International Journal of Engineering, Mathematical and Physical Sciences
ISSN: 2517-9934
Vol:9, No:12, 2015

Generic Model for Timetabling Problems by Integer Linear Programming Approach

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***Abstract*—**The agenda of showing the scheduled time for performing certain tasks is known as timetabling. It is widely used in many departments such as transportation, education, and production. Some difficulties arise to ensure all tasks happen in the time and place allocated. Therefore, many researchers invented various programming models to solve the scheduling problems from several fields. However, the studies in developing the general integer programming model for many timetabling problems are still questionable. Meanwhile, this thesis describes about creating a general model which solves different types of timetabling problems by considering the basic constraints. Initially, the common basic constraints from five different fields are selected and analyzed. A general basic integer programming model was created and then verified by using the medium set of data obtained randomly which is much similar to realistic data. The mathematical software, AIMMS with CPLEX as a solver has been used to solve the model. The model obtained is significant in solving many timetabling problems easily since it is modifiable to all types of scheduling problems which have same basic constraints.

***Keywords*—**AIMMS mathematical software, integer linear programming, scheduling problems, timetabling.

I. Introduction

TIMETABLING is the key to run an organization effectively. In many real world scheduling problems such as educational timetabling, crew scheduling and machine scheduling, several criteria must be taken into account when creating the timetables in order to maintain the quality. Among these criteria are: length of schedule, utilization of resources, preferences of customer and complication with regulation [15]. Therefore, the timetable which satisfies customer demands and provide maximum income by using limited supply is consider to be more effective and optimal.

Given a set of resources with capacities, a set of activities with processing time and specific requirement, and a set of constraints between activities, scheduling problems consist of determining when to execute each activity, so that both hard constraints and soft constraints are satisfied [16]. Several scheduling problems that included in this research are university courses scheduling, examination scheduling, nurse scheduling, flight-gate scheduling and bus crew scheduling.

Educational timetabling problems which are university course scheduling problem and examination timetabling problem are major administrative activity at most universities. It analyses how courses and examinations are allocated into the available classrooms and exam halls respectively at proper timeslots and places, subject to constraints based on lecturers, students and invigilators preferences. Thus, this will improve universities management system and students' performance. Besides that, nurse scheduling is also one of the hard and time-consuming task disregarding recent significant technological advances in the field. This is because it must consider employee's request, satisfy organization regulations as well as ensure continuous service for patients without wasting the manpower [3]. An approach to overcome this problem is to develop better nurse rostering systems which employ resources more efficiently while fulfil hospitals, patients and nurses demands. Moreover, flight-gate scheduling problems and bus crew scheduling problems play an important role in transportation industry. The problems arise in flight-gate scheduling when thousands of flights are assigned to different gates at a specific period of time and place under various compatibility constraints while the bus crew scheduling problems involves finding the most suitable way of allocating drivers to available set of buses at respective shifts. These problems should be solved to ensure the satisfaction of passengers and create an organized transport system. Previous study has shown the ability of a general model to solve university course timetabling problem that is applicable across different institutions with similar constraints [1]. There is a potential for a general basic model to solve all related problems that is flexible to all kind of timetabling categories. It is therefore our ultimate target in this study to develop this model. Meanwhile, the efficiently of the model is tested using Advanced Interactive Multidimensional Modelling Software (AIMMS).

II. General Timetabling Problem

Basically, scheduling problems are solved based on their specific field with respective constraints. Each of the fields has its own model and constraints due to the different on situation involved. For instance, a research of flight-gate scheduling problems will concentrate only the constraints exists in the problem and yield a model which can be only used to clear up the area of research. Ultimately, this model is controlled in the same problem. The problem in repetition is occurred in the process of solving and finding optimal solution for each of the scheduling problems separately since the scope of scheduling does not have a model that can be applied to numerous scheduling problems.

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There are two types of method to solve scheduling problems which are exact and heuristic methods for optimization. Exact optimization guarantee finding an optimal solution with high effort while heuristics optimization have no guarantee on solution quality but often able to find reasonable solution in relatively short time [14]. Exact methods consist of branch and bound, dynamic programming, Lagrangian relaxation based method, linear and integer programming based methods such as branch-and-cut and branch-and-price. On the other hand, heuristic methods consist of simulated annealing, tabu search, iterated local search, variable neighbourhood search and various population based model such as memetic algorithm [13].

Previously, graph colouring and heuristic methods were broadly investigated since it used simple techniques. Eventually it tends to be impossible for complex problems. There is still the need to solve the problem in systematic way that facilitates the real world variants of the problems solution of large scale problems. Therefore, linear and integer programming model were used to solve school and university timetabling problems [8]. However, exact solution method tends to be expensive when solving large timetabling problems. Over the years, meta-heuristic method becomes very successful on a variety of timetabling problems [9].

Numerous researches had been done on university exam, course, flight-gate, nurse and bus crew scheduling problems to obtain the similar constraints involved in these five fields that can be used as the general constraints. Constraints that are involved in Examination timetabling problem can be found in [4] and [2]; University Course Scheduling Problem as in [1], [12] and [5]; Flight-Gate Scheduling Problem in [10] and [11]; Nurse Scheduling Problem can be found in [6] and [3]; and Bus Crew Scheduling Problem in [7]. We selected the constraints by choosing the similar requirement from all these research to be compiled and formulated into one general basic model.

III. Basic Constraints

By referring to the previous research, the common basics constraints that identified, investigated and modified from all the scheduling problems are listed. By including the basic constraints, a general model that represent the respective university course, examination, nurse, flight-gate and bus crew scheduling are formulated using standard MILP format. The common constraints used in modelling the timetable are listed below:

A. Hard Constraints

1) All courses/exams/nurses/flights/crews must be assigned to respective place in the timetable.
2) No student/passengers take more than one course/exam/flight in respective place at a timeslot and no nurses/crews are allowed to work more than one shift every day and they must be at most one place per day.
3) Students/passengers take less than the consecutive courses/exams/flights preferred that assigned to places while the nurses/crews must not work more than consecutive shift preferred within their working days. There also must be a rest day for nurses and crews.
4) The minimum number of courses/ exams/ nurses/ flights/ crews and places must be fulfilled for each shift/timeslot.
5) Students/nurses/flights/crews unattended at certain shift/timeslot at any places but their name in schedule.
6) Each courses/exams/nurses/flights/crews must be at specific place/gate/ bus at specific shift/timeslot.

B. Soft Constraints

1) The nurses'/students'/flights'/crews' preferences must be fulfilled.

IV. Modelling

A. Mixed Integer Linear Programming

By observing a few research methods, Mixed Integer Linear Programming is an ideal method to solve the scheduling problems. It is the most convenient and clear technique to formulate the problems. This is because the variables represent quantities that can be real-valued and integer-valued. Furthermore, the variables also could be represented by the value of 0 and 1.

B. Notation

a. Sets:
- C: Total number of courses/exams/nurses/flights/crews to be scheduled.
- T : Total number of timeslots/shifts
- I : Total places

b. Index:
- c - Courses/exams/nurses/flights/crews
- t - Timeslot/shift
- i - Places

c. Parameters:
- n – maximum number of timeslots per time period
- q – number of consecutive timeslots preferred
- $C_{t,i}$ –minimum number of courses/exams/nurses/flights/ crews and places per shift/timeslot
- $P_{c,t,i}$ – preference of courses/exams/nurses/flights/crews to be assigned to places at shift/timeslot.

d. Decision variable:

$$X_{c,t,i} = \begin{cases} 1 & \begin{array}{l}\text{if courses or exams or nurses or flights} \\ \text{or crews, c are assigned in places, i} \\ \text{at timeslot or shift, t}\end{array} \\ 0 & \text{0therwise} \end{cases}$$

C. Model

Maximize;

$$Z = \sum_{c}^{C} \sum_{t}^{T} \sum_{i}^{I} P_{c,t,i} X_{c,t,i} \tag{1}$$

subject to

$$\sum_t^T \sum_i^I X_{c,t,i} = 1 \qquad \forall c \tag{2}$$

$$\sum_c^C \sum_i^I X_{c,t,i} \leq 1 \qquad \forall t \tag{3}$$

$$\sum_c^C \sum_i^I \begin{pmatrix} X_{c,t,i} + X_{c,t+1,i} \\ + \ldots + X_{c,t+q,i} \end{pmatrix} \leq q \forall t \in \{1,2,\ldots,n-q\} \tag{4}$$

$$\sum_c^C \sum_i^I X_{c,t,i} \geq C_{t,i} \qquad \forall t \tag{5}$$

$$\sum_c^C X_{c,t,i} = 0 \quad \forall t \text{ and } \forall i \tag{6}$$

$$\sum_c^C X_{c,t,i} \leq 1 \ \forall t \text{ and } \forall i \tag{7}$$

$$X_{c,t,i} \in \{0,1\} \tag{8}$$

In our modelling approach, (1) is the objective function that maximizes the preference of assigning courses or exams or nurses or flights or crews to places at shifts or timeslots. Equation (2) is the constraint of completeness that ensures all the courses, exams, nurses and flights or crews must be assigned to places and time. Meanwhile, (3) ensures no students and gates take more than one course, exam and flight in respectively at a timeslot. It also states no nurses and crews are allowed to work more than one shift every day and they must be at most one place per day. Besides that, (4) ensures no student takes more than the consecutive courses or exams preferred and all gates take consecutive flights preferred within the timeslots. Furthermore, the nurses and crews must not work more than consecutive shift preferred within their working days. There also must be a rest day for nurses and crews per week. On the other hand, the constraint in (5) shows that the minimum number of courses, exams, nurses, flights, crews and places must be fulfilled for each shift or timeslot. The absent of students/nurses/flights/crews at certain shift/timeslot in any places is formulated in (6).

Equation (7) ensures that each courses, exams, nurses, flights and crews must be at specific place, gates and bus respectively at specific shift or timeslot. Lastly, (8) ensures that all the value of decision variable should be either zero or one. All the hard constraints discussed will be satisfied in order to obtain feasible solution. Even though, soft constraint is not necessary to be solved always, this model will not violate it as possible.

C. Data

From the analysis of previous work, many ways are conducted to collect data which almost similar to real database. This is because to ensure the high quality of the result and reliability of the model proposed. In this case, a randomly generated data using software AIMMS is used to test the model. The suitable logical expression is mentioned in the definition of the parameter to create random data. Each scheduling problems are provided with a table that summarize necessary information about the problem. The data then inserted into main sets, parameters and constraint of the model.

TABLE I
DATA FOR EXAMINATION TIMETABLING

Sets	Notation	Data
Total number of examinations	C	36
Total number of student's group sitting for examination	csn $n = \{1,\ldots,9\}$	9
Total number of examinations taken by each student group	$\{1..4\} \in$ cs1 $\{5..8\} \in$ cs2 $\{9..12\} \in$ cs3 $\{13..16\} \in$ cs4 $\{17..20\} \in$ cs5 $\{21..24\} \in$ cs6 $\{25..28\} \in$ cs7 $\{29..32\} \in$ cs8 $\{33..36\} \in$ cs9	
Total number of timeslots	t	8
Total number of places	i	6
Parameters	Notation	Data
Preference of examinations to places at timeslots	$P_{c,t,i}$	Range: (1,5) 1- not preferred 5- most preferred
Maximum number of timeslot per time period	n	4
Minimum number of examinations and places for every timeslots	$C_{t,i}$	4
Number of consecutive timeslot preferred	q	2
Constraints	Notation	Data
Unattended student group at any places	cs1, t1, i	Student group one on first day

V. RESULT AND DISCUSSION

Generally, all the courses, exams and flights are divided into several groups while nurses and crews are presented as individual person. Consequently, this will ease the process of problem solving. Each of the timetabling problems is analyzed and solved. Table VI shows the overall results for all the timetabling problems studied. Meanwhile, Figs. 1-5 are the schedules for each of the timetabling problem.

Referring to Table VI, it can be seen that all the scheduling problems studied had achieved practical and feasible solutions. All the exams, courses, flights, nurses and bus crews are assigned based on their highest preferences for each time and places. Meanwhile, the time taken to solve each of the problems is considered fast but nurse and bus crew scheduling required more time to compute the results. This is because the problems are complex as it involved shifts. Besides that, the number of iterations for all the timetabling problems also varies depending on the difficulty of the problems. Figs. 1-5 show the schedules of examination timetabling, university courses timetabling, flight-gate scheduling, nurse scheduling and bus crew scheduling.

TABLE II
DATA FOR UNIVERSITY COURSE TIMETABLING

Sets	Notation	Data
Total number of university courses	C	48
Total number of student's group sitting for examination	cln $n = \{1,...,12\}$	12
Total number of courses taken by each student group	$\{1..4\} \in cl1$ $\{5..8\} \in cl2$ $\{9..12\} \in cl3$ $\{13..16\} \in cl4$ $\{17..20\} \in cl5$ $\{21..24\} \in cl6$ $\{25..28\} \in cl7$ $\{29..32\} \in cl8$ $\{37..40\} \in cl10$ $\{41..44\} \in cl11$ $\{45..48\} \in cl12$	
Total number of timeslots	t	11
Total number of places	i	5
Parameters	**Notation**	**Data**
Preference of courses to places at timeslots	$P_{c,t,i}$	Range: (1,5) 1- not preferred 5- most preferred
Maximum number of timeslot per day	n	4
Minimum number of courses and places for every timeslots	$C_{t,i}$	4
Number of consecutive timeslot preferred	q	2
Constraints	**Notation**	**Data**
Unattended student group at any places	$cl1, t1, i$	Student group one on first timeslot

Based on Fig. 1, all the thirty-six examinations are divided into nine student groups at where each group indicates exams that registered by same students. Moreover, each of the exams will be held in a day at a place among the total of eight days and six places. After analyzing the schedule, it proved that all the examinations are scheduled to respective places at every timeslots. Furthermore, no student from same group takes more than one exam that held at a place at a timeslot. They also do not have more than two consecutive exams at the respective places. Moreover, it shows that no students from group one are assigned to first day as requested in the constraint. The minimum number of exams and places per timeslot is maintained. The schedule also shows that each exam is assigned to specific place at specific timeslot based on estimated preference scores of students. Overall, the schedule obtained is optimal as the exams are allocated in effective way.

Based on Fig. 2, an appropriate solution is determined which reflects the preferences of student's for allocating each of the courses to respective timeslot provided. There are twelve groups of courses such that each group consists of four courses that taken by same students. Furthermore, all the eleven timeslots and five places was occupied by every courses. Besides that, the schedule obtained shows that no student from each group takes two or more courses that conducted in respective classes at a timeslot. They also do not have more than two consecutive courses to be attended within the timeslots. Additionally, no students from group one have attend any courses on first timeslot (8am) as stated in the data. Moreover, the minimum number of courses at respective places per timeslot is satisfied. Each course is assigned to a specific timeslot at a specific place.

TABLE III
DATA OF FLIGHT-GATE SCHEDULING

Sets	Notation	Data
Total number of flights	C	48
Total number of flight groups	cfn $n = \{1,...,12\}$	12
Total number of flights allocated to each group	$\{1..4\} \in cf1$ $\{5..8\} \in cf2$ $\{9..12\} \in cf3$ $\{13..16\} \in cf4$ $\{17..20\} \in cf5$ $\{21..24\} \in cf6$ $\{25..28\} \in cf7$ $\{29..32\} \in cf8$ $\{33..36\} \in cf9$ $\{37..40\} \in cf10$ $\{41..44\} \in cf11$ $\{45..48\} \in cf12$	4
Total number of timeslots	t	11
Total number of gates	i	5
Parameters	Notation	Data
Preference of flight to gates at timeslots	$P_{c,t,i}$	Range: (1,5) 1- not preferred 5- most preferred
Maximum number of timeslot per time period	n	4
Minimum number of flight and gates for every timeslots	$C_{t,i}$	4
Number of consecutive timeslot preferred	q	2
Constraints	Notation	Data
Unattended flight at any gates	$cf1, t1, i$	Flight from group one on first timeslot

Based on Fig. 3, all the forty-eight flights are equally distributed among twelve flight groups. Each group represents the flights with common passengers. For example, group one consist of passengers from a country. Then, there are approximately eight to eleven flights at each gate. On the other hand, each of the flights is well arranged to their specific timeslots and gates. All the flights are in the timetable. There are no two or more flights from same gates that assigned to a timeslot. Furthermore, no gates take three consecutive flights within the timeslots. The minimum number of flights and gates used per timeslot also considered. There are no flights from group one that assigned to the first timeslot as mentioned in the data. To conclude, the schedule formed is systematic since it does not violate any hard constraints and soft constraint.

TABLE IV
DATA FOR NURSE SCHEDULING

Sets	Notation	Data
Total number of nurses	C	60
Total number of nurses to be assigned (each nurse works six times per week)	cnn $n = \{1,...,10\}$ $\{1..6\} \in cn1$ $\{7..12\} \in cn2$ $\{13..18\} \in cn3$ $\{19..24\} \in cn4$ $\{25..30\} \in cn5$ $\{31..36\} \in cn6$ $\{37..42\} \in cn7$ $\{43..48\} \in cn8$ $\{49..54\} \in cn9$ $\{54..60\} \in cf10$	10
Total number of time (days x shifts)	t	21
Total number of days	dn $n = \{1,...,7\}$	7
Total number of shift per day		3
Total number of wads	i	11
Parameters	Notation	Data
Preference of nurses to wads at timeslots	$P_{c,t,i}$	Range: (1,5) 1- not preferred 5- most preferred
Maximum number of working day	n	6
Minimum number of nurses and places for every shift	$C_{t,i}$	2
Number of consecutive shift preferred	q	3
Constraints	Notation	Data
Unattended nurse at any wads	cn1, t1, i	Nurse 1 at first shift of first day

Figs. 4 and 5 show the schedules of nurses and crews respectively. There are ten nurses and crews chosen to work in eleven wards and buses respectively at three shifts among a week. Similarly, all the nurses and crews are arranged systematically to wards and buses respectively in the timetable without violating any of the constraints. No nurses and crews have more than one shift and place per day. Besides that, there are not more than three consecutive similar shifts within seven days period. All the nurses and crews are given one off day in a week. Furthermore, the minimum number of nurse, crews and wards, buses for every shift are achieved. Nurse 1 and crew 1 are not working on the first shift (morning) of first day (Monday) as stated in the constraints. Beyond that, each of them is assigned to respective place based on their preferences to work at each place accordingly. Therefore, there is always continuous supply of employees to every wards and buses.

TABLE V
DATA FOR BUS CREW SCHEDULING

Sets	Notation	Data
Total number of crews	C	60
Total number of crews to be assigned (each crew works six times per week)	cbn $n = \{1,...,10\}$ $\{1..6\} \in cb1$ $\{7..12\} \in cb2$ $\{13..18\} \in cb3$ $\{19..24\} \in cb4$ $\{25..30\} \in cb5$ $\{31..36\} \in cb6$ $\{37..42\} \in cb7$ $\{43..48\} \in cb8$ $\{49..54\} \in cb9$ $\{54..60\} \in cb10$	10
Total number of time (days x shifts)	t	21
Total number of days	dn $n = \{1,...,7\}$	7
Total number of shift per day		3
Total number of buses	i	11
Parameters	Notation	Data
Preference of crew to buses at timeslots	$P_{c,t,i}$	Range: (1,5) 1- not preferred 5- most preferred
Maximum number of working day	n	6
Minimum number of crews and places for every shift	$C_{t,i}$	2
Number of consecutive shift preferred	q	3
Constraints	Notation	Data
Unattended crew at any buses	cb1, t1, i	Crew 1 at first shift of first day

A. Overall Discussion

Timetabling involves constructing a plan to carry out number of activities over a period of time at respective places by considering the resources required which are limited and there are various rules that need to be satisfied while optimizing one or more objective functions. Several criteria that must consider from the results gained are satisfaction of the people involved in the scheduling problems, optimization of the objectives and also computation time to complete the problems which mostly depends on size of the problems. From this research, the results obtained are considers to be optimal since it satisfies all hard constraints and also allocate all the events to its best preference. The time taken to solve all the problems is in between 0.06 seconds to 0.78 seconds which reflects it as best computational time. Moreover, multiple trials were done in order to obtain these results. The model is modified for every trial to achieve best results. On the other hand, the process of solving nurse scheduling problem and bus crew scheduling problem are very different from other three problems studied since the requirement is more complicated. The schedules for both of the problems are seemed to be imbalance within the time period. Some of the workers did not have work all the three shifts in the time period. This is the only difficulty that encountered while combining the nurses and bus crews problem to other problem in order to form a general model. Overall, the results reached the satisfaction level in most of the aspects.

TABLE VI
OVERALL RESULTS FOR TIMETABLING PROBLEMS

Results	Examination timetabling	University courses timetabling	Flight-gate scheduling	Nurse scheduling	Bus crew scheduling
Objective function	180	240	240	300	300
Solving time	0.06 sec	0.09 sec	0.09 sec	0.78 sec	0.61sec
Iterations	154	149	166	274	296

Days / Courses	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Day 8
Group 1		BIO 3201			BIO 3202	BIO 3204		BIO 3203
Group 2		KIM 3203		KIM 3201	KIM 3204		KIM 3202	
Group 3		MTK 3201		MTK 3203		MTK 3202	MTK 3204	
Group 4	BBB 3204			BBB 3201	BBB 3201		BBB 3203	
Group 5	SOC 3201		SOC 3203		SOC 3202		SOC 3204	
Group 6	ACC 3201		ACC 3203	ACC 3202				ACC 3204
Group 7			FIZ 3204	FIZ 3203		FIZ 3201		FIZ 3202
Group 8	EKN 3203	EKN 3201		EKN 3202			EKN 3204	
Group 9	FDT 3201		FDT 3202			FDT 3203		FDT 3203

KK1 KK2 KK3 KK4 KK5 KK6

Fig. 1 Examination schedule

Time / Groups	8am	9am	10am	11am	12am	1pm	2pm	3pm	4pm	5pm	6pm
Group 1		BIO 3201					BIO 3202	BIO 3203			BIO 3204
Group 2	MTK 3201				MTK 3202					MTK 3203	MTK 3204
Group 3			KIM 3202			KIM 3201		KIM 3203	KIM 3204		
Group 4	FIN 3203			FIN 3204		FIN 3201			FIN 3202		
Group 5		SOC 3202			SOC 3203		SOC 3204			SOC 3201	
Group 6	EKN 3203	EKN 3204		EKN 3201		EKN 3202					
Group 7		BBB 3201		BBB 3204	BBB 3202		BBB 3203				
Group 8					FIZ 3201		FIZ 3203	FIZ 3202		FIZ 3204	
Group 9	CSF 3203		CSF 3201	CSF 3202						CSF 3204	
Group 10		KEJ 3203	KEJ 3201						KEJ 3204		KEJ 3202
Group 11	FEI 3204		FEI 3202	FEI 3203		FEI 3201					
Group 12					KAS 3204			KAS 3203	KAS 3201		KAS 3203

KK1 KK2 KK3 KK4 KK5

Fig. 2 University courses schedule per day

Time / Groups	8.20 am	9.20 am	10.20 am	11.20 am	12.20 pm	1.20 pm	2.20 pm	3.20 pm	4.20 pm	5.20 pm	6.20 pm
Group 1		F2			F4			F3			F1
Group 2	F5	F6				F7				F8	
Group 3			F10			F11		F9			F12
Group 4	F13				F16			F14		F15	
Group 5	F18			F19		F17		F20			
Group 6	F24			F23			F21		F22		
Group 7		F25			F28		F26		F27		
Group 8		F32			F29	F30		F31			
Group 9			F33	F36					F34	F35	
Group 10		F37	F39							F40	F38
Group 11	F41			F44			F43		F42		
Group 12			F47	F45			F46				F48

GATE A GATE B GATE C GATE D GATE E

Fig. 3 Flight-gate schedule per day

Days / Nurses	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Nurse 1	A	A	N	N	A	OFF	M
Nurse 2	M	M	OFF	M	M	A	N
Nurse 3	A	OFF	A	N	N	A	M
Nurse 4	OFF	M	M	N	M	N	M
Nurse 5	N	M	OFF	M	N	A	N
Nurse 6	N	A	N	OFF	N	M	M
Nurse 7	M	N	M	OFF	N	M	A
Nurse 8	M	A	A	M	M	N	OFF
Nurse 9	M	A	N	A	A	OFF	N
Nurse 10	M	N	A	A	OFF	A	A

WARD 1 WARD 3 WARD 5 WARD 7 WARD 9 WARD 11
WARD 2 WARD 4 WARD 6 WARD 8 WARD 10

M- morning shift **A- afternoon shift** **N- night shift** **OFF- on leave**

Fig. 4 Nurse schedule

Days / Crews	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Crew 1	A	A	OFF	N	M	M	A
Crew 2	M	A	A	M	N	N	OFF
Crew 3	A	N	M	M	N	OFF	M
Crew 4	A	OFF	M	M	A	N	A
Crew 5	M	M	OFF	A	A	M	N
Crew 6	M	M	A	M	M	A	OFF
Crew 7	N	OFF	N	A	A	A	M
Crew 8	N	N	N	M	OFF	M	M
Crew 9	A	M	M	N	M	OFF	N
Crew 10	M	OFF	A	M	A	N	A

BUS 1 BUS 3 BUS 5 BUS 7 BUS 9 BUS 11
BUS 2 BUS 4 BUS 6 BUS 8 BUS 10

M- morning shift **A- afternoon shift** **N- night shift** **OFF- on leave**

Fig. 5 Bus crew schedule

VI. Conclusion

In this paper, we presented new approach to solve multiple scheduling problems by generating a general model which is applicable and flexible to all the scheduling problems studied. The model is created by MILP method as it is efficient and straightforward to compute. A variety of rules are illustrated in the model with suitable constraints provided by this formulation. Furthermore, the choice made for the objective function permits the introduction of some preferences regarding timeslots, places and days, so that the timetables can be improved to better quality measures.

There are five types of scheduling problems investigated which are university courses timetabling, examination scheduling, nurses timetabling, flight-gate scheduling and bus crew scheduling. The main objective of the model which is maximize the preferences of each courses, examinations, nurses, flights and bus crews to be assigned at corresponding timeslots and places. The objective is achieved by considering six basic hard constraints and a soft constraint from the scheduling problems. On the other hand, the total number of courses, examinations, nurses, flights, and bus crews are combined into groups respectively which then allocated to time and places. This technique was very useful in satisfying each of the constraints to find feasible solution. In addition, usage of AIMMS to solve the problems also helps to obtain result quickly. Ultimately, creating timetables for different scheduling problems is a tedious process; however it can be easily done by using this general model since the single model cover multiple timetabling problems.

Recommendation

From this research, it can be known that only medium size data is used to test and validate the model. Therefore, the computational time for each scheduling problems are quite fast although there was some complexity during model formation. The process of obtaining the best timetables by considering all the constraints for large instances is still a challenging approach. So, it is recommended to test the model using small and large size data in order to increase the quality of the timetables since it closely related to real world problems. Furthermore, it is possible to generalize the model further by analyzing and adding more constraints and types of scheduling problem. Consequently, the timetables produced can be applicable to varies fields. Besides that, the timeslots can be included in the examination timetabling problems for further enhancement since only days are considered in this research. In addition, it is suggested to include the activities such as arrival, departure and parking time in flight-gate scheduling problems for future work.

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