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An Approach to Noise Variance Estimation in Very Low Signal-to-Noise Ratio Stochastic Signals

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Abstract**—**This paper describes a method for AWGN (Additive White Gaussian Noise) variance estimation in noisy stochastic signals, referred to as Multiplicative-Noising Variance Estimation (MNVE). The aim was to develop an estimation algorithm with minimal number of assumptions on the original signal structure. The provided MATLAB simulation and results analysis of the method applied on speech signals showed more accuracy than standardized AR (autoregressive) modeling noise estimation technique. In addition, great performance was observed on very low signal-to-noise ratios, which in general represents the worst case scenario for signal denoising methods. High execution time appears to be the only disadvantage of MNVE. After close examination of all the observed features of the proposed algorithm, it was concluded it is worth of exploring and that with some further adjustments and improvements can be enviably powerful.

Keywords**—**Noise, signal-to-noise ratio, stochastic signals, variance estimation.

I. Introduction

MUCH has been done in the field of signal denoising in recent years. A useful overview of the subject is provided in [1]. Knowing noise variance is a prerequisite for working with and denoising AWGN (Additive White Gaussian Noise) noisy signals. Common methods of noise variance estimation rely on linear modeling of stochastic signals [2]. This paper tries to make minimal number of assumptions on the original signal (e.g. linear structure) and aims to be general in the matter of application (speech signals, biomedical, etc.). The noise variance estimation algorithm proposed in this paper assumes stationarity of the original signal and its statistical independence with white Gaussian noise, and *zero-mean* Gaussian nature of noise. The algorithm shall be denoted by an abbreviation MNVE, which stands for Multiplicative-Noising Variance Estimation. Its name highlights the basic step of the method – multiplication of noisy signal and synthetic AWGN noise. Every noise estimation algorithm works better for greater values of signal-to-noise ratio. In the case of greater SNR (signal-to-noise ratio), the difference between signal and noise values is greater, and consequently, more accessible to algorithms for discriminating the main signal and noise sequence. However, MNVE shows considerable performance at very low SNR values which was explained by theory of random variables.

The following analysis of MNVE, as well as comparison to AR (autoregressive) modeling variance estimation, shows great expectations for the proposed algorithm. Its accuracy and wide range of signals it can be applied on make it worth of further research.

This paper consists of several key parts. First, theoretical background is presented justifying the algorithm steps; then the method itself is described. Next, simulation is performed on speech signals to confirm efficiency of MNVE. After discussion of the simulation results, a conclusion has been provided and a few proposals for future work were made.

II. Theoretical Background

Mathematical expressions describing stochastic signals shall provide theoretical background for deriving the proposed algorithm [3], [4]. The problem presented is estimating variance of zero-meaned noise random variable Ω, when it is added to the original random variable X, resulting in random variable Y.

$$Y = X + \Omega \quad (1)$$

Random variable M is also introduced. The basis of the method is calculating variance of Y previously corrupted by M as multiplicative noise, mathematically

$$Var[Y \cdot M] = Var[X \cdot M + \Omega \cdot M]. \quad (2)$$

Thus, (2) transforms to (3) after considering the property of variance operator *Var* that variance of the sum of random variables equals to sum of variances of these variables separately and double covariance of the variables.

$$\begin{aligned} Var[Y \cdot M] &= Var[X \cdot M] + Var[\Omega \cdot M] \\ &+ 2Cov[X \cdot M,\ \Omega \cdot M] \end{aligned} \quad (3)$$

Next, the assumption is made on statistical independence of variables X and M, which makes sense knowing that X is real recording and M is pseudorandom generated sequence. Variance of product of two independent variables can be calculated via

$$Var[A \cdot B] = E[A^2] \cdot E[B^2] - E[A]^2 \cdot E[B]^2, \quad (4)$$

where A and B are independent variables, and E operator represents expected value. Another assumption is that variable M is zero-meaned, written in (5), so the expression in (6) can be derived.

$$E[M] = 0 \quad (5)$$

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$$Var[Y \cdot M] = E[X^2] \cdot E[M^2] + Var[\Omega \cdot M] + 2Cov[X \cdot M,\ \Omega \cdot M] \quad (6)$$

Several conclusions can be derived through analyzing (6). The first term of the summation is linearly dependent on $E[M^2]$, which is, because of the zero mean, variance $Var[M]$. The second term is the variance of product of two random noise variables. They are both white Gaussian noise and zero-meaned. Intuitively, variance should be greater as the two variables differ more, i.e. come from distributions with greater difference of variance parameters. The actual algorithm uses this fact rephrased as following. The function $Var[\Omega{\cdot}M]$ should have a minimum in the case of

$$D[\Omega] \equiv D[M], \quad (7)$$

where operator D denotes the underlying distribution of the random variable as argument. In the case of normally distributed Ω and M, this means

$$\sigma_{\Omega}^{2} = \sigma_{M}^{2}, \quad (8)$$

i.e. variance parameters σ_{Ω}^2 and σ_M^2 of Gaussian distributions of variables Ω and M are equal.

The last term in (6) is interpreted as error of estimation method. It becomes greater as signal-to-noise ratio increases, that is, as underlying distributions of X and Ω, and thus, of $X{\cdot}M$ and $\Omega{\cdot}M$, are more distinct. This is why MNVE works better at low SNR. Namely, low SNR means X and Ω (original signal and noise applied to it) are more similar; hence, the estimation error is lower.

III. Method Description

The MNVE algorithm consists of several steps. Input data are samples of noisy signal y, and the output is noise variance estimate σ_{Ω}^2.

- *Step 1*: Signal y is normalized to unit variance and zero mean.
- *Step 2*: Zero-meaned Gaussian random noise signal is generated for different linearly distributed values of sample variance estimate. Each of these noise signals are multiplicated by y, after which the sample variance estimate of all product signals is calculated. As a result of this step, discrete function of variance with multiplicative noise variance as argument is taken.
- *Step 3*: Remove linear trend from the variance function.
- *Step 4*: Find multiplicative noise variance at which detrended variance function has minimum and multiply it by sample variance estimate of y (denormalization).
- *Step 5*: Steps 1 to 4 are repeated on corresponding surrogate signals instead of y.
- *Step 6*: Noise variance estimate is found as minimum of all minimums calculated in repeated steps 4.

Normalization in step 1 makes the algorithm general because if noisy signal has unit variance, linear change of variance in step 2 is always in range from 0 to 1. Also, zero-meaning renders the signal less nonstationary. The assumption of stationarity of X in deriving (6) is made, so in the case of input signal that has more prominent linear trend (or moving average) in time domain, zero-meaning yields in estimation accuracy.

Sample variance estimate of multiplicative noise signals and product signals (step 2) is the unbiased variance estimate *V;*

$$V = \frac{1}{N-1}\sum_{i=1}^{N}\left|A_i - \frac{1}{N}\sum_{i=1}^{N}A_i\right|^2, \quad (9)$$

where, N is length of vector variable A for which the estimate is calculated.

Detrending in step 3 is done to compensate for the linear term in (6). The method applied is subtracting the least-square fit of a straight line to the given signal (variance function).

Finding the minimum of variance function corresponds to finding the case described in (8). However, the estimate is not equal to the real value of noise variance because of the error term in (6). Namely, minimum of the variance function is not equal, but close to minimum of its second term. In order to partially compensate the error, repetition of previous steps is done on several surrogate signals of noisy original. Surrogates are generated by a simple method of random permutations of time series [5]. It was shown that the error makes an estimate slightly greater than real value, so compensation is done by taking the minimum estimate from all the surrogate cases (step 6).

IV. Simulation

A. Data and Simulation Features

For the evaluation of the algorithm, NOIZEUS (A noisy speech corpus for evaluation of speech enhancement algorithms [6]) database was used. Two noiseless IEEE sentences were chosen. Those are “The birch canoe slid on the smooth planks” and “He knew the skill of the great young actress”. Two nonsilent sections of 125ms (1000 samples) in length were extracted from these recordings and used as input signal for MNVE algorithm. Number of surrogates generated per noisy signal (step 5 of the algorithm) is set to 50. Fig. 1 shows time domain evolution of the two chosen speech signals *a* and *b*.

Noise is pseudorandom generated from zero-mean Gaussian distribution. Noise variance was previously calculated from sample variance of the original signal and set value of SNR. The formula for SNR used in these calculations is

$$SNR = 10\log_{10}\frac{V[x]}{V[\omega]}, \quad (10)$$

where x and ω are original (speech) signal and generated noise, respectively, and V operator represents unbiased sample variance estimate (9). Simulation was performed for a range of values of SNR on the same original recording, as well as for different multiplicative noise on the same recording and SNR.

Multiplicative noise was also pseudorandom generated from zero-mean Gaussian distribution. Variance of this noise is set in range 0.001 to 1 with step 0.01.

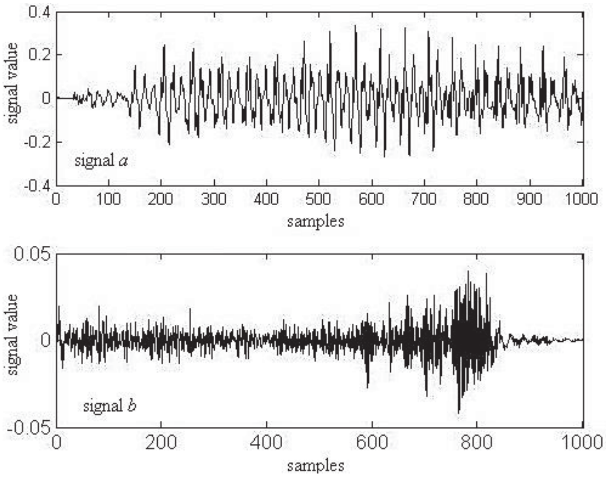

Fig. 1 Waveforms of two input speech signals *a* and *b*

For purpose of comparison, another noise variance estimation method was performed. It is the AR model (of order 10) estimation that include solving Yule-Walker equations by Levinson-Durbin recursion. Noise estimate is calculated from model coefficients by a simple formula although there are more complex and efficient methods, such as the one based on an overdetermined system modeling [7].

B. Simulation Results

Noise variance estimation was performed with proposed method and AR method on signals *a* and *b* for a set of SNR values. Vector of SNRs ranges from -2dB to 2dB with step of 0.2dB. The results can be observed on Figs. 2 and 3 for signals *a* and *b*, respectively.

Values yielded by MNVE are marked on graphs with asterisks and values yielded by AR modeling are marked with diamonds. "Real" values of noise variance, i.e. unbiased sample variance estimates of generated noise sequences are marked with circles.

In order to make a comparison between applied methods, a graph representing absolute errors of variance estimates is shown in Fig. 4. Two plots, upper and lower, correspond to signals *a* and *b*, respectively, whereas solid line denotes error of AR modeling method, and dotted line error of the MNVE.

Estimation procedure was repeated for 20 times on signal *b* for SNR equal to 1dB so that consistency of MNVE method could be checked. Results can be seen in Fig. 5. Again, asterisks mark MNVE estimates, diamonds AR estimates, whereas straight line denotes real value of noise variance.

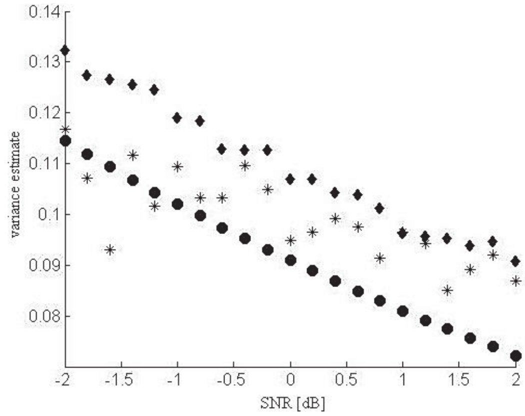

Fig. 2 Values of noise variance estimate on noised signal *a* using MNVE method (asterisks), AR method (diamonds), and unbiased noise estimate of generated noise signal (circles)

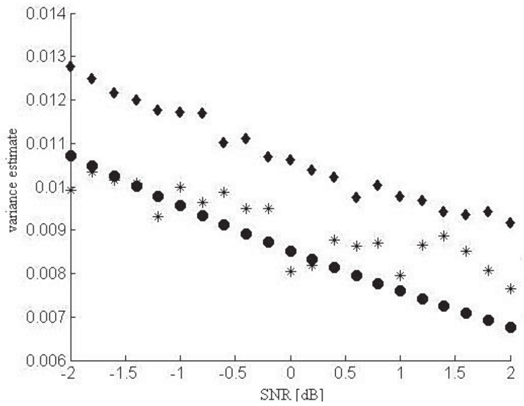

Fig. 3 Values of noise variance estimate on noised signal *b* using MNVE method (asterisks), AR method (diamonds), and unbiased noise estimate of generated noise signal (circles)

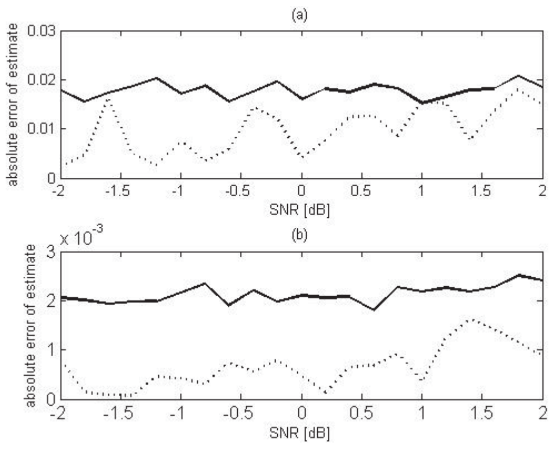

Fig. 4 (a) Absolute errors of noise variance estimates on noised signal *a* for AR method (solid line) and proposed method (dotted line) (b) Absolute errors of noise variance estimates on noised signal *b* for AR method (solid line) and proposed method (dotted line)

V. Discussion

Results of the simulation show the performance of the algorithm. It is clear that the noise variance estimate due to proposed method is closer to the real value than the estimate acquired by AR modeling. This is confirmed with absolute error values in Fig. 4. The only case where error of AR method (0.0152) is less is when SNR equals to 1dB, but it is not much smaller than the error of proposed method, which equals 0.0155. It was calculated that mean errors for signal *a* of MNVE and AR are 0.0097 and 0.0178, respectively, for the SNR range from -2dB to 2dB. For signal *b*, these are $0.66 \cdot 10^{-3}$ and $2.1 \cdot 10^{-3}$. Therefore, high precision is attributed to MNVE as the most promising feature.

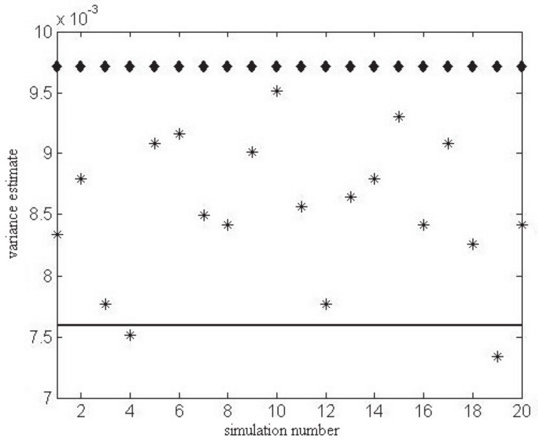

Fig. 5 Noise variance estimates on noised signals *b* with 20 different noise sequences at same SNR (1 dB) for AR method (diamonds) and MNVE (asterisks), and real variance value (line)

As opposed to AR method it can be observed that MNVE gives dispersed results. More linear correlation is present between AR estimates and real variances in Figs. 1 and 2 than between MNVE and real variances. These values were detrended to remove the influence of linear change of SNR, after which standard deviation is calculated on residuals. Results show 0.0018 deviation for AR, and 0.0051 for MNVE, for signal *a*, whereas for signal *b*in case of AR, deviation is $0.186 \cdot 10^{-3}$, and in case of MNVE $0.416 \cdot 10^{-3}$. In both cases the measure of dispersion of estimate is close to two times greater in MNVE. This can also be observed in Fig. 5 where MNVE was applied multiple times on the same noisy signal. Standard deviation of AR estimates is $1.78 \cdot 10^{-18}$, what can be considered a computational error, and deviation of MNVE estimates is $0.59 \cdot 10^{-3}$, which is considerably high comparing to real variance value of $7.6 \cdot 10^{-3}$. This denotes the fact that MNVE is not consistent in calculations. The explanation can be found in pseudorandom generation of multiplicative noise. Each generated sequence in repeated estimation algorithm can be different, which affects the product signal variance function. Surrogate signals also make a difference since the algorithm generates 50 of them in each execution and there exist much greater number of them (factorial of length of the signal – in case of signals *a* and *b* 1000!). However, this is not considered a major drawback because the accuracy of the method is still greater. It can even be enhanced by taking the mean of all results. In the case of simulation in Fig. 5, the overall noise variance estimate would be $8.5 \cdot 10^{-3}$.

If multiple randomizations of surrogate generation are taken into account, it can be inferred that execution of MNVE algorithm requires much time, which was confirmed during simulations in this paper. What can also be slowing down the estimation is repeating the algorithm to decrease the effect of estimation inconsistency. On the other hand, AR model technique is much faster. In order to find a tradeoff between speed and accuracy, number of surrogates and optionally number of repetitions in the MNVE algorithm must be chosen with great care.

VI. Conclusions and Future Work

In this paper it was shown that the proposed algorithm works well on speech signals. However, two facts should be emphasized in given analysis. First, speech signals are considered to consist of a deterministic and a stochastic part. Pure stochastic signals should also be checked for noise recognition by MNVE, such as stock market signals, biomedical signals, etc. The method was also proven to work with some biomedical signals but the corresponding results were not included in this paper due to difficulties of acquiring permission for publishing the usage of particular medical data.

Further work can be aimed towards improving the algorithm in the matter of speed. Particularly, more sophisticated algorithm for surrogate generation could be applied, or another method of removing error term in (6) may be derived. Also, different real background noise signal should be acquired for variance estimation.

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