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A Bi-Objective Model to Address Simultaneous Formulation of Project Scheduling and Material Ordering

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Abstract**—**Concurrent planning of project scheduling and material ordering has been increasingly addressed within last decades as an approach to improve the project execution costs. Therefore, we have taken the problem into consideration in this paper, aiming to maximize schedules quality robustness, in addition to minimize the relevant costs. In this regard, a bi-objective mathematical model is developed to formulate the problem. Moreover, it is possible to utilize the all-unit discount for materials purchasing. The problem is then solved by the E-constraint method, and the Pareto front is obtained for a variety of robustness values. The applicability and efficiency of the proposed model is tested by different numerical instances, finally.

Keywords**—**E-constraint method, material ordering, project management, project scheduling.

I. Introduction

In traditional planning methods, project scheduling and material ordering issues were treated as separate problems. This approach yielded to neglect of trade-off consideration of the project corresponding costs, mainly including the ordering, holding, and penalty (reward payments) costs for late (early) project completion.

To the best of our knowledge, [1] introduced the integrated problem for the first time by presenting a hybrid model of the critical path method with material requirement planning. Afterwards, [2] developed an improved version of the problem by a heuristic scheduling for large-sized projects based on the least slack rule. Smith-Daniels and Smith-Daniels [3] addressed fixed duration for the activities and found that the latest starting time schedule could lead to an optimal solution. Their proposed objective function (OF) included minimization of total costs corresponding to the inventory holding, material ordering, completed activities holding, and project delay.

Dodin and Elimam [4] extended the problem by total costs minimization under activity crashing possibility, rewards for early completion, and materials quantity discounts. Schmitt and Faaland [5] considered a heuristic algorithm for scheduling a recurrent construction to the net present value maximization of cash flows, in which an initial schedule is constructed and worker teams are dispatched to the tasks for backlogged products. In another research, [6] used a genetic algorithm (GA) to solve an extended version of [4]. However, the crashing cost had been presumed to follow a constant slope for every activity.

Regarding the aforementioned notes, this paper aims to develop a mathematical model to address simultaneous planning of the project scheduling and material ordering. The proposed model incorporates the project execution costs minimization and schedules quality maximization, as well.

The rest of the paper is organized as follows. The mathematical model is described in Section II. The next Section III discusses the solution methodology and experimental results are presented in Section IV. Finally, the conclusions and future research directions are mentioned in Section V.

II. Problem Definition

This section addresses the problem definition, in addition to the mathematical formulation. The proposed model for simultaneous project scheduling and material ordering consists of two OFs aiming to maximize the schedule flexibility, in case of unexpected incidents, and minimize the corresponding costs to perform a project.

Different researches have addressed the schedules quality in terms of slacks time maximization. For instance, [7] accounted for different total slack (TS)-based measures in assessment of project scheduling robustness. They utilized the Monte Carlo Simulation to test robustness measures by generation of a random realization set of activity durations. They concluded that weighted slack and project buffer size could provide the best results in the case of disruptions. Equation (1) shows the selected weighted slack criterion, in which NS_j and TS_j stand for the number of successors and total slack of the jth activity, respectively. Considering the weighted slack maximization objective, the rest of the problem formulation is described next.

$$WS = \sum_{j=1}^{N} NS_j . TS_j \tag{1}$$

A. Mathematical Model Formulation

The mathematical model, the indices, parameters, and decision variables are introduced, as follows.

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TABLE I
INDICES

Parameters	Random distribution function
j=1,2,…,N	index of project activities
m=1,2,…,M	index of required materials
t=0,1,…,lN	index of time
k=1,2,…,Km	index of price discount ranges

TABLE II
PARAMETERS

Parameters	Random distribution function
P_j	Set of activities preceding *j*.
es_j	Earliest start time of activity *j*.
ls_j	Latest start time of activity *j*.
NS_j	Successor numbers of activity *j*.
TS_j	Total slack of activity *j*.
Dd	Project due date.
H	Project planning horizon.
Pe	Penalty amount paid for later completion of the project than the due time.
Re	Reward amount received for earlier completion of the project than the due time.
δ_{mk}	Unit cost of material *m* in quantity range *k* purchased from.
R_{jm}	Requirement amount of activity *j* to material *m*.
G_m	Ordering cost of material *m*.
h_m	Holding cost of material *m*.
d_j	Duration time of activity *j*.
α_{mk}	Limit on quantity range *k* of material *m*.
K_m	Number of quantity discount ranges for material *m*.
L_m	Lead time of material *m*

TABLE III
DECISION VARIABLES

Parameters	Random distribution function
x_{jt}	1 if activity *j* is started at time *t* and 0, otherwise.
λ_{mkt}	1 if material *m* is ordered within quantity range *k* in period *t* and 0, otherwise.
P_{mktj}	1 if material *m* of activity *j* is ordered in period *t* within quantity range *k* and 0, otherwise.
I_{mt}	Inventory amount of material *m* in period *t*.

Now, the proposed model can be formulated according to the following mixed-integer programming model.

$$Max \quad \sum_{j=1}^{N} NS_j . TS_j$$
$$Min \quad \sum_{t=Dd}^{H} Pe(t-Dd)x_{Nt} - \sum_{t=es_N}^{Dd} \mathrm{Re}(Dd-t)x_{Nt} \tag{2}$$
$$+\sum_{m=1}^{M}\sum_{k=1}^{K_m}\sum_{t=1}^{H-L_{ms}-d_j+1}\sum_{j=1}^{N}\delta_{mk}.R_{jm}.P_{mktj} + \sum_{m=1}^{M}\sum_{t=1}^{H-L_{ms}-d_j+1} G_m \sum_{k=1}^{K_m}\lambda_{mkt}$$
$$+\sum_{m=1}^{M}\sum_{t=1}^{H-1} h_m . I_{mt}$$

S.t:

$$\sum_{t=es_i}^{ls_i} t.x_{it} + d_i \le \sum_{t=es_j}^{ls_j} t.x_{jt} \quad ;\forall i \in P_j \tag{3}$$

$$\sum_{t=es_j}^{ls_j} x_{jt} = 1 \quad ;\forall j \in 1,2,...,N \tag{4}$$

$$I_{mt} = I_{m(t-1)} + \sum_{k=1}^{K_m}\sum_{j=1}^{N} R_{jm} P_{mk(t-L_m)j} - \sum_{j=1}^{N}\sum_{\tau=Max(t-d_j+1,es_j)}^{Min(t,ls_j)} R_{jm}\, x_{j\tau} \quad ;\forall m = 1,2,...,M, \forall t = 1,2,...,l_N \tag{5}$$

$$\alpha_{m(k-1)}\lambda_{mkt} \le \sum_{j=1}^{N} R_{jm} P_{mktj} \le \alpha_{mk}\lambda_{mkt} \quad ;\forall m = 1,2,...,M \quad \forall t = 1,2,...,H, \forall k \in 1,2,...,K_m \tag{6}$$

$$\sum_{k=1}^{K_m} \lambda_{mkt} \le 1 \quad ;\forall m = 1,2,...,M, \forall t \in 1,2,...,H \tag{7}$$

$$\sum_{k=1}^{K_m}\sum_{t=1}^{H-1} P_{mktj} = 1 \quad ;\forall j = 1,2,...,N, \forall m \in 1,2,...,M \tag{8}$$

$$\sum_{k=1}^{K_m}\sum_{t=1}^{H-1} t.P_{mktj} + L_m \le \sum_{t=es_j}^{ls_j} t.x_{jt} \quad ;\forall j = 1,2,...,N, \forall m \in 1,2,...,M \tag{9}$$

$$x_{jt}, \lambda_{mkt}, P_{mktj} \in [0,1], \quad I_{mt} \ge 0 \tag{10}$$

The OFs are written by (2), in which the former and latter point to the schedule robustness maximization and project costs minimization, respectively. The encompassed costs consist of the completion time, materials purchasing, ordering, and holding, respectively. Equation (3) guarantees the precedence relations to schedule a project. Equation (4) reiterates that start time of each activity is embedded within the earliest and finish start bounds. The inventory level of each material is calculated through (5). Equation (6) indicates discount quantity intervals for each material. Equation (7) states that the order can be accommodated just in a single discount interval. Equation (8) promises that just one order is put for each activity regarding the required materials. The materials lead time is also taken into consideration by (9), which binds an activity to start after the arrival of its required resources. Finally, (10) reveals the domain of decision variables.

III. SOLUTION METHODOLOGY

The concept of multi-objective optimization has been addressed at the outset. Afterwards, the $\varepsilon-$constraint method is taken into account as the solution approach.

The Pareto dominance and Pareto front constitute the two fundamental concepts of a multi-objective optimization. Accordingly, a Pareto front of solutions is found for the problem, instead of a unique optimal solution. The front includes a set of non-dominated Pareto solutions. In a broader sense, a solution $x = [x_1, x_2, ..., x_n]$ dominates solution $y= [y_1, y_2, ..., y_n]$ if and only if y is not better than x for any objective $i = 1,2,...,n$, and there exists at least one objective, x_i, better than the corresponding objective for y, in which n represents the number of the objectives. On the contrary, two solutions x and y are non-dominated if none of them dominates the other one.

A. $\varepsilon -$ Constraint Method

This method can transform a multi-objective problem into a single-objective one with additional constraints, where the objective with the highest priority is maintained as the OF and the others are transformed into the constraints. For instance, application of the $\varepsilon -$constraint method can be written through (11), as follows, for the given problem. As can be seen, W_2 has been remained in the OF, representing the schedule costs, and W_1 has been transformed into the constraints. Model (11) equations consist of (3)-(10).

$$Min \;\; W = W_2$$

S.t.

$$W_1 \le \varepsilon \tag{11}$$

The $\varepsilon -$constraint method can be efficiently applied for non-convex problems; however, there is a significant dependency on the ε [8]. In other words, we can obtain an efficient trade-off surface by setting appropriate constraint vector choices.

IV. Computational Experiments

To graphically present the solution method performance, a typical network with thirty activities has been taken into consideration. In this regard, the total project costs are calculated with respect to each of the given slack times. According to Fig. 1, it can be seen that how the execution costs increase along with the slacks rise.

Furthermore, the applicability and efficiency of the proposed model is tested through a set of instances for thirty-activity projects with different structures. However, the required parameters' values are randomly generated through Table IV. Finally, Table V accommodates the obtained results, in which the OF values are reported according to the selected TS times. The numerical investigation has been addressed in terms of 2, 3, and 4 different materials. As can be seen, the costs of projects increase for higher degrees of quality robustness. The issue indicates that decision makers should identify to which limit of schedules robustness, it is reasonable to increase the project's costs.

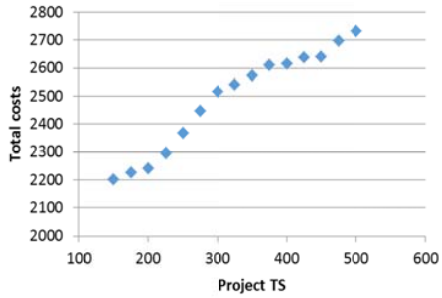

Fig. 1 The obtained Pareto front for the ε -constrained method

TABLE IV
DATA GENERATION METHOD

Parameters	Random distribution function
Pe	~ U [250, 350]
Re	~ U [60, 100]
δ_{mk}	~ U [4, 10]
G_m	~ U [6, 12]
h_m	~ U [1, 4]
Dd	~ U [0.2, 0.5] $\times es_N$
H	~ U [0.2, 0.4] $\times Dd$
α_{mk}	~ U [5, 20]
R_{jm}	~ U [1, 5]
d_j	~ U [1, 10]
L_m	~ U [1, 10]
K_m	~ U [1, 3]

TABLE V
PROJECTS' COSTS REGARDING THE TS TIMES

Instance No.	Number of materials (M)	TS value				
		200	300	400	500	600
1	2	2234	2376	2483	2668	2717
2	2	2226	2297	2376	2588	2630
3	2	2305	2378	2414	2602	2715
4	2	2191	2237	2558	2624	2686
5	2	2316	2480	2576	2695	2743
6	3	2638	2708	2796	2943	2973
7	3	2590	2673	2740	2866	2920
8	3	2673	2780	2815	2899	2967
9	3	2684	2749	2861	2943	3021
10	3	2690	2740	2865	2926	2968
11	4	2916	3025	3246	3347	3422
12	4	3011	3180	3276	3480	3496
13	4	3021	3178	3245	3393	3418
14	4	2913	2981	3170	3260	3374
15	4	2878	2967	3140	3268	3315

V. Conclusions

A bi-objective mixed-integer programming model was developed in this paper to consider the project scheduling and material-ordering problem at the same time. The OFs consisted of the costs minimization and schedules robustness maximization of the project, respectively. We also took all-unit discount strategy into account to purchase the materials. To check the applicability of the mathematical model in practice, we used the $\varepsilon -$constraint method. However, the proposed solution methodology could not deal with larger instances, i.e., projects' networks with more activities. Hence, it is needed to apply efficient heuristics to solve larger-sized cases, regarding the NP-hard nature of the resource constraint project-scheduling problem, in further studies. On the other hand, development of the problem with respect to the broader purchasing and execution site conditions can be considered as other future research interests.

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