. Stoneley waves provide a possible approach for non-destructive testing on defects or binding status at interfaces, and thus have drawn much attention since then. After the theories of the existence and uniqueness of Stoneley waves were demonstrated [2], [3], the effect of interface bonding status and media properties on basic properties of Stoneley waves has been studied thoroughly [4]-[6]. However, the investigation of Stoneley waves always remains in the range of the two half space.

With the development of society, application of multilayered structures has become an inevitable trend in the engineering field. As an interest and important research topic, guided waves propagation theories in multilayered structures have been widely explored for over a century, and have been reported in the public literatures [7]. Typically, two matrix techniques, the transfer matrix approach and the global matrix method, are always adopted to the solution of guided waves [8]. The mature theories of guided waves in multilayered structures guarantee and promote further application of guided waves in non-destructive testing fields. However, the interface defect, one most common and concern defect in multilayered structures, is still hard to be detected effectively.

Stoneley waves are sensitive to the interface defects [9], [10], whereas the difference and complexity of boundary conditions for Stoneley waves seriously hinders the development of the specific theories of Stoneley waves from two half space to multilayered structure. For further research on properties of Stoneley waves and promoting its application in multilayered structures, the theoretical solution of Stoneley waves in a multilayered structure is an imperative but challenging problem to be solved.

In this study, the characteristic equation and properties of Stoneley waves in an elastic multilayered plate are investigated. Taking a perspective of bulk wave [7], the potential functions of Stoneley waves in the *n*th-layer medium are determined, which realizes that the theories of Stoneley waves are expanded from two half space (given by R. Stoneley in 1924) to multilayered structures. Then taking three-layered plate as an example, the characteristic equation, dispersion curves and displacement distribution are presented and discussed.

II. Theoretical Solution

A. Model Definition and Description

An elastic multilayered plate is considered as shown in Fig. 1. It is noted that layer 1 and layer *N* are defined as two half space (*N* is the number of all layers). Each individual layer is identified by an integer n from 1 to *N*. It is assumed that every two contiguous layers are in rigid connection. The origin of coordinate system is set at interface 1. The downward and rightward directions are the positive directions of *z* and *x* axis, respectively. Thus *y* axis is vertical to and out of *x-z* plane based on Cartesian coordinate system.

The elastic multilayered plate can be analyzed by the assumption of plain strain, where elastic waves propagate along axis x and are unlimited distribution along *y* direction. The density and Lamé constants of nth layer material are separately defined as ρ^n 、 λ^n and μ^n, where λ and μ are real numbers for elastic layers and can be approximated to complex numbers for viscoelastic layers. The thickness of the *n*th layer is h_n (*n*=2, 3 … *N*-1). All field variables of the *n*th layer are denoted with a superscript *n* in analysis thereafter.

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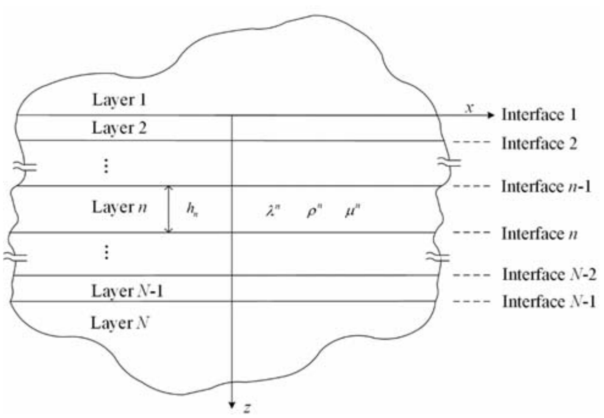


Fig. 1 Analysis model of the *N*-layered plate

B. Assumption of Potential Function

The displacement field of the *n*th layer should satisfy the Navier equation of motion, which is given as

$$\mu^n \nabla^2 u^n + \left(\lambda^n + \mu^n\right)\nabla\left(\nabla \cdot u^n\right) = \rho^n \frac{\partial^2 u^n}{\partial t^2} \tag{1}$$

where

$$\nabla = \frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k \; ; \; \nabla^2 = \nabla \cdot \nabla \cdot$$

In addition, the displacement field of the *n*th layer can be decomposed into two parts of the expansion wave and shear wave

$$u^n = \nabla \phi^n + \nabla \times \psi^n \tag{2}$$

where ϕ^n and ψ^n are the potential functions of the expansion wave and shear wave, respectively.

There are some assumptions that $\psi_x^n = \psi_z^n \equiv 0$, $\psi_y^n = \psi_y^n(x,z)$ and $\phi^n = \phi^n(x,z)$ for general plane strain problem, which can guarantee the displacement component u_y^n is zero, and the other two displacement components are function of the variables x and z. In this paper, the potential function ψ_y^n, nonzero component of ψ^n, is represented by ψ^n.

Since the energy of Stoneley waves is considered to concentrate at interfaces and exponentially decay away from interfaces, the potential function ϕ^n of the expansion wave and potential function ψ^n of the shear wave in each layer can be written as:

$$\begin{cases} \phi^1 = C_2^1 e^{k\alpha_1 z} e^{i(kx - wt)} \\ \psi^1 = C_4^1 e^{k\beta_1 z} e^{i(kx - wt)} \end{cases}; \tag{3}$$

$$\begin{cases} \phi^n = \left[C_1^n e^{-k\alpha_n (z - H_{n-1})} + C_2^n e^{k\alpha_n (z - H_n)}\right] e^{i(kx - wt)} \\ \psi^n = \left[C_3^n e^{-k\beta_n (z - H_{n-1})} + C_4^n e^{k\beta_n (z - H_n)}\right] e^{i(kx - wt)} \end{cases} (n=2, 3 \ldots N-1); \tag{4}$$

$$\begin{cases} \phi^N = C_1^N e^{-k\alpha_N (z - H_{N-1})} e^{i(kx - wt)} \\ \psi^N = C_3^N e^{-k\beta_N (z - H_{N-1})} e^{i(kx - wt)} \end{cases} \tag{5}$$

where k is real wave number; $\{C_1^n, C_2^n, C_3^n, C_4^n$ (n=1, 2 … N)$\}$ are unknown coefficients; $\alpha_n^2 = 1 - c^2/c_L^n$; $\beta_n^2 = 1 - c^2/c_T^n$.

C. Theoretical Dispersion Characteristics

Referring to derivation in [8], the displacement components and stress components should satisfy the continuity boundary conditions at each interface, thus there are eight homogeneous equations. These equations can be rewritten in the matrix form

$$\begin{bmatrix} A_{1,1} & A_{1,2} & \cdots & A_{1,8} \\ A_{2,1} & A_{2,2} & \cdots & A_{2,8} \\ \vdots & \vdots & \ddots & \vdots \\ A_{8,1} & A_{8,2} & \cdots & A_{8,8} \end{bmatrix} \begin{bmatrix} C_2^1 \\ C_4^1 \\ \vdots \\ C_3^3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}. \tag{6}$$

Setting the determinant of coefficient matrix be zero, the characteristic equation of Stoneley waves in a three-layered plate is given as

$$\begin{vmatrix} [A_1] & [A_2] \\ [A_3] & [A_4] \end{vmatrix} = 0 \tag{7}$$

where

$$[A_1] = \begin{bmatrix} 1 & \beta_1 & -1 & -e_1 \\ \alpha_1 & 1 & \alpha_2 & -\alpha_2 e_1 \\ \left(1+\beta_1^2\right) & 2\beta_1 & -g_1\left(1+\beta_2^2\right) & -g_1\left(1+\beta_2^2\right)e_1 \\ 2\alpha_1 & \left(1+\beta_1^2\right) & 2g_1\alpha_2 & -2g_1\alpha_2 e_1 \end{bmatrix};$$

$$[A_2] = \begin{bmatrix} \beta_2 & -\beta_2 e_2 & 0 & 0 \\ -1 & -e_2 & 0 & 0 \\ 2g_1\beta_2 & -2g_1\beta_2 e_2 & 0 & 0 \\ -g_1\left(1+\beta_2^2\right) & -g_1\left(1+\beta_2^2\right)e_2 & 0 & 0 \end{bmatrix};$$

$$[A_3] = \begin{bmatrix} 0 & 0 & e_1 & 1 \\ 0 & 0 & -\alpha_2 e_1 & \alpha_2 \\ 0 & 0 & g_2\left(1+\beta_2^2\right)e_1 & g_2\left(1+\beta_2^2\right) \\ 0 & 0 & -2g_2\alpha_2 e_1 & 2g_2\alpha_2 \end{bmatrix};$$

$$[A_4] = \begin{bmatrix} -\beta_2 e_2 & \beta_2 & -1 & \beta_3 \\ e_2 & 1 & \alpha_3 & -1 \\ -2g_2\beta_2 e_2 & 2g_2\beta_2 & -\left(1+\beta_3^2\right) & 2\beta_3 \\ g_2\left(1+\beta_2^2\right)e_2 & g_2\left(1+\beta_2^2\right) & 2\alpha_3 & -\left(1+\beta_3^2\right) \end{bmatrix};$$

$$e_1 = e^{-k\alpha_2 h_2}; \; e_2 = e^{-k\beta_2 h_2}; \; g_1 = \mu^2/\mu^1; \; g_2 = \mu^2/\mu^3$$

Given the material parameters (λ^n, μ^n, n=1, 2, 3) and geometric parameter (h_2), (7) could be an implicit function of ω and k. Instead of wave number k with phase velocity c

(where $k = \omega / c$), the dispersion relation of c and ω will be obtained.

Once the phase velocity of one Stoneley wave mode with the given frequency has been solved by (7), the corresponding unknowns coefficients (C_1^n, C_2^n, C_3^n, C_4^n) will be determined by (6). Thus the displacement components of each wave mode can be obtained explicitly by (2).

III. Results

The dispersion characteristic and wave structures are two key properties for guided waves to be investigated. In this section, the dispersion curves and wave structures of Stoneley waves in three-layered plate are presented and discussed. It is imperative to declare that both the two interfaces in the three-layered plate satisfy the existence criteria of Stoneley waves [1], [7].

A. Dispersion Curves

The numerical results of the dispersion curves defined by (7) can be solved by the graphical edge method [11], and the simulation results are implemented by ABAQUS/Explicit with good accuracy on an appropriate mesh. The comparison between theoretical and simulation dispersion curves of Stoneley waves in an Al-Steel-Ti plate is displayed in Fig. 2. The simulation curves are expressed as a dashed line and numerical curves are shown using a solid line. The material parameters and geometric parameters of the media involved in this paper are listed in Table I.

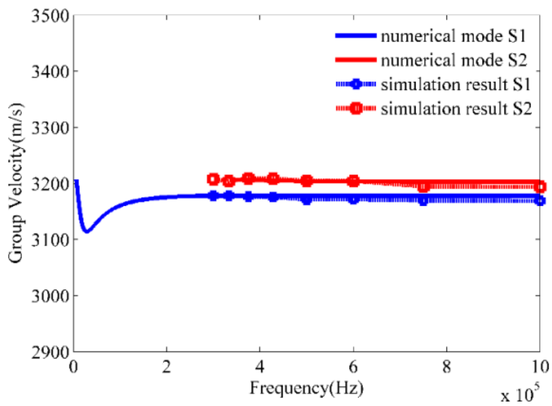


Fig. 2 Dispersion curves of Stoneley waves in an Al-Steel-Ti plate

TABLE I
The Material Parameters and Geometric Parameters of Media

Media	ρ(kg/m3)	E(Pa)	μ	C_L(m/s)	C_T(m/s)	h(m)
Al	2700	7.9e10	0.33	6584	3316	∞
Steel	7850	2.1e11	0.3	6001	3208	0.04
Ti	4500	1.25e11	0.33	6415	3231	∞

In Fig. 2, there are two Stoneley wave modes (which are termed S1 and S2 modes) in the three-layered plate. The conspicuous dispersion occurs on low frequency band, and the phase velocities tend to be constant on high frequency band. In addition, the velocities of two Stoneley waves modes approach the corresponding velocities of interface waves in two half space respectively (the velocity of Stoneley waves at interface between Al and Steel is 3178m/s, and that at interface between Steel and Ti is 3203m/s). The limiting velocity of each Stoneley wave mode only depends on the material parameters of consisting media at the interface and is independent of the geometric parameter.

The effect of the geometric parameter on dispersion curves of Stoneley waves is presented in Fig. 3. Two set of three-layered plates with identical material parameters and different thickness (0.04m, 0.02m) of the middle layer are considered. The solid line presents the dispersion curves with thickness h=0.04m, and the dashed line presents the dispersion curves with thickness h=0.02m. It is obvious that the range of conspicuous dispersion on low frequency band is mostly related to the thickness of the middle layer. The bandwidth where conspicuous dispersion of phase velocity exists, decreases with the thickness h. For example, the dispersion curve of Stoneley wave mode S1 with thickness h=0.04m approaches steady when the frequency is higher than 2.5MHz, and the dispersion curve with h=0.02m becomes essentially constant when the frequency is higher than 5MHz.

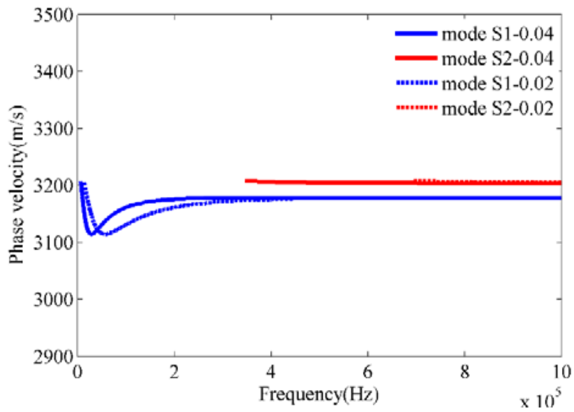
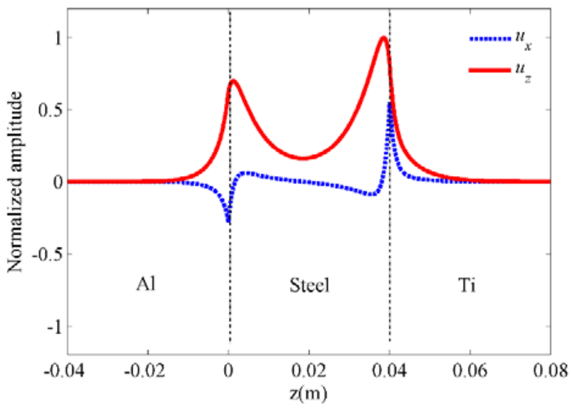


Fig. 3 Dispersion curves of Stoneley waves with the different thicknesses of the middle layer

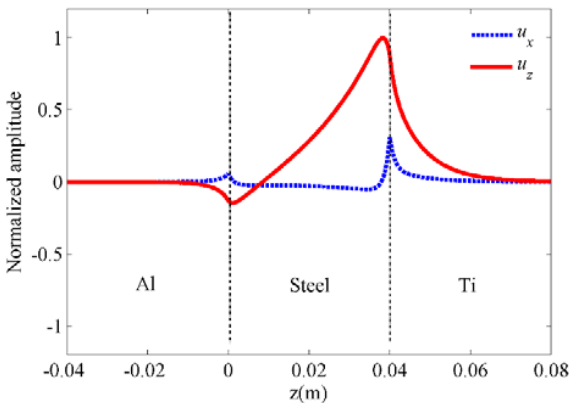
B. Numerical Wave Structures

Wave structures of guided waves reflect the particle displacement components distribution of guided waves in the structure, which is necessary for wave mode and excitation frequency optimum selecting in engineering application. For each Stoneley wave mode with certain phase velocity and frequency, the displacement components defined by (2) can be obtained. The displacement distribution curves of mode S1 and mode S2 with frequency 500 kHz are displayed in Fig. 4. The solid and dashed lines present the amplitudes of the normal and tangential displacement components (denoted u_z and u_x), respectively. The curves are normalized against the maximum amplitudes of two Stoneley wave modes.

Notice that in Fig. 4, the normal and tangential displacement components of each Stoneley wave mode are relatively high at two interfaces in the three-layered plate, and decay sharply with increasing distance away from each interface. The maximum amplitude always occurs in Steel medium and is very close to the interface. For each Stoneley wave mode, the difference between the amplitudes at two interfaces indicates the energy of Stoneley waves has an uneven distribution in the structure. Such wave structures provide Stoneley waves the outstanding performance in detecting interface defects and binding states than other guided waves with relatively low in-plane displacement at the concern interface.



(a)



(b)

Fig. 4 Wave structures in an Al-Steel-Ti plate. (a) Stoneley wave mode S1; (b) Stoneley wave mode S2

IV. Conclusion

In this study, the characteristic equation, dispersion curves and displacement distribution of Stoneley waves in an elastic multilayered plate are presented. Supposing the specific potential function forms of the expansion wave and shear wave in *n*th layer medium, the characteristic equation of the three-layered plate was given in a determinant form. The characteristic equation indicates that the phase velocity of Stoneley waves in three-layered plates depends on material and geometric parameters (λ , μ , h) of all layers and wave number k.

The dispersion properties of Stoneley waves in three-layered plates were investigated. Both the numerical and simulation dispersion curves were obtained, and the good agreement between numerical and simulation results demonstrates the correctness of theories of Stoneley waves in multilayered plates. There are two Stoneley wave modes in a three-layered plate. For mode S1, there is conspicuous dispersion on low frequency band, and the asymptotic phase velocity is 3178m/s (the velocity of Stoneley waves at interface between Al and Steel). For mode S2, the asymptotic phase velocity is 3203m/s (the velocity of Stoneley waves at interface between Steel and Ti). The asymptotic phase velocity of each wave mode only depend on the material parameters of all layers, and is independent of the geometric parameters. The geometric parameter (the thickness h) could produce a significant effect on the frequency bandwidth where the conspicuous dispersion exists.

The wave structures of Stoneley waves in three-layered plates were displayed subsequently. The wave structures reveal the in-plane displacement of Stoneley waves is relatively high at interfaces, and decay sharply with increasing distance away from each interface. The difference between the amplitudes of each Stoneley wave mode at two interfaces indicates the energy of Stoneley waves has an uneven distribution in the structure.

In this study, the dispersion properties and wave structures of Stoneley waves reveal the potential for Stoneley waves to interface defects detection in multilayered structures, which may be of value for further research.

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References

[1] R. Stoneley, “Elastic waves at the surface of separation of two solids,” *Proc. R. Soc. London*, vol. 106, pp. 416-428, 1924.
[2] J. G. Scholte, “The range and existence of Rayleigh and Stoneley waves,” *Mon. Not. R. Astron. Soc.*, vol. 5, pp. 120-126, 1947.
[3] K. Sezawa, K. Kanai, “The range of possible existence of Stoneley waves and some related problems,” *Bull. Earthq. Res. Inst. Tokyo Univ.*, 17, pp. 1-8, 1939.
[4] G. S. Murty, “A theoretical model for the attenuation and dispersion of Stoneley waves at the loosely bonded interface of elastic half space,” *Phys. Earth Planet. Interiors,* vol. 11, no. 1, pp. 65-79, 1975.
[5] M. A. Goda, “The effect of inhomogeneity and anisotropy on Stoneley waves,” *Acta Mech.*, vol. 93, p. 89-98, 1992.
[6] Vinh, Pham Chi, Giang, “Uniqueness of Stoneley waves in pre-stressed incompressible elastic media,” *Int. J. Non-Linear Mech.*, vol. 47, pp. 128-134, 2012.
[7] J. L. Rose, *Ultrasonic waves in solid media.* Cambridge university press, 2004, ch. 13.
[8] M. J. S. Lowe, “Matrix techniques for modeling ultrasonic waves in multilayered media,” *IEEE Trans. Ultrason. Ferroelectr. Freq. Control,* vol. 42, pp. 525-542, 1995.
[9] B. Li, L. Qiang, T. Lu, “A Stoneley wave method to detect interlaminar damage of metal layer composite pipe,” *Frontiers of Mechanical Engineering,* vol. 10, no. 1, pp. 89-94, 2015.
[10] B. Li, X. Geng, T. Lu, M. Li, “Experimental verification of the interface wave method to detect interlaminar damage of a metal multilayer structure,” *Frontiers of Mechanical Engineering,* vol. 10, no. 4, pp. 380-391, 2015.
[11] B. Li, P. Duan, L. Qiang, J. Zhuo, “A graphical edge method to solve dispersion Equation of Lamb waves,” *Proceedings of the First Symposium on Aviation Maintenance and Management-Volume I*, pp. 471-479, 2014.