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Finite Element Method for Modal Analysis of FGM

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Abstract**—**Modal analysis of a FGM plate containing the ceramic phase of Al_2O_3 and metal phase of stainless steel 304 was performed using ABAQUS, with the assumptions that the material has an elastic mechanical behavior and its Young modulus and density are varying in thickness direction. For this purpose, a subroutine was written in FOTRAN and linked with ABAQUS. First, a simulation was performed in accordance to other researcher's model, and then after comparing the obtained results, the accuracy of the present study was verified. The obtained results for natural frequency and mode shapes indicate good performance of user-written subroutine as well as FEM model used in present study. After verification of obtained results, the effect of clamping condition and the material type (i.e. the parameter *n*) was investigated. In this respect, finite element analysis was carried out in fully clamped condition for different values of *n*. The results indicate that the natural frequency decreases with increase of *n*, since with increase of *n*, the amount of ceramic phase in FGM plate decreases, while the amount of metal phase increases, leading to decrease of the plate stiffness and hence, natural frequency, as the Young modulus of Al_2O_3 is equal to 380 GPa and the Young modulus of stainless steel 304 is equal to 207 GPa.

Keywords**—**FGM plates, Modal analysis, Natural frequency, Finite element method.

I. Introduction

FGM products are as a certain type of advanced composite materials with heterogeneous microstructure [1] that was introduced for the first time in 1984 followed by materials scientists trying to obtain materials with high thermal resistance by Koizmi [2] in Japan. Practically instead of making all the pieces by valuable and sometimes expensive materials which have not good resistance to unfavorable boundary conditions such as excessive heat or high mechanical loads and etc., only more resistant and expensive materials can be used in areas near borders with adverse conditions along with coating other pieces by cheaper materials or proper physical properties. microscopic properties of FGMs changes in accordance with the changes in the distribution of materials and as a result, mechanical properties will change gradually, slowly and continuously in order of preference and in accordance with designing requirements including modulus of elasticity, density, coefficient of conductive heat transfer and other properties. This important feature that distinguishes these products from other substances makes to use from materials as are needed and not as they are. Mechanical properties of the material changes from one side of the structure to the other as needed. These gradual changes in the properties that are caused by changing the volume fraction of structural materials may be in one dimension or multiple dimensions. For example, in a FGM plate, the changes of properties can only be in line with thickness or it can be changed in another direction such as length in addition to the thickness direction in which, it is called the bi-directional FGM [3].

As important advantages of using these materials are that there are all the required properties (such as flexibility and heat resistance) without sacrificing for another one. Metals have ductility, high strength and machinability and on the other hand, ceramics have special properties such as high resistance to heat and corrosion. Other highlighted points of these materials are the possibility of optimizing the stress changes by appropriate change in the profile of structural material changes. Any rupture or separation is not observed at any point of the microstructure materials and the changes alter quite uniformly with an identified rate. Because there is no internal or boundary split within it, stress peaks in the FGM structures damp when an external force is applied to them and as a result, it prevents failure due to internal discontinuity and stress concentration and it has more effective mechanical properties compared to layered composite materials. The extensive use of these materials is because of continuity in the composition of its ingredients which distinguishes them from layered composite materials. These materials in comparison with the layered composite materials have the mismatch problem in mechanical properties at the junction of layers. Due to discontinuity of the mechanical properties in the borders of layers in layered composite materials, there is a possibility of separation of layers at high temperatures and growth of cracks at the junction of layers and spreading of them in the weaker sections of the material. Moreover, due to differences in thermal expansion coefficient of layers, the presence of residual stress is inevitable that these adverse effects in the FGMs have been resolved by continuous change of volume fraction and therefore the properties of matter; resulting in production of highly efficient materials that meet all the expected needs.

Generally, it can be said on structures with FGMs that these structures or materials are monolayer and multilayer unlike other composites and material properties in each layer are unique to that layer, these materials are monolayer, but the composition of the material is variable in this layer [4].

Common type of FGMs is made from a combination of ceramic and metal structural materials and usually by powder metallurgy. Change in metal and ceramic from one level to another is totally continuous, so that the surface of the plate that is exposed to high temperature is ceramic and other surface is pure metal. There is a continuous combination of

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both materials between two surfaces. Mechanical properties have also continuous changes in the thickness direction regarding the composition. The ceramic component in these materials shows high thermal resistance due to low heat transfer coefficient and high resistance to heat and on the other hand, the metal component makes it very flexible and prevents the crack and breakage of the material due to very high heat stresses. Another method of making FGMs is during an industrial process in which high-speed centrifugal casting forms thin layers with different density in the radial direction. Alternative method is by depositing layers of ceramic materials on the metal foundation.

Ramu and Mohanty performed modal analysis of FGM plates using finite element method in 2014. At first they coded a program in MATLAB software for FGM plates. Then some examples were solved by it and the results were compared with the results of available experimental data. They obtained natural frequencies of FGM rectangular plates for different boundary conditions. Finally, they stated that the results obtained by the finite element method were acceptably similar to the available experimental data [5].

Kadoli et al. conducted the analysis using higher-order shear deformation theory for static FGM beams. They obtained field of displacement based on the higher-order shear deformation theory by studying the static behavior of metal - ceramics beams at room temperature, and used the minimum potential energy principle for analysis and obtained the mechanical properties of the analysis for different volume percentages of metal - ceramic using exponential distribution resulting in two stiffness matrixes. One of the matrixes reflects the impact of normal rotation and another showed the effect of shearing rotations. They obtained numerical results for a central transverse bending and shear stresses on distributed loads of FGM for intermediate thickness of uniform beams for simple boundary conditions. They also evaluated the effect of different volume percentages of metal - ceramics on the bending and stress [8].

Zenkour investigated an extensive analysis of glazing and stresses, bending, free vibration and buckling of FGM plates in 2005. He presented a two-dimensional solution based on sinusoidal shear deformation theory for bending analysis. Boundary conditions were considered as simple and it was achieved critical load for buckling and several natural frequencies of vibration. He also reviewed the effect of the transverse shear deformation, the ratio of length to width of plates, ratio of width to thickness and volume fraction distribution on bending, tension and vibrations of plates [9].

Ng et al. investigated the effects of materials in FGM plates on parametric resonance of theses plates in 2010. For this purpose, they analyzed parametric resonance of rectangular FGM plates under harmonic forces. They formulated the problem based on the Hamilton’s principle. It was concluded from the research that the origin of unstable areas can be easily controlled by changing the amount of Power-low [10].

The natural frequencies and mode shapes of a piece can be determined using modal analysis. So, in this article, it was expressed how to perform modal analysis for a plate made up of functionally graded materials by ABAQUS finite element software. A sub-program was written in FORTRAN programming language and to model Young's modulus of FGM plates (which has a variable Young's modulus along with the thickness). Then the following sub-program written in FORTRAN would be linked to ABAQUS finite element software. Finally, this link into the ABAQUS software would be analyzed by the software to determine the natural frequency of FGM plate. Vibrational modes of FGM plate was also analyzed and reviewed with finite element method (ABAQUS software). For this purpose, the assumptions on the finite element model were expressed at first and how to analysis of the frequency of FGM plates and the results were explained finally.

II. Hypotheses

- The desired FGM plate was a combination of a ceramic material (Al2O3) and a metal (stainless steel 304); such that the lower layer of plate was from the steel and as moving upwards in the thickness direction, the percentage of steel has been increased and the percentage of ceramic material has been added and the upper layer was generally made of ceramic.
- The behavior of material was considered as elastic.
- Elastic properties of material (Young's modulus and density) were variable in line with plate thickness but Poisson's ratio was considered constant (equal to 0.3).

III. Modal analysis of FGM plate

The first step in modal analysis of FGM plate is geometric modeling of the plate and in this research, in order to be able to compare the results with the results of other researchers, the geometric dimensions of the plate were selected like as reference [5] that the plate had length and width of 1m and 0 thickness of 0.01m (Fig. 1).

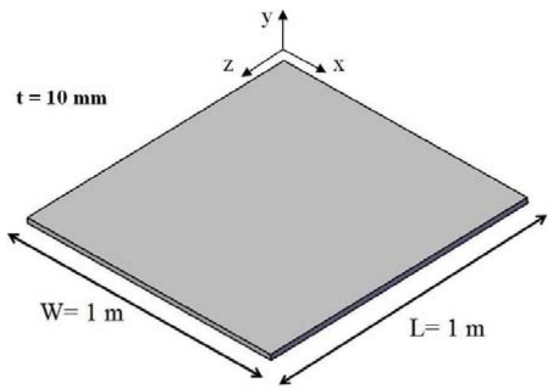

Fig. 1 Geometric dimensions of FGM plate

Mechanical properties of a FGM plate were defined after the geometric modeling of plates. As mentioned earlier, it was assumed that Poisson's ratio is fixed in the thickness direction but the Young's modulus and density are variable and a power function was used to define this variable properties, according to (1) [5]. Where V_c is volume fraction of ceramic material, V_m is volume fraction of steel material, h is thickness of the plate, E and ρ are the Young's modulus and density respectively, E_c and ρ_c are Young's modulus and density of ceramic material, E_m and ρ_m are Young's modulus and density of steel material, n is the power of power function and y is the distance from the geometric center of the plate (Fig. 2).

$$V_c(y) = \left(\frac{2y+h}{2h}\right)^n, \; V_m(y) = 1 - V_c(y) \\ E(y) = E_c * V_c + E_m * V_m \\ E(y) = \rho_c * V_c + \rho_m * V_m \qquad (1)$$

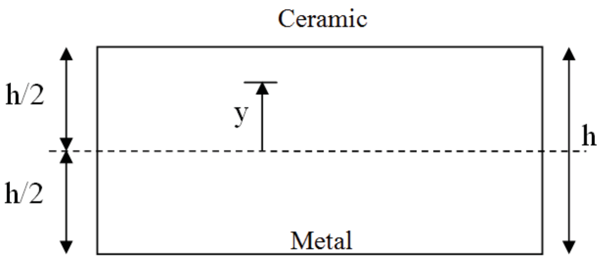

Fig. 2 Schematic representation of FGM plate

TABLE I
MECHANICAL PROPERTIES OF INGREDIENTS OF FGM PLATE [5]

Material	Properties		
	Young's modulus E (N/m^2)	Poisson's ratio ν	Density ρ (kg/m^3)
SUS304 stainless steel	$207*10^9$	0.3	8166
Al2O3 ceramic	$380*10^9$	0.3	3800

The values of the mechanical properties of the ingredients for FGM plate (SUS304 stainless steel and Al2O3 ceramic) has been selected from [5] (Table I). In addition, the type of analysis has been chosen and for the purpose, option of frequency analysis was selected from the linear chaos in Step modules of software and it was determined the number of modes of vibration and the maximum frequency. Boundary condition and the type of supports of the piece was defined that the type of clamped (CCCC) mode were selected in this study. Equation (2) shows boundary condition for this type of supports with respect to the coordinate system shown in Fig. 1.

$$Clamped\ mode\ (CCCC): \\ u = v = w = \theta_x = \theta_y = \theta_z = 0, at\ x = 0, W\ and\ z = 0, L \qquad (2)$$

where u, v and w are displacements in the direction of x, y and z axes. θ_x, θ_y and θ_z are also rotation around the x, y and z axes.

FGM plate was meshed after defining the boundary conditions. In this regard, the shape and type of elements was determined at first, so that the shape of elements was cubic and the type of elements was continuum 3D 8 nodded reduced integration (C3D8R). Finally, after the meshing of the workpiece, modal analysis was performed to determine the natural frequencies and mode shapes. In order to define the variable elastic properties, a sub-program was written in FORTRAN programming language and was linked with ABAQUS software. It is worth noting that summon of the sub-program written in FORTRAN programming language and linking it with ABAQUS software took place in this field (Job Module).

IV. RESULTS

One of the most efficient methods to verify the accuracy of results of a finite element analysis is to compare its results with the results of other researchers. The results of [5] was used for validation of the results in this thesis because of the impossibility of making FGM plate and experimental modal analysis; for this purpose, modal analysis was carried out in accordance with the parameters of [5] and the results were compared. For this purpose, a FGM plate composed of (SUS304 / Al2O3) in dimensions of 1 m * 1 m * 0.01 m was selected and has been analyzed in the cases of boundary condition clamped (CCCC) and it was obtained natural frequencies of the first four modes with the mode shapes. Results related to natural frequencies obtained from this study and its comparison with results of [5] are represented in Table II. Fig. 3 also shows first four vibrational modes obtained from this study in clamped mode. As mentioned in the introduction, the vibration mode shape shows the shape of the piece in the maximum amount of deformation and as a result, vibrations of different points of piece per resonance frequency (natural frequency).

According to Table II and Fig. 3, it can be seen a good agreement between the results and the results of [5], therefore, finite element analysis was performed in this study had sufficient accuracy for modal analysis of FGM plate and prediction of natural frequencies and mode shapes.

TABLE II
THE NATURAL FREQUENCY AT THE CCCC CASE FOR N = 1

Vibration mode		The first mode	The second mode	The third mode	The fourth mode
The natural frequency (Hertz)	Recent study	108	214	214	318
	Reference [5]	103	194	247	332

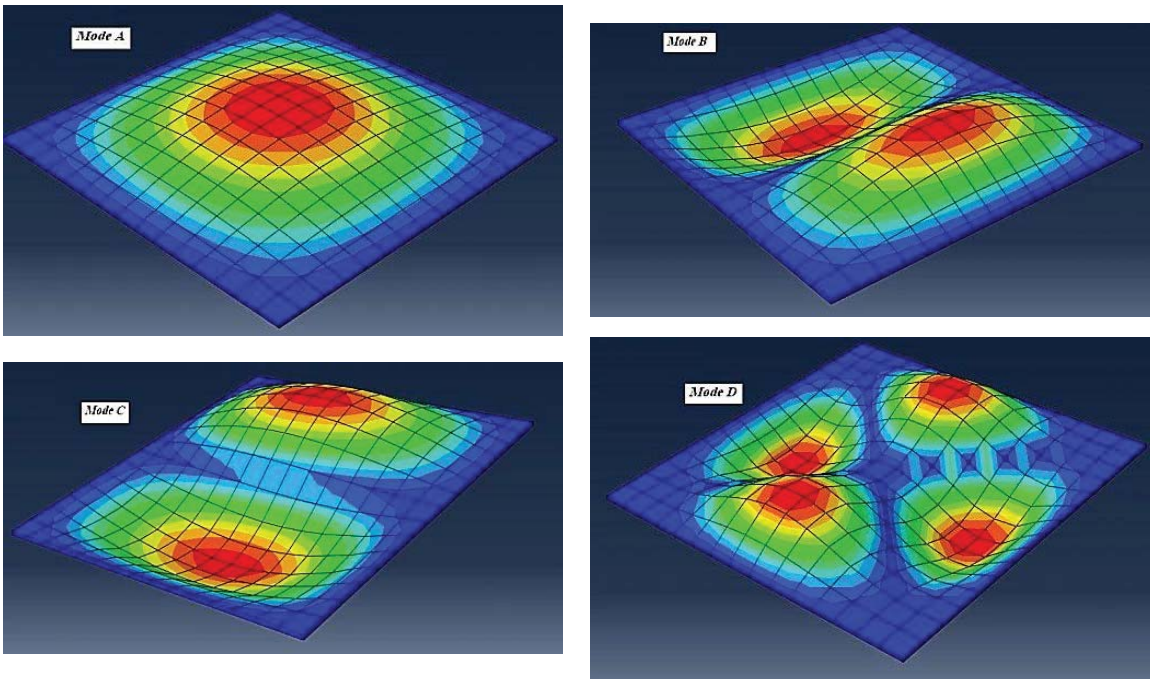

Fig. 3 The first four vibration modes at the CCCC case and n = 1

V. The Effect of N Parameter

The effect of power of the power function (n) on the natural frequency was evaluated that according to (1), n parameter specifies the influence of material, so that for n = 0, FGM plate is entirely of ceramic but whatever the value of n increases, the share of ceramics has been reduced and the share of steel material increases. The results of modal analysis for various n in boundary condition of CCCC at are summarized in Table III, and Fig. 4. The result is ϖ dimensionless frequency parameter that this parameter is usually used to express the natural frequency in the literature [6], [7]. Equation (3) shows how to calculate ϖ:

$$\varpi = \omega\sqrt{\frac{12\,(1-\nu^2)\rho_c L^2 W^2}{\pi^4 E_c h^2}} \qquad (3)$$

where ω is the natural frequency, L is the length of plate, W is width and h is its thickness. Other parameters have been described previously.

TABLE III
Dimensionless Frequency Parameter (ϖ) for CCCC in Different N

n		The first mode	The second mode	The third mode	The fourth mode
0 (ceramic)	Recent study	3.55	7.02	7.02	10.45
	Reference [5]	3.5	7.1	7.1	10.5
1	Recent study	2.27	4.50	4.50	6.68
	Reference [5]	2.2	4.7	4.7	7
5	Recent study	1.99	3.97	3.97	5.91
	Reference [5]	2	3.9	3.9	5.8
10	Recent study	1.78	3.53	3.53	5.25
	Reference [5]	1.8	3.6	3.6	5.1

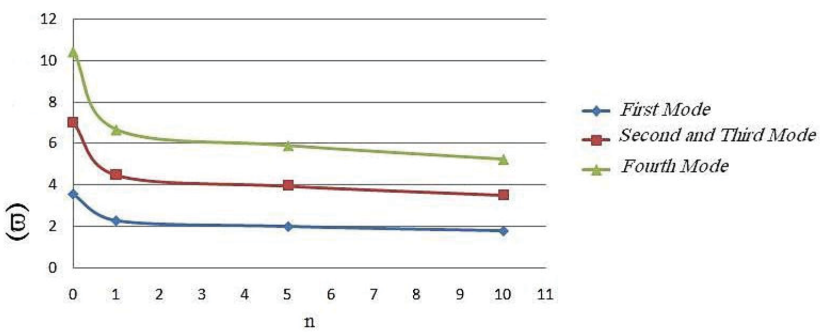

Fig. 4 Changes of ϖ for different values of n in the CCCC case

About the impact of n parameter which shows the effect of material on the natural frequency, it can be said that the natural frequency decreases with respect to the results shown in Fig. 4 with increasing n; because as stated previously, by increasing in the share of n, the share of ceramic phase in FGM plate decreases and share of steel phase increases leading to a reduction in stiffness of FGM plates and thereby reduce in the natural frequency. Because the Young's modulus of Al2O3 ceramic was 380 GPa and Young's modulus of SUS304 steel was 207 GPa. This is true in studies of other researchers [5]-[7].

VI. Conclusion

Due to the adverse effects of resonance phenomenon which led to large tensions and displacement resulting in disintegration of the components of a mechanical structure, modal analysis is necessary to calculate the mode shapes and values of natural frequencies and to prevent the occurrence of resonance phenomenon. In this thesis, modal analysis of a FGM plate consisting of Al2O3 ceramic phase and 304 stainless steel metallic phases was performed by ABAQUS software with the assumption that the behavior of the material is elastic and mechanical properties (Young's modulus and density) are variable in the thickness direction. Therefore, a sub-program was written in FORTRAN programming language and was linked with ABAQUS software. For modal analysis, a finite element analysis was carried out similar to other researchers at first and the accuracy of results was evaluated after comparison of results. Then it was evaluated the effect of material (n parameter) on the natural frequency. In this regard, finite element analysis was conducted for different values of n (0, 1, 5, 10) in clamped modes. A summary of the results of this research is the following:

- Comparison of natural frequencies and mode shapes of the study with the results of other researchers indicated conformity and resulted in high performance of program written in FORTRAN and high accuracy of finite element model used in this research.
- To compare the results, a dimensionless frequency parameter (ϖ) was defined as a function of the mechanical properties of FGM plate (density, Young's modulus and Poisson's ratio), geometric dimensions of the plate (length, width and height) and its natural frequency by software.
- The effect of n parameter that indicates the effect of material on the natural frequency, it can be said that the natural frequency decreases as n increases, because by increasing n, the share of ceramic phase in FGM plate has decreased and the share of steel phase decreased leading to reduce the stiffness of FGM plate and thereby reduce in the natural frequency. Because the Young's modulus of Al2O3 ceramic was 380 GPa and Young's modulus of SUS304 steel was 207 GPa.

VII. Offers

- Modal analysis of FGM plate by ANSYS software and comparison of the results with the results of ABAQUS software.
- Calculation of natural frequencies of FGM plate by analytic methods.
- Calculation of natural frequencies of FGM plate by coding the MATLAB program.
- Investigating the effect of different materials of FGM plate (different ceramic and metal phases).
- Modal analysis on various components of FGM such as beams, shells, braking disc and so on.

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