Generating Arabic Fonts Using Rational Cubic Ball Functions

Fakharuddin Ibrahim, Jamaludin Md. Ali, Ahmad Ramli

Abstract—In this paper, we will discuss about the data interpolation by using the rational cubic Ball curve. To generate a curve with a better and satisfactory smoothness, the curve segments must be connected with a certain amount of continuity. The continuity that we will consider is of type G¹ continuity. The conditions considered are known as the G¹ Hermite condition. A simple application of the proposed method is to generate an Arabic font satisfying the required continuity.

Keywords—Continuity, data interpolation, Hermite condition, rational Ball curve.

I. INTRODUCTION

CURVE reconstruction is a significant field in computer graphics, particularly for the curves which are not easily computed. The smoothness and accurateness of the curve are the main challenge in curve reconstruction technique. The Ball bases are one of the influent polynomial and important tool for interpolation of the curve. One of the most important properties of Ball bases is it formed a curve which is far from the middle control polygon and it helps in curve fitting. Control points and free parameters are used to control rational Ball interpolant [1].

Ball bases indicate two advantages when compared to Bezier curves. Firstly, there exists a robust time effective algorithm to evaluate the Ball Curve as compared to the evaluating Bezier Curve using the Casteljau algorithm [2]. Secondly, generalized Ball bases are said to suit much better in degree elevation and reduction [3]. This point is imperative when it comes to data transfer among Computer Aided Design (CAD) systems. To add, it also can handle conic sections in the form of rational cubic. Thus, the developed algorithm can be utilized for product design with numerical controls, NCs, whereby the curvature value of the interpolating points is controllable [4].

The Ball curve interpolation is the fixed interpolation which means that the shape of the interpolating curve is fixed for the given interpolating data and control polygon, since the interpolating function is unique for the control points. If the user wishes to modify the shape of the interpolating curve, the control points need to be changed.

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In geometric modelling, and computer graphics, rational parametric curve have gained attention due to its advantage/superiority. Especially because this rational curve can represent a conic curve and is good at generating a free curve. Rational curves are usually used in designing object shapes. For example, in designing boats and cars. In addition, complex shapes made from the piecewise rational curve are used because a single piece of the curve is not adequate. It is important to be aware of the continuity and smoothness from one piece to another when it is combined [5]. Therefore, in this paper work the discussion will be about the continuity and smoothness of pieces that are combined. The paper discussion is about getting the condition to connect two rational parametric cubic Ball curve. The commonly emphasized continuity is G^1 and the condition considered are G^1 Hermite condition.
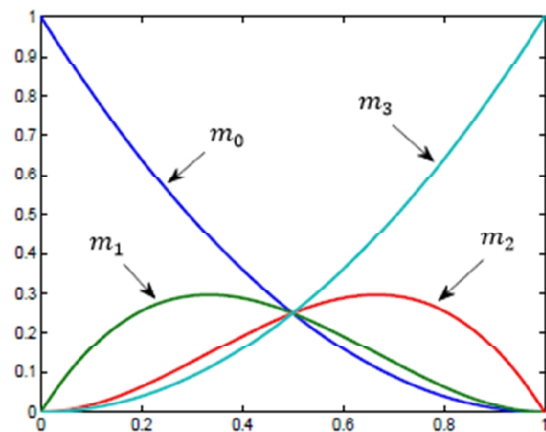

Fig. 1 Ball Basis Functions

II. BALL CUBIC

The Ball basis (see Fig. 1) for cubic polynomials on $[\,0,1\,]$ is given by:

$$m_0(t) = (1-t)^2, m_1(t) = 2t(1-t)^2, \ m_2(t) = 2t^2(1-t),$$
$$m_3(t) = t^2$$

where;

$$\sum_{t=0}^{3} m_i(t) = 1$$

and so we can write the equation of this Ball curve as:

$$P(t) = (1-t)^2 P_0 + 2t(1-t)^2 P_1 + 2t^2(1-t)P_2 + t^2 P_3.$$

The variable $m_i(t)$ are usually called blending functions and the locations P_i are known as control points. The blending functions, in the case of Ball curves, are known as Bernstein polynomials.

The properties of Ball curve are:
- Coordinate system independent,
- Obeys convex hull property, CHP,
- Obeys variation diminishing property, VHP,
- Symmetry,
- It takes invariant from under affine transformation,
- It interpolates the end points.

A. Rational Cubic Ball Curves

Rational Ball cubic curve is a parametric curve $R(t)$ which can be defined as

$$R(t) = \frac{(1-t)^2 P_0 + 2t(1-t)^2 w_1 P_1 + 2t^2(1-t)w_2 P_2 + t^2 P_3}{(1-t)^2 + 2t(1-t)^2 w_1 + 2t^2(1-t)w_2 + t^2)} \quad (1)$$

where $0 \leq t \leq 1$ and P_i are the control points of a cubic Ball curve. Beside the four control points P_i, a rational cubic Ball curve also controlled by the weights, w_1 and w_2 where $w_1, w_2 \geq 0$.

B. G^1 Hermite Condition

Beside the control points P_i, a rational cubic ball curve is also controlled by the respective unit tangent vectors, T_i. This paper is all about the interpolating points, P_i and its respective unit tangent vectors, T_i by using G^1 Hermite condition. The advantage of this approach is that one may control the direction of the Ball curve by only controlling the given value of unit tangent vector.

Let $P_0 = (x_0, y_o), P_1 = (x_1, y_1)$ as the end point and $T_0 = (m_0, n_0), T_1 = (m_1, n_1)$ as the tangents corresponding to it, then the rational cubic ball curve given as

$$R(t) = \frac{(1-t)^2 P_0 + 2t(1-t)^2 w_1 Q_1 + 2t^2(1-t)w_2 Q_2 + t^2 P_1}{(1-t)^2 + 2t(1-t)^2 w_1 + 2t^2(1-t)w_2 + t^2)} \quad (2)$$

$$t \in [0,1] \, a \quad w_1, w_2 \geq 0$$

$$R(0) = P_0 \text{ and } R(1) = P_1 ;$$

$$R'(0) = T_0 \text{ and } R'(1) = T_1$$

with the given condition, we get

$$Q_1 = P_0 + \frac{T_0}{2W_1} \text{ and } Q_2 = P_1 - \frac{T_1}{2w_2} \quad (3)$$

Fig. 2 shows the curve that we have when we change the tangent direction but still keep its weight as 0.5.
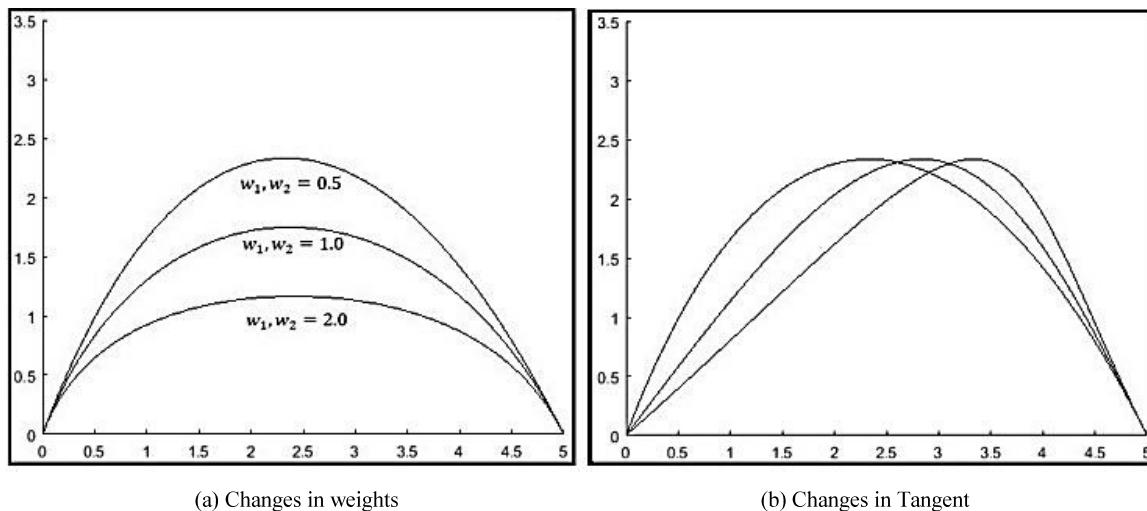

(a) Changes in weights

(b) Changes in Tangent

Fig. 2 Effects weight and tangent on rational ball curve

- Remark 1: If the weight is approaching infinity, the interpolation of Ball curve will form a straight line. When the weight is infinitely bigger, (2) will end up with two points left, which are P_0 and P_1, then connected as straight line by the Matlab.
- Remark 2: If the weight is zero, curve interpolation is not formed. When zero weight applied, the points Q_1 and Q_2 on (3) will be at infinity.

C. The Convex Hull Property of Ball Curves

The convex hull property is an important property for curve design; for any control polygon, the curve always lies in the convex hull of the control polygon [6]. The convex hull can be envisioned by pounding an imaginary nail into each control point, stretching an imaginary rubber band so that it surrounds the group of nails, and then collapsing that rubber band around the nails. The polygon created by that imaginary rubber band is the convex hull. Since all of the control points lie on one

side of an edge of the convex hull, it is impossible for those control points to lie on the other side of the line.

D. Continuity

There have several types of continuity which is parametric continuity and geometric continuity. C is referring to parametric continuity where G is geometric continuity. Two curve segments $P_{[t_0,t_1]}$ and $Q_{[t_1,t_2]}$ are said to be C^k continuous (or, to have k^{th} order parametric continuity) if

$$P(t_1) = Q(t_1), P'(t_1) = Q'(t_1), \dots, P^{(k)}(t_1) = Q^{(k)}(t_1)$$

Thus, C^0 means simply that the two adjacent curves share a common endpoint. C^1 means that the two curves not only share the same endpoint, but also that they have the same tangent vector at their shared endpoint, in magnitude as well as in the direction. C^2 means that two curves are C^1 and in addition that they have the same second order parametric derivatives at their shared endpoint, both in magnitude and in direction.

A second method for describing the continuity of two curves that is independent of their parametrization, is called geometric continuity and is denoted G^k. The conditions for geometric continuity (also known as *visual* continuity) are less strict than for parametric continuity. For G^0 continuity, we simply require that the two curves have a common endpoint, but we do not require that they have the same parameter value at that common point.

For G^1, we require that line segments $p_2 - p_3$ and $q_0 - q_1$ are collinear, but they need not be of equal length. This means that they have a common tangent line, though the magnitude of the tangent vector may be different. G^2 (second order geometric continuity) means that the two neighboring curves have the same tangent line and also the same center of curvature at their common boundary. Two curves which are C^n are also G^n, as long as equation below holds.

$$P(t_1) = Q(t_1), P'(t_1) = Q'(t_1), \dots, P^{(k)}(t_1) = Q^{(k)}(t_1) \neq 0$$

Second order continuity ($C^2 o$ G^2) is often desirable for both physical and aesthetic reasons. One reason that cubic NURBS curves and surfaces are an industry standard in CAD is that they are C^2. Two surfaces that are G^1 but not G^2 do not have smooth reflection lines. Thus, a car body made up of G^1 surfaces will not look smooth in a show room. Railroad cars will jerk where G^1 curves meet.

III. ALGORITHM TO GENERATE A G^1 BALL CURVE

G^1 data are a set of points and unit tangent vectors. In G^1 interpolation, the curve passes through the given points and matches given unit tangent vectors at the perspective points.
- Choose manually any computer application image and print it out.
- Coordinate the points from the edges of the image.
- Create the Ball curve code in MATLAB software corresponding to rational Ball curves and its continuity.

- Enter the points that had been coordinated from the image and run it out for the result.

IV. GENERATED G^1 ARABIC FONT

Convex hull already discussed in Section II as well as the G^1 Hermite condition in Section II enable us to generate various font designs such as letters or words using cubic Ball curves. The Arabic character "shin" in Figs. 3 and 4, Arabic character "ain" and words "sabar" in Fig. 6 are generated using the piecewise cubic ball function.
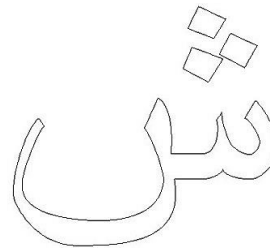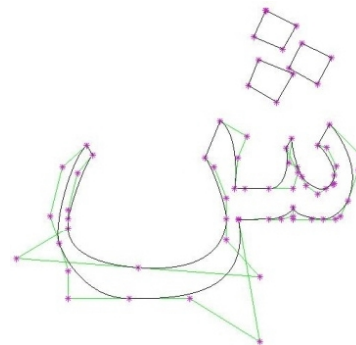

Fig. 3 Arabic letter "shin"

Fig. 4 Arabic letter without control polygon "shin" and its control polygon
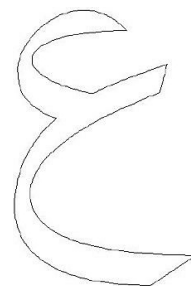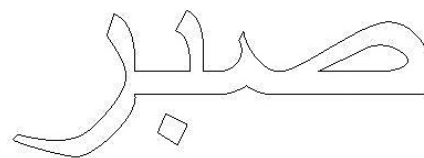

Fig. 5 Arabic letter "ain"

Fig. 6 Arabic words "sabar"

This algorithm also can be applied on any alphabets and any images or logo. Figs. 7 and 8 are showing that the logo batman and twitter that generated by using the piecewise cubic ball function.
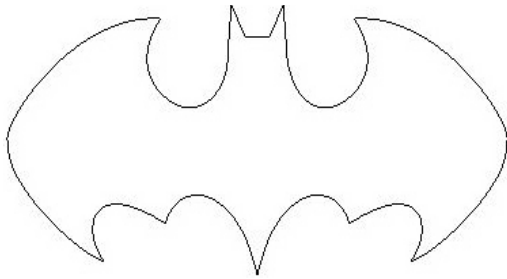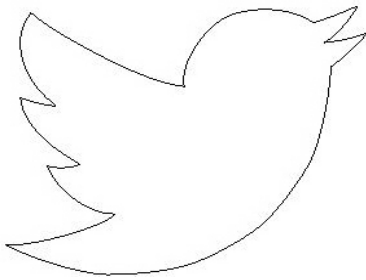

Fig. 7 Batman logo

Fig. 8 Twitter logo

V. CONCLUSION

In this paper, we have used the construction proposed by Allan Ball which was based on the construction of Ball curve to generate Arabic fonts which satisfy the G^1 Hermite condition. To get the desired curves, users must manipulate the tangent direction wisely. For the future research, the interpolation by rational Ball curves can be explored more. It is easy and interesting interpolation for those who love in modelling.

ACKNOWLEDGMENTS

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