

# Optical Flow Technique for Supersonic Jet Measurements

H. D. Lim, Jie Wu, T. H. New, Shengxian Shi

**Abstract**—This paper outlines the development of an experimental technique in quantifying supersonic jet flows, in an attempt to avoid seeding particle problems frequently associated with particle-image velocimetry (PIV) techniques at high Mach numbers. Based on optical flow algorithms, the idea behind the technique involves using high speed cameras to capture Schlieren images of the supersonic jet shear layers, before they are subjected to an adapted optical flow algorithm based on the Horn-Schnuck method to determine the associated flow fields. The proposed method is capable of offering full-field unsteady flow information with potentially higher accuracy and resolution than existing point-measurements or PIV techniques. Preliminary study via numerical simulations of a circular de Laval jet nozzle successfully reveals flow and shock structures typically associated with supersonic jet flows, which serve as useful data for subsequent validation of the optical flow based experimental results. For experimental technique, a Z-type Schlieren setup is proposed with supersonic jet operated in cold mode, stagnation pressure of 4 bar and exit Mach of 1.5. High-speed single-frame or double-frame cameras are used to capture successive Schlieren images. As implementation of optical flow technique to supersonic flows remains rare, the current focus revolves around methodology validation through synthetic images. The results of validation test offers valuable insight into how the optical flow algorithm can be further improved to improve robustness and accuracy. Despite these challenges however, this supersonic flow measurement technique may potentially offer a simpler way to identify and quantify the fine spatial structures within the shock shear layer.

**Keywords**—Schlieren, optical flow, supersonic jets, shock shear layer.

## I. INTRODUCTION

EXPERIMENTAL measurements in supersonic flows are always invaluable due to difficulties in numerical models that allow accurate predictions of flow transitions, turbulence and shock interactions with the shear layer, just to name a few. The need to develop increasingly more accurate experimental methods in supersonic flows arises naturally and is crucial in the pursuit of better understanding of supersonic flow phenomenon. Traditional techniques in determining fluid flow velocity involve employing specialized pitot pressure tubes or hot wire anemometry, which relies on relationships between

fluid flow velocity and certain properties such as pressure or temperature. While such techniques are suitable for most applications, they are ill-suited for supersonic flow measurements due to their intrusive nature or lack of robustness of said techniques. Strong possibilities of induced shock waves can occur, and velocity measurements obtained remain point-measurements. A more suitable non-intrusive measurement technique would be particle-image velocimetry (PIV), whereby significant progress in the past 20 years have made it possible to become the preferred technique for a wide range of different research applications. It is able to provide global velocity measurements, as well as avoiding generation of extraneous shocks. However, the selection of seeding particles and ensuring uniform distribution of such particles under supersonic flow conditions remain challenging. In the case of supersonic jets with strong shocks generated, there has always been a genuine concern over whether the seeding particles can track the air particles faithfully. While selection of smaller particles can reduce such problems, they tend to have lower light scattering efficiency and extremely powerful lasers are needed, which might not be pragmatic. Strong density changes associated with shocks also result in large refractive index variation, causing reflected light from seeding particles to undergo significant bending thus distorting any PIV images captured. Naturally, limitations imposed by seeding particles mean other measurement techniques such as laser Doppler velocimetry (LDV) which relies on seeding particles will face similar issues.

An experimental technique involving the use of optical flow algorithms with Schlieren images to obtain completely non-intrusive measurements of supersonic fluid flows is hereby proposed. This approach is able to avoid the intrinsic limitations associated with seeding particles, and offers full-field non-intrusive velocity measurements with high accuracy and spatial resolutions. Optical flow techniques first originated from the computer vision community and was introduced by Horn and Schunck [1] for rigid body motions. Consequently, optical flow techniques have been adapted to quantify subsonic flows by the fluid mechanics community. Later, [2] used optical flow for PIV to determine flow velocity of water flows based on a dynamic programming approach, thus introducing optical flow as a serious contender in place of correlation methods traditionally used in PIV techniques. Subsequently, [3] used the continuity equation to derive the governing optical flow equations for a few different experimental techniques, including Schlieren imagery. Reference [4] evaluated the suitability of optical flow in Schlieren imaging systems as compared to using cross-

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correlation methods in Schlieren imaging, and concluded optical flow having a much lower endpoint error for both laminar and turbulent subsonic flows. A similar research was also conducted by [5] whereby optical flow and cross-correlation methods on PIV images were compared, and optical flow was concluded to have higher accuracy and spatial resolution than cross-correlation techniques. Reference [6] used optical flow on cloud image pairs to showcase velocity jumps across shockwaves in supersonic flows. These literatures highlight the possibility of using optical flow measurements with Schlieren images for supersonic flow research. Nevertheless, there are certain limitations and challenges associated with the proposed experimental technique. Schlieren imagery relies on light refraction which is caused by changes in refractive index due to density variation. Hence, regions with constant density will be invisible and no information will be available. Turbulent flows are also problematic for optical flow estimation, as increased turbulence reduces the robustness of optical flow algorithm. As illustrated by [4], wrong choice of parameter values can lead to severe errors, and there is often a tradeoff between excessive smoothing and low robustness. Depending on the choice of penalty function, a non-convex minimization problem can result as well, whereby obtaining the solution is non-trivial. Furthermore, challenges of optical flow technique towards supersonic jet flows involve correctly differentiating and handling discontinuities due to boundaries and flow phenomenon, and quantifying complex shock interactions with the jet shear layer.

To obtain quantitative information of supersonic jets through optical flow technique, it is proposed that an experimental setup based on high-speed cameras and Schlieren systems be used. Optical flow algorithms based on the original Horn-Schunck approach will be adapted to process the Schlieren images obtained for the supersonic jets. To validate the experimental results, corresponding numerical simulations will be used to generate results for comparisons. The primary motivation behind implementing optical flow technique onto a supersonic jet flow stems from the interest in extending earlier works by one of the authors into the supersonic flow regime [7]-[17].

## II. EXPERIMENTAL METHODS

### A. Schlieren System

Schlieren imaging systems rely on principles of light refraction in inhomogeneous transparent medium, and the use of a knife-edge to create intensity patterns that characterize a particular flow field. Inherently a non-intrusive approach, the intensity pattern provides information on the first derivative of density of the test region, and Schlieren images used in conjunction with optical flow algorithms are able to provide quantitative information of the flow field. In this experimental setup, a Z-type schlieren arrangement is proposed to be used and this is illustrated in Fig. 1. For the present supersonic jet flows, they will be exhausting with a nominal Mach number of 1.5, operated in cold mode with stagnation pressure rated at 4

bar. High-speed single-frame or double-frame mode cameras will be used to capture successive Schlieren images. A circular de Laval nozzle will be used and its design is depicted in Fig. 2.

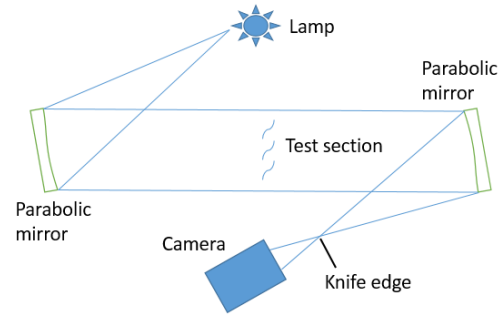


Fig. 1 Z-type Schlieren system arrangement

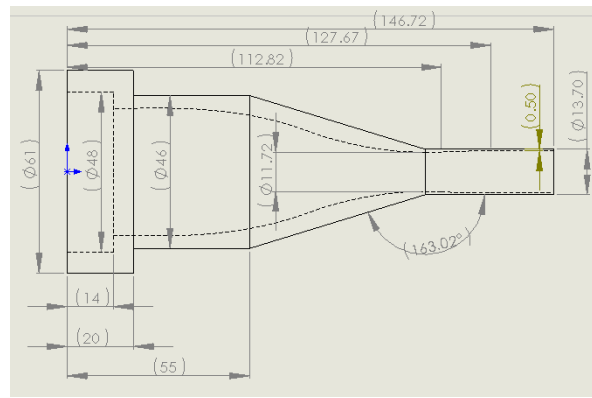


Fig. 2 Geometry of De Laval nozzle

### B. Optical Flow Algorithm for Velocity Determination

The reconstruction of image velocity from two consecutive image frames is coined as optical flow techniques by the computer vision community. Knowledge of image velocity and the use of perspective projection can then allow the true velocity of an object to be determined. An optical flow algorithm was first introduced by Horn and Schunck [1] from the computer vision community for rigid body motions. By assuming constant image intensity and smooth variation of the velocity field, the optical flow equation can be shown to be

$$\varepsilon_T^2 = \iint \underbrace{(E_x u + E_y v + E_t)^2}_{\text{Brightness constancy}} + \underbrace{\alpha^2 (\nabla^2 u + \nabla^2 v)}_{\text{Smoothness constraint}} dx dy \quad (1)$$

where  $E$  refers to image intensity, subscripts refers to the derivative in the corresponding direction, and  $\alpha^2$  refers to a weighted factor for the smoothness constraint. The brightness constancy term is usually known as the data term, while the smoothness constraint is usually known as the regularization term. Minimization of the above cost function will allow for velocity components  $u, v$  to be determined.

It is proposed that the algorithm to be developed for the present supersonic jet flow Schlieren images to be largely based on the work of [1], [18], [19]. The data term will be

based on a continuity equation which is better suited for fluid flow images as compared to the original brightness constancy data term. In order to account for large displacements, an integrated continuity equation will be used. The cost function for data term is given by [19]

$$F_1(d_i) = \iint f_1 \left( E(x_i + d_i(x_i), t + \Delta t) \exp(\nabla \cdot d_i(x_i)) - E(x_i, t) \right) dx_i \quad (2)$$

where  $d_i$  is the displacement of a point located at  $x_i$  from time  $t$  to  $t + \Delta t$  and  $f_1$  is the associated penalty function. Since this is a non-linear function, a successive coarse-to-fine multi-resolution scheme can be used for minimization. For the regularization term, a second-order div-curl regularizer will be used to allow for better estimation of vorticity and compressibility effects of fluid. The cost function for regularization term is given by [19]

$$F_2(d_i) = \iint f_2 \left( |\text{div}(d_i(x_i))|^2 + |\text{curl}(d_i(x_i))|^2 \right) dx_i \quad (3)$$

where  $f_2$  is the associated penalty function. In order to obtain the velocity estimates, the displacement field must first be obtained by minimizing the total cost function

$$F_1(\Delta d) + \alpha F_2(\Delta d). \quad (4)$$

Current progress of the optical flow algorithm involves replicating the work of [18], [19], and exploring the use of different data term, regularization term, penalty function and smoothness parameter.

### C. Significance

Schlieren imaging is inherently a non-intrusive method. This ensures the original flow field remains undisturbed, which is crucial in supersonic flow as formation of shock waves is highly sensitive to external disturbances. Seeding particles and image blurring issues highlighted earlier can also be avoided since Schlieren technique relies on light refractions to create intensity patterns. Optical flow technique has also proven to offer better accuracy and resolution than correlation techniques in PIV, with a spatial resolution of one velocity vector per pixel. This allows for detection of very fine spatial structures in highly turbulent supersonic flows. This is in contrast to correlation methods used in PIV, which require several seeding particles in an interrogation window to generate a velocity vector. Full-field unsteady flow information can also be obtained if multiple successive Schlieren images are available. This is very useful for supersonic flow investigations and potentially superior to point-measurement techniques such as pitot tubes or hot wire anemometry.

## III. NUMERICAL VALIDATION

In order to validate experimental results, numerical methods will be used. Reynolds Averaged Navier-Stokes (RANS) solutions for a circular nozzle have been generated through

ANSYS CFX. Three-dimensional structured mesh was generated using ANSYS ICEM-CFD. In order to reduce computational cost while achieving reasonable accuracy, a dense grid was generated for the jet core and shear layer, while relatively coarser grid was used for flow outside the shear layer and very downstream of the jet plume. The region of interest lies in the shear layer and jet core, thus justifying the use of a coarser mesh beyond these regions. Fig. 3 depicts the mesh density used while Fig. 4 shows the mesh for the entire domain. Inlet conditions used are: total temperature 300 K, velocity 19 ms<sup>-1</sup>, NPR 4 with reference to ambient pressure, and viscous wall for external and internal walls of nozzle. Design Mach number was set at 1.5 and the numerical results are shown in Fig. 5.

As seen in Fig. 5, the supersonic flow starts to expand as it emerges from the nozzle exit, indicating an underexpanded flow condition. Due to constant pressure along the jet boundary, the flow is bent towards the jet axis. Thus, expansion waves are sent back into the flow as compression waves, which coalesce to form the jet shock also known as intercepting shock. Between the two intercepting shocks a Mach disk can be observed. The intercepting shocks penetrate each other and continue to propagate towards the jet boundary. As the shocks reach the jet boundary, they are reflected off and continue to decay within the core of the jet flow.

In Fig. 6, the velocity profile along the jet axis was extracted. This offers an estimation for the velocity that is to be obtained from experimental techniques.

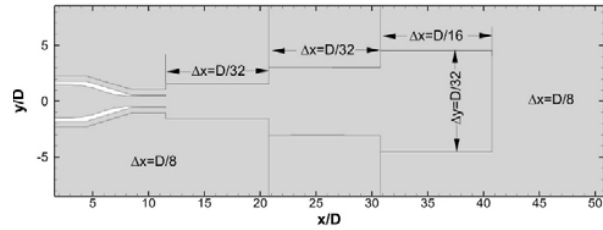


Fig. 3 Mesh density for nozzle

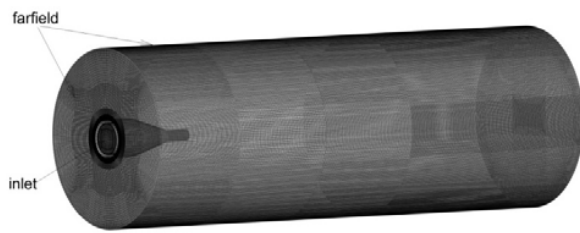


Fig. 4 Meshing for entire computational domain

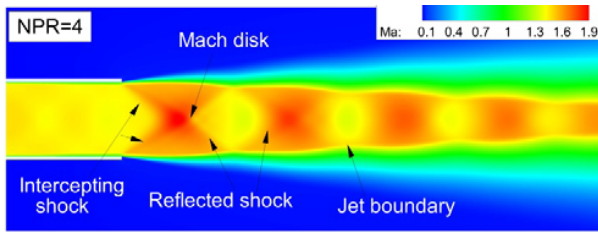


Fig. 5 Numerical results for NPR 4 circular jet

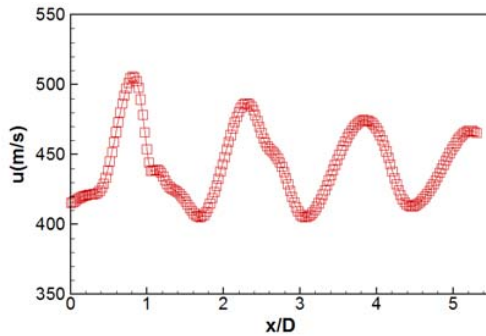


Fig. 6 Velocity profile along jet axis, downstream of jet exit

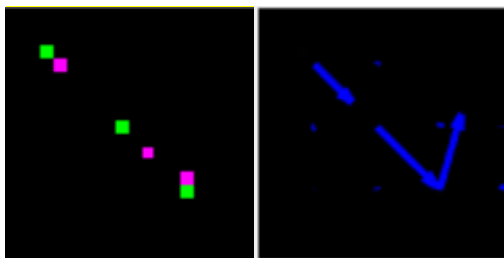


Fig. 7 (a) Moving blob (b) Associated velocity vector

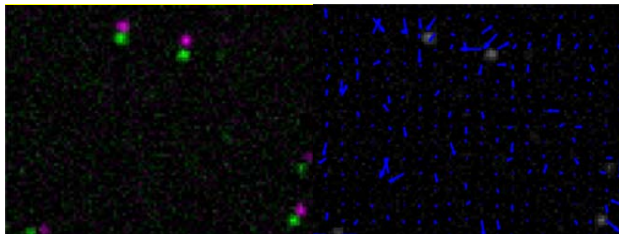


Fig. 8 (a) Olseen vortex (b) Associated velocity vector

#### IV. EXPERIMENTAL APPROACH

The original Horn-Schunck optical flow algorithm was adapted to include the continuity equation in its derivative form. Results from its imposition upon synthetic image pairs are shown in Fig. 7 and 8.

Results from the blob test in Fig. 7 are encouraging while the Olseen vortex test in Fig. 8 shows significant work remains to be done. The large errors observed in Olseen vortex test can be attributed to several reasons. Firstly, the data term which incorporates the continuity equation in its derivative form is unable to accommodate large displacements. Secondly, in order to improve robustness of the algorithm,

smoothing and range validation was conducted. It is possible that excessive smoothing was imposed and the range chosen for the range validation was not that accurate. The choice of discretization schemes and parameter values could have contributed to the large errors seen here as well. As mentioned earlier, the authors are currently working on replicating the work of [19], and subsequently intends to modify appropriate terms in the optical flow equation to achieve quantitative information regarding the supersonic jet velocity field.

#### V. CONCLUSIONS

Due to the formation of shock waves in a supersonic flow, current flow measurement methods such as PIV that uses seeding particles suffer from intrinsic limitations. To avoid such problems, an alternative approach of using Schlieren imagery with optical flow algorithm has been proposed. The present paper reports upon the progress and challenges encountered during its implementation, where numerical simulations of the supersonic jet flow concerned are used for benchmarking purposes. Development of optical flow algorithm for supersonic jets is at its early stages and there remains much work to be done to refine the algorithms. Nevertheless, once perfected, this new technique can avoid limitations faced by other experimental techniques and offer a viable alternative in quantitative supersonic flow measurements.

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