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Reinforced Concrete Slab under Static and Dynamic Loadings

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Abstract**—**In this study, static and dynamic responses of a typical reinforced concrete solid slab, designed to British Standard (BS 8110: 1997) and under self and live loadings for dance halls are reported. Linear perturbation analysis using finite element method was employed for modal, impulse loading and frequency response analyses of the slab under the aforementioned loading condition. Results from the static and dynamic analyses, comprising of the slab fundamental frequencies and mode shapes, dynamic amplification factor, maximum deflection, stress distributions among other valuable outcomes are presented and discussed. These were gauged with the limiting provisions in the design code with a view of justifying valid optimization objective function for the structure that can ensure both adequate strength and economical section for large clear span slabs. This is necessary owing to the continued increase in cost of erecting building structures and the squeeze on public finance globally.

Keywords**—**Economical design, Finite element method, Modal dynamics, Reinforced concrete, Slab.

I. Introduction

STEEL rods embedded in concrete, also known as Reinforced Concrete (RC) have been widely reported to have good compressive and tensile strengths owing to the complimentary strength contribution of the two constituents. This excellent property of reinforced concrete, among others, has for many years been reason for its widely used in construction of structural elements. Tensile strength of plain concrete is reported to be only about 10 per cent of its compressive strength [1]; hence in design of RC structures the embedded reinforcements are generally assumed to carry tensile forces which are transferred to it by bond between interfaces of the two materials.

Research to improve concrete durability [2], strength and cost [3], largely carried out by partial replacement of the traditional constituents materials with cheaper alternatives have yielded some interesting outcomes. Considering however, the usual high volume of concrete used in buildings, the cost of producing concrete can still be said to be high. Thus, further research dwelling on optimum utilization of concrete in buildings and other structures is essential and hence this study.

Designs of RC structural elements are mostly based on either national or international standards (codes), which are often put together to ensure provision of sections with adequate strength to resist both self and live loads during the expected service life of the structure.

Limits for RC materials as well as its response to load are normally set; chief among the limiting factors especially for flexural elements is the maximum allowable deflection- a provision that ensures RC section of adequate stiffness to prevent both structural damage and serviceability concerns. Various codes have different approach to this: Eurocode (EC 2) and BS 8110 set span-to-depth ratio for continues two-way solid slabs at 26, which can be modified upwards dependent on the reinforcement density in the concrete [4]. Other codes require detailed estimation of deflections as against the empirical limits of the span-depth ratios.

In design practice, using the BS 8110 and allied codes, the span-depth ratio is the bases for sizing of RC slabs and beams. This tradition, though very convenient in practical design of residential and commercial buildings is however, tantamount to having a generous sections that undermines the essence of having an economic section/design. This paper thus, considers analysis of a commonly employed two-way solid slab for dance halls or event centres under self and dynamic live loading allowance of 5kN/m^2 as stipulated in BS 8110.

Static and dynamic analyses of the structural element under wide conditions were carried out using Abaqus commercial finite element code, employing the general static and linear perturbation analysis respectively. This is with a view to estimate static instantaneous maximum deflection of the slab as well as modal dynamic deflections over a wide range of forcing function frequencies. Results from the aforementioned analysis were compared with the allowable limits to justify or otherwise state the effect of application of the span-depth ratio in design of slabs.

II. Governing Equations

Structures under static loading can be described by the equation of equilibrium (1) which can be linear or non-linear depending on the structural material and the loading regime.

$$\mathbf{KU} = \mathbf{F} \tag{1}$$

Under dynamic loading condition however, this is described using the equation of motion given by:

$$\mathbf{M}\ddot{\mathbf{U}} + \mathbf{C}\dot{\mathbf{U}} + \mathbf{KU} = \mathbf{F}(t) \tag{2}$$

where in both cases, **K** is the stiffness matrix, **U** is the displacement vector, (**KU**-internal force), **M** is the mass matrix and **C** is the viscous damping matrix of the structure.

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Under free vibration, natural frequencies and mode shapes of a structure are computed using the generalised eigenvalue problem given by:

$$[\mathbf{K} - \omega^2\mathbf{M}]\mathrm{U} = 0 \tag{3}$$

where (3) is an nth order polynomial of ω^2 (rotation frequency) in which n solutions (roots) or eigenvalues and their corresponding normal modes can be found.

Equation (3) has normal modes $\mathbf{U}_i$, that satisfies:

$$\mathbf{U}_i^T\mathbf{K}\mathbf{U}_j = 0$$
$$\mathbf{U}_i^T\mathbf{M}\mathbf{U}_j = 0 \tag{4}$$

and

$$\mathbf{U}_i^T\mathbf{K}\mathbf{U}_i = \omega_i^2$$
$$\mathbf{U}_i^T\mathbf{M}\mathbf{U}_i = 1$$

From (3)

$$\Phi_i^T\mathbf{K}\Phi_i = \Omega = \begin{bmatrix} \omega_1^2 & 0 & 0 \\ 0 & \omega_2^2 & 0 \\ 0 & 0 & \omega_3^2 \end{bmatrix} \tag{5}$$
$$\Phi_i^T\mathbf{M}\Phi_i = \mathbf{I}$$

where

$$\Phi_{(nxn)} = U_1 \quad U_2 \quad . \quad . \quad . \quad U_n$$

Transformation of the displacement vector is defined as:

$$\mathbf{U} = Z_1U_1 + Z_2U_2 + \ldots\ldots\ldots + Z_nU_n = \Phi\mathbf{Z} \tag{6}$$

where

$$\mathbf{Z} = [Z_1(t), Z_2(t), \ldots\ldots\ldots Z_n(t)]^T$$

Substituting (6) in (2) yields:

$$\mathbf{M}\Phi\ddot{\mathbf{Z}} + \mathbf{C}\Phi\dot{\mathbf{Z}} + \mathbf{K}\Phi\mathbf{Z} = \mathbf{F}(t) \tag{7}$$

Hence, the modal dynamic equation becomes:

$$\ddot{Z}_i + 2\xi_i\omega_i\dot{Z}_i + \omega_i^2 Z_i = F_i(t), \quad i=1,2,..\mathrm{n} \tag{8}$$

Frequency response on the other hand utilises (2), where the forcing function is a harmonic force given by:

$$\mathbf{M}\ddot{\mathbf{U}} + \mathbf{C}\dot{\mathbf{U}} + \mathbf{K}\mathbf{U} = \mathbf{F}\sin\omega t \tag{9}$$

Applying the modal equation, leads to:

$$\ddot{Z}_i + 2\xi_i\omega_i\dot{Z}_i + \omega_i^2 Z_i = F_i \sin\omega t \qquad \mathrm{i}=1,2,\text{---}\mathrm{n} \tag{10}$$

where

$$Z_i(t) = \frac{F_i/\omega_i^2}{\sqrt{(1-\eta^2)^2 + (2\xi_i\eta_i)^2}}\sin(\omega t - \theta_i)$$

phase angle $\theta_i = \arctan\dfrac{2\xi_i\eta_i}{1-\eta_i^2}$, $\eta_i = \omega/\omega_i$ and damping ratio $\xi_i = \dfrac{c}{c_c} = \dfrac{c_i}{2m\omega_i}$, and **U** is recovered from Z using (6).

III. Geometry and Design of the RC slab Based on BS8110

Solid slab of large clear span are often employed in office or commercial buildings. In line with this tradition; in this study, a two-way solid slab of dimensions 6 m by 6 m is considered. Design loads were generated from live loads and self-weight of the reinforced concrete, inclusive of finishes and partition allowances of 1.2 kN/m^2 and 1 kN/m^2 respectively. Hence, factored/ultimate design load

$$(n) = 1.4gk + 1.6qk \tag{11}$$

i.e. n = 1.4 x 6.28 + 1.6 x 5 = 16.79 kN/m² where gk and qk are the dead and live loads factors respectively.

Design of the slab based on BS 8110-1997 [4] to resist moments and deflection from the above loadings was carried out. This requires 297 mm2 of main reinforcement and a concrete of 170 mm thick as shown in Fig. 1 (a). Reinforcement however provided, as can be seen from Fig. 1 (b) is of 12 mm diameter, high yield steel positioned at 250 mm centres, representing 452 mm2 in both directions in line with conventional practice.

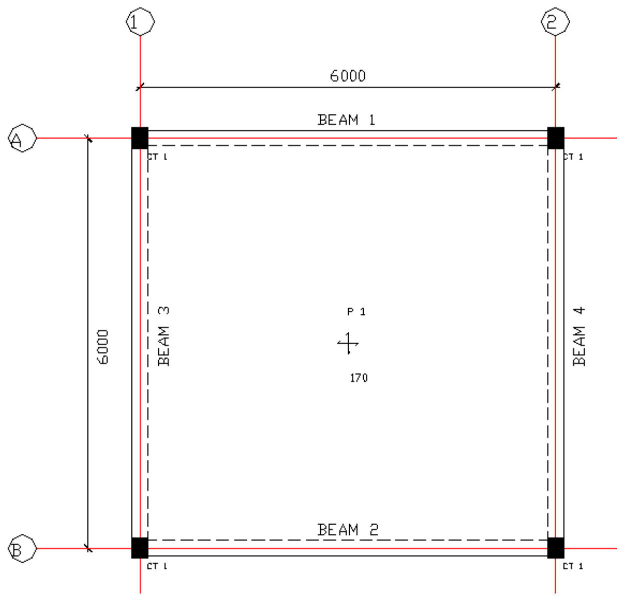

Fig. 1 (a) General arrangement of the floor slab

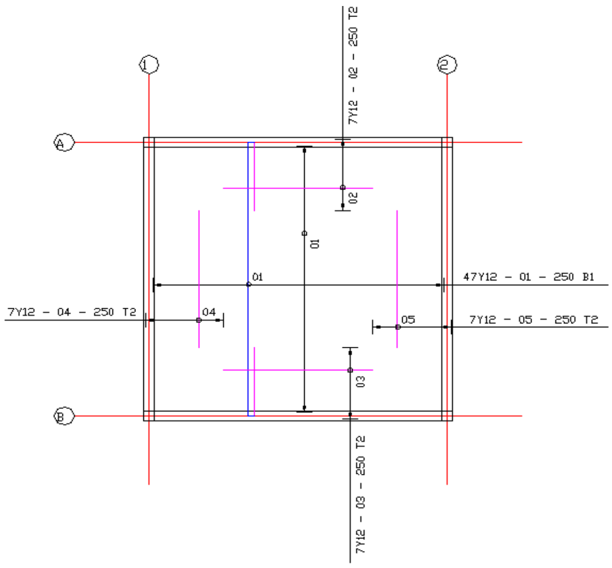

Fig. 1 (b) Designed slab reinforcement details

IV. Materials and Structural Model

A. Material Properties

Material properties typical for concrete and steel rebar used in this study are presented in Table I.

V. Finite Element Model

The concrete slab presented in Fig. 1 was modelled using 57600 volume elements (C3D8R), completely restrained at the support ends. The high yield steel was also modelled using rebar on 9600 surface elements (SFM3D4) and is embedded in the concrete material via the constraint condition in Abaqus FE.

The structural model validity was checked via mesh convergence tests to a satisfactory level.

TABLE I
Material Properties for Concrete and Steel

Material	Parameters	Quantities
Concrete	Young's Modulus	41000 MPa
	Density	2300 kg/m^3
	Poisson ratio	0.15
	Compressive strength	40 N/mm^2
	Tensile strength	4 N/mm^2
	Ultimate compressive strain	0.0035
	Cracking strain	0.00015
Steel	Young's modulus	200 GPa
	Density	7850 kg/m^3
	Poisson ratio	0.3
	Yield strength	410 N/mm^2

VI. Results and Discussion

A. Static Analysis

Results of static analysis of the slab are presented in Figs. 2-4. Fig. 2 presents the displacement distribution of the slab where maximum displacement recorded was 3.5 mm. This compares well with the exact displacement (δ) of 3.4 mm computed (assuming an un-cracked concrete section) using the relation:

$$\delta = \frac{wl^4}{384EI} \tag{12}$$

where w and l are respectively the design load and span of the slab, E is the young's modulus and I the second moment of area.

The central/maximum displacement recorded from the static analysis is well below the allowable displacement in the code. This is set at the lesser of either 20 mm or span-to-500 ratio for the purpose of avoiding serviceability and structural damage [1].

Results of the reaction force distribution and stresses in the slab as presented in Figs. 3, 4. These are also marginal and well below failure state of the two constituents as presented in Table I.

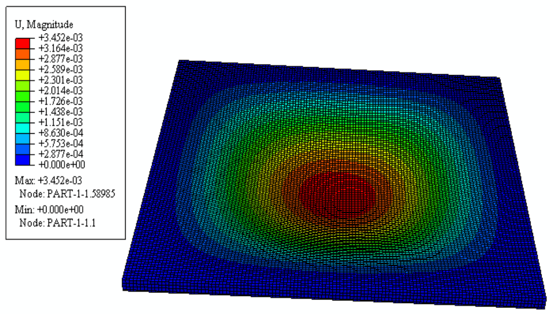

Fig. 2 Displacement Distribution of the Two-way Slab under Static Loading

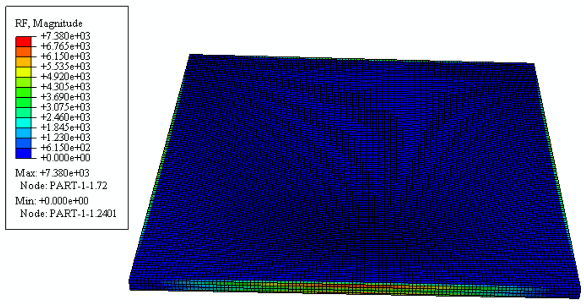

Fig. 3 Reaction Force Distribution of the Two-way Slab under Static Loading

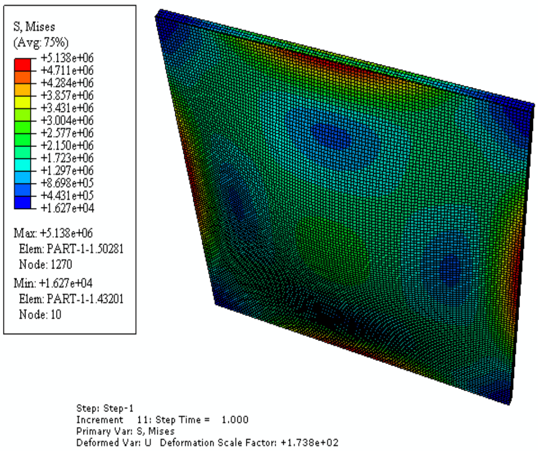

Fig. 4 Mises Stress Distribution of the Two-way Slab under Static Loading

VII. Dynamic Analysis

A. Modal Analysis

From the modal analysis carried out, Figs. 5-10 show the first six eigen-vectors of the slab and their corresponding fundamental frequencies of 21.96 Hz, 44.52 Hz, 44.61 Hz, 65.15 Hz, 79.65 Hz and 80.01 Hz respectively. These frequencies are well above normal walking (foot drop) rates, which is estimated to be about 2.5 Hz at the most. Human sensitivity to vibration is reported to be within the range of 4 to 8 Hz [5]. Vibration forcing function frequencies range however, for other loadings on building structures such as wind, acoustic, machinery outside and inside of a building plus that due to impact as reported in ISO 4866: 1990 [6] are presented in Table II. These are through and above the fundamental frequencies computed and hence necessitating a further check on response of the structure to excitations at the various fundamental frequencies.

Vibrating machines and other harmonic loading through a range of frequencies (as previously presented) on the slab have the effect of amplifying the static displacements shown in Fig. 2. Results of response of the slab through a range of exciting frequencies is shown in Fig. 11 (a) and scaled-up in Fig. 11 (b) for clarity. The result shows a general increase in central displacements of the slab with resonance amplitude at the first fundamental frequency of 15 mm and devastating amplitude of 450 mm at about the second and third fundamental frequencies.

TABLE II
Amplitude-Frequency Range of Typical Loading Conditions

Loading	Frequency range (Hz)	Amplitude range (mx10^{-6})
Wind	0.1 to 10	10 to 10^5
Impact	0.1 to 100	100 to 500
Machinery outside	1 to 300	10 to 1000
Machinery inside	1 to 1000	1 to 100
Acoustic	10 to 250	1 to 1100

Forcing function of magnitude as that of the design load considered in this study with harmonic frequency similar to the first fundamental frequency of the slab amplified the central displacement to 15 mm. Hence the dynamic amplification factors of the structural system as defined in the literature [7] range from 1.2 to 4.4.

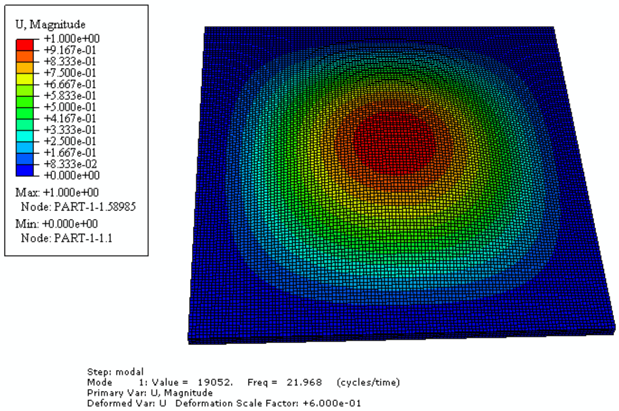

Fig. 5 First mode of vibration of the slab (21.96 Hz)

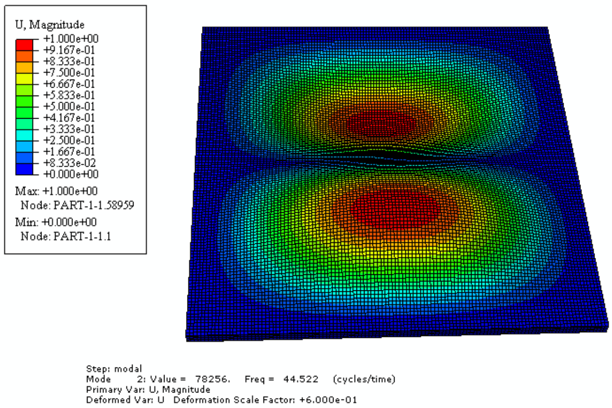

Fig. 6 Second mode of vibration of the slab (44.52 Hz)

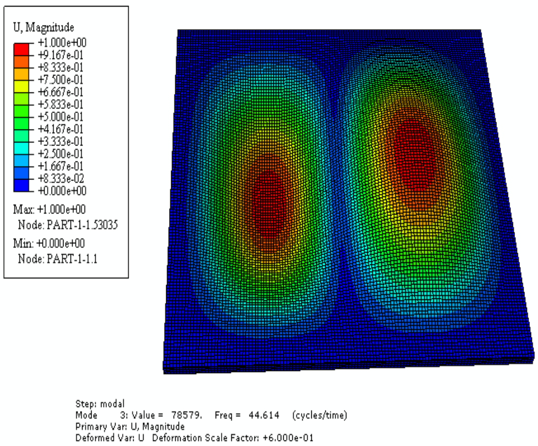

Fig. 7 Third mode of vibration of the slab (44.61 Hz)

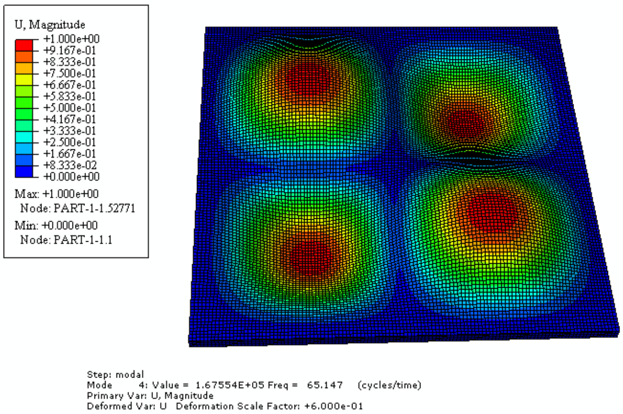

Fig. 8 Fourth mode of vibration of the slab (65.15 Hz)

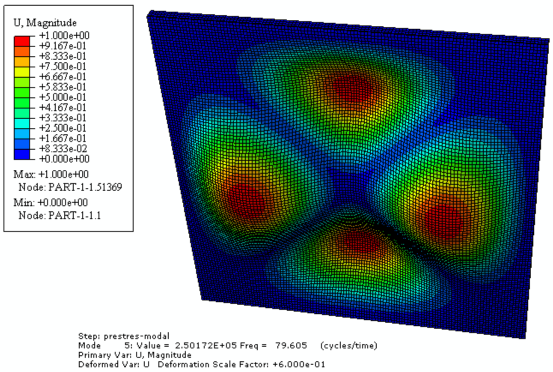

Fig. 9 Fifth mode of vibration of the slab (79.65 Hz)

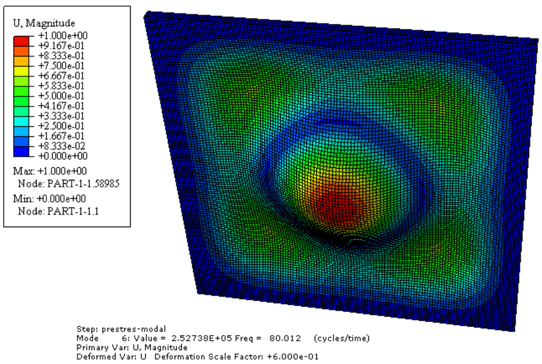

Fig. 10 Sixth mode of vibration of the slab (80.01 Hz)

VIII. Impulse Loading

The design load under impulse within 0.02 seconds yields a central displacement of 5.5 mm as shown in Fig. 12 and an increased in stresses of the slab (Fig. 13) when compared to the results under static loading. This loading effect has a dynamic amplification factor of 1.6 and stress level that is relatively low and much lower than the failure stress of the RC slab.

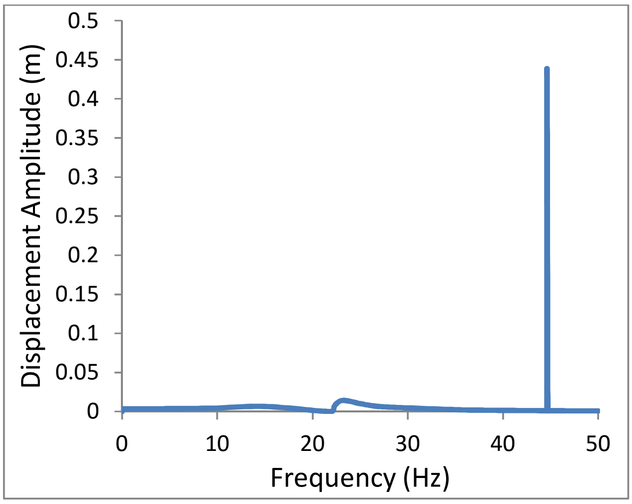

Fig. 11 (a) Slab response to varying exciting frequencies

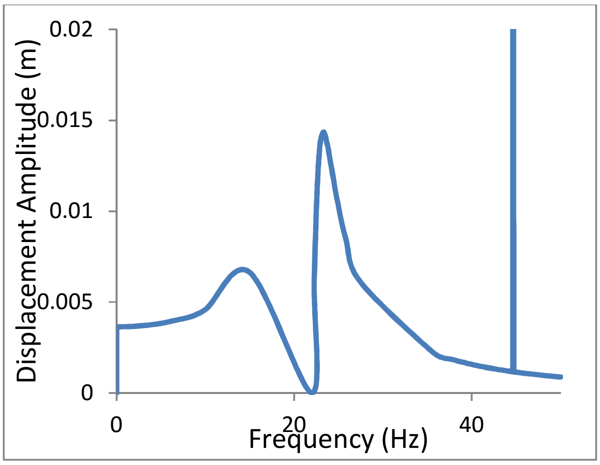

Fig. 11 (b) Slab response to varying exciting frequencies

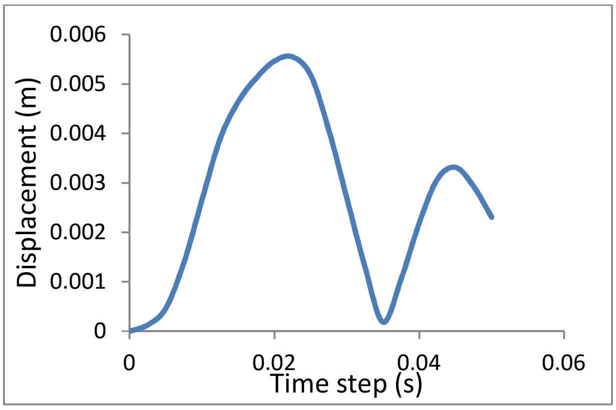

Fig. 12 Central displacement of the slab under impulse loading

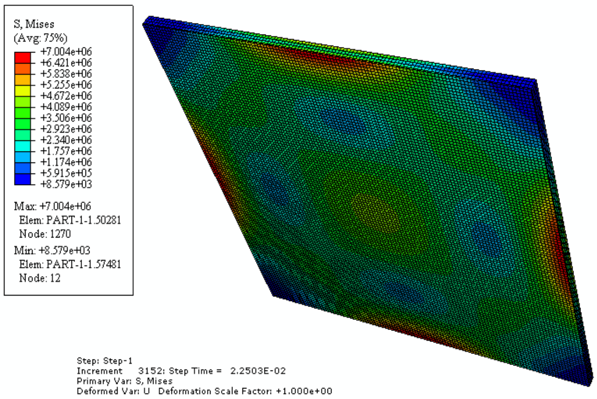

Fig. 13 Stress distribution of the slab under impulse loading

IX. Conclusion

In this study, static and likely vibration responses of a solid reinforced concrete slab, designed to BS 8110:1997 and under varied dynamic forcing function and impulse loadings were examined. The following findings were observed:

- Dynamic amplification factor of the slab for all likely loading scenarios ranges from 1.1 to 1.6.
- Span-to-depth ratio utilised in BS 8110 for sizing RC slabs leads to sections that are highly stiff and hence marginal stresses and moments as well as deflections for both static and dynamic loading conditions.

Hence to ensure cost effective/optimised section/design of the RC slab, the slab thickness determined from the span-to-depth ratio should be reviewed. In the meantime, optimization of design of the slab may be carried out where the chief objective function is to reduce the slab thickness.

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