

# CSTR Control by Using Model Reference Adaptive Control and PSO

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**Abstract**—This paper presents a comparative analysis of continuously stirred tank reactor (CSTR) control based on adaptive control and optimal tuning of PID control based on particle swarm optimization. In the design of adaptive control, Model reference adaptive control (MRAC) scheme is used, in which the adaptation law have been developed by MIT rule & Lyapunov's rule. In PSO control parameters of PID controller is tuned by using the concept of particle swarm optimization to get optimized operating point for minimum integral square error (ISE) condition. The results show the adjustment of PID parameters converting into the optimal operating point and the good control response can be obtained by the PSO technique.

**Keywords**—Model reference adaptive control (MRAC), optimal control, particle swarm optimization (PSO).

## I. INTRODUCTION

DURING the past decades, there has been a great advancement in the process control. Various control methods such as Adaptive control, neural control, fuzzy control and optimal control have been studied. In this paper two methods i.e. adaptive control and optimal control by using PSO has been discussed.

An adaptive controller is a mechanism of some system identification to obtain a model and then design a controller. Adaptive controller can be apply for modify its behavior in response to the changing dynamics of the process and the character of the disturbances. The base element of all the approaches is that they have the ability to adapt the controller to accommodate changes in the process. This permits the controller to maintain a required level of performance in spite of any noise, nonlinearity or fluctuation in the process. An adaptive system has maximum application when the plant undergoes transitions or exhibits non-linear behavior and when the structure of the plant is unknown. Adaptive is called a control system, which can adjust its parameter automatically to compensate for variations in the characteristics of the process it control.

Particle Swarm Optimization (PSO), first introduced by Kennedy and Eberhart, is one of the modern heuristic algorithms. It was developed through simulation of a simplified social system. It has been found to be robust in solving continuous nonlinear optimization problems. The PSO technique is used to generate a high-quality solution within shorter calculation time and stable convergence characteristic than other stochastic methods. PSO method is an excellent

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optimization methodology and a promising approach for solving the optimal PID controller parameters. Therefore, this study develops the PSO-PID [1], [10].

## II. DEVELOPMENT OF MATHEMATICAL MODELING

A perfectly mixed continuously stirred tank reactor (CSTR) as shown in Fig. 1 with first order exothermic irreversible reaction  $A \rightarrow B$  is considered. In this a fluid stream is continuously fed to the reactor and other fluid stream is continuously removed from the reactor. A jacket surrounding the reactor has in feed and exit streams. The jacket is assumed to be perfectly mixed and at lower temperature than the reactor [4]-[6].

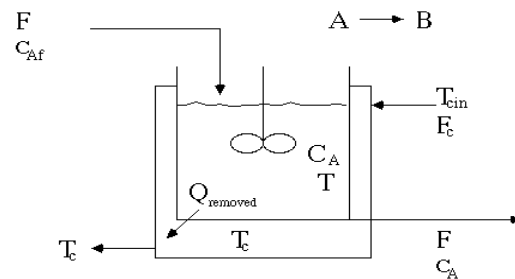


Fig. 1 Continuously Stirred Tank Reactor

Following assumptions has been made for CSTR:

- Perfect mixing (product stream values are the same as the bulk reactor fluid)
- Constant volume
- Constant parameter values

### A. State Variable form of Dynamic Equations

In state variable form equations can be written as:

$$f_1(C_A, T) = \frac{dC_A}{dt} = \frac{F}{V}(C_{Af} - C_A) - r \quad (1)$$

$$f_2(C_A, T) = \frac{dT}{dt} = \frac{F}{V}(T_f - T) + \left(\frac{-\Delta H}{\rho c_p}\right)r - \frac{UA}{V\rho c_p}(T - T_j) \quad (2)$$

The reaction rate per unit volume (Arrhenius expression) is:

$$r = k_o \exp\left(\frac{-\Delta E}{RT}\right) C_A$$

where it is assumed that the reaction is first-order [5].

### B. Steady-State Solution

The steady-state solution is obtained when  $\frac{dC_A}{dt} = 0$  and  $\frac{dT}{dt} = 0$ , that is:

$$f_1(C_A, T) = 0 = \frac{F}{V}(C_{Af} - C_A) - k_0 e^{-\Delta E/RT} C_A$$

The linear model of the system is obtained as:

$$X = \begin{bmatrix} -\frac{F}{V} - k_s & -C_{AS}k'_s \\ -\frac{\Delta H}{\rho c_p} k_s & -\frac{F}{V} - \frac{UA}{V\rho c_p} + \left(\frac{-\Delta H}{\rho c_p}\right) C_{AS}k'_s \end{bmatrix} \begin{bmatrix} C_A \\ T \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{UA}{V\rho c_p} \end{bmatrix} [T_f] \quad (3)$$

TABLE I  
REACTOR PARAMETERS

Reactor Parameter	Description	Values
F/V (hr <sup>-1</sup> )	Flow rate*reactor volume of tank	1
K <sub>0</sub> (hr <sup>-1</sup> )	Exponential factor	10e <sup>15</sup> e <sup>12</sup>
-ΔH (kcal/kmol)	Heat of reaction	6000
E(kcal/kmol)	Activation energy	12189
ρC <sub>p</sub> (BTU/ft <sup>3</sup> )	Density*heat capacity	500
T <sub>f</sub> (K)	Feed temperature	312
C <sub>Af</sub> (lbmol/ft <sup>3</sup> )	Concentration of feed stream	10
$\frac{UA}{V}$	Overall heat transfer coefficient/reactor volume	1451
T <sub>j</sub> (K)	Coolant Temperature	300

### III. MODEL REFERENCE ADAPTIVE CONTROL

Model reference adaptive system is an important adaptive controller. It can be considered as an adaptive servo system in which the desired performance is expressed in terms of a reference model, which gives the desired response to a command signal as shown in Fig. 2. The system has two loops. One is normal feedback loop which consist of the process and the controller and other is parameter adjustment loop that changes the controller parameters. The parameters are changed on the basis of feedback from error, which is the difference between the output of system and the output of reference model [2]. The ordinary feedback loop is called the inner loop and the parameter adjustment loop is called the outer loop. The mechanism for adjusting the parameters in a model reference adaptive system can be obtained in two ways by using a gradient method or by applying Lyapunov stability theory [3]. MRAC is composed of four parts: a plant containing unknown parameters, a reference model for compactly specifying the desired output of control system, a feedback control law containing adjustable parameters [1], [3].

#### A. MIT Rule

The MIT rule, also known as the gradient method, changes the parameters based upon the gradient of the error, with respect to that parameter. The parameters are changed in the direction of the negative gradient of the error. This means that if the error, with respect to a specified parameter, is increasing then by the MIT rule the value of that parameter will be decreased. This control system consists of a reference model, an adjustment mechanism and a controller. The reference model describes the desired input/output dynamics of the closed loop. The controller derives the control signal so that the plant's closed-loop characteristics from the command signal to the plant output are the same as the dynamics of the reference model. The convergence of the modeling error to

zero for any given command signal is assured when plant output exactly follows the output of the model.

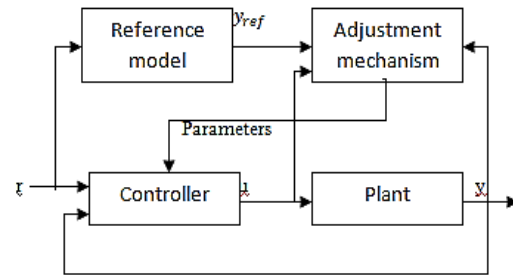


Fig. 2 Model Reference Adaptive Control

The modeling error  $e$  is given by:

$$e = Y - Y_m \quad (4)$$

The controller parameters are adjusted with the loss function  $J(\theta)$ :

$$J(\theta) = \frac{1}{2} e^2$$

To minimize  $J$ , the parameters can be changed in the direction of negative gradient of  $J$ . The rate of change of controller parameters ( $\theta$ ) with respect to time is defined by (5) where the adaptation gain is defined by  $\gamma$  [2], [3], [6], [7].

$$\frac{d\theta}{dt} = -\gamma \frac{dJ}{d\theta} = -\gamma e \frac{\partial e}{\partial \theta}$$

#### A. Lyapunov Rule

An alternative approach to the MIT rule is to use a Lyapunov based method, which avoids the stability problems present in the gradient approaches. In order to derive an update law using Lyapunov theory, the following Lyapunov function is defined as:

$$V = \frac{1}{2} \gamma e^2 + \frac{1}{2b} (b\theta_1 - b_m)^2 + \frac{1}{2b} (b\theta_2 + a - ba_m)^2 \quad (6)$$

The time derivative of  $V$  can be found as:

$$\dot{V} = \gamma e \dot{e} + \dot{\theta}_1 (b\theta_1 - b_m)^2 + \dot{\theta}_2 (b\theta_2 + a - ba_m) \quad (7)$$

Moreover, its negative definiteness would guarantee that the tracking error converge to zero along the system trajectories [6], [7].

#### B. Adaption Law

The adaption law attempts to find a set of parameters that minimize the error between the plant and the reference model outputs. For this purpose the parameters of the controller are incrementally adjusted until the error has reduced to zero. In this



abrupt movement toward or past target regions. Generally, the inertia weight  $w$  is set according to (8):

$$w = \frac{w_{\max} - w_{\min}}{iter_{\max}} * iter \quad (8)$$

Suitable selection of  $w$  provides a balance between global and local explorations, thus it requires less iteration on average to find a sufficiently optimal solution where  $iter_{\max}$  is the maximum number of iterations or generations and  $iter$  is the current number of iterations.

It is a very simple concept, and paradigms can be implemented in a few lines of computer code. It requires only primitive mathematical operators, and is computationally inexpensive in terms of both memory requirements and speed. Early testing has found the implementation to be effective with several kinds of problems [8]-[10], [13]-[16].

#### A. Optimal Tuning of PID Controller Using PSO

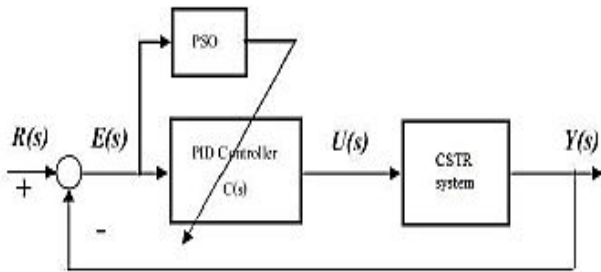


Fig. 5 Block diagram of optimal PID controllers with PSO for CSTR

The parameters of PID controller are tuned by PSO to get optimal PID parameters and by applying PSO-PID controller an excellent output response can be obtained.

Performance characteristic of evaluation function includes overshoot, rise time, settling time and static error time. The evaluation function is defined by (9), to compute the evaluation value of each particle in swarm according to control performance.

The sequence of steps to study the PSO-PID controller for the CSTR system is given below:

- Step 1. Specify the lower and upper bounds of  $K_p$ ,  $K_i$ , and  $K_d$  also randomly initialize the particles of the swarm including swarm size, iteration, acceleration constant, inertia weight factor, the position matrix  $x_j$  and the velocity matrix  $v_j$  and so on.
- Step 2. Calculate the evaluation value of each particle using the evaluation function given in 9.
- Step 3. Compare each particle's new fitness value with its personal best position fitness value, and update the personal best position  $pbest$ .
- Step 4. Search for the best position among all particles personal best position, and denote the best position as  $gbest$ .
- Step 5. Update the velocity  $v_j$  of each particle according to (6), and update the particle position matrix according to (7).
- Step 6. Update control parameter.

Step 7. If the number of iterations reaches the maximum, then stop. The latest  $gbest$  is regarded as the optimal PID controller parameter. Otherwise, go to step 2 [11], [12].

#### B. Performance Indices

A performance index is a quantitative measure of the performance of the system. A system is considered an optimal control system when the system parameters are adjusted so that the index reaches an extreme value, commonly a minimum value [7].

A suitable performance index is the integral of the square of the error, ISE, which is defined as:

$$ISE = \int_0^T e^2(t) dt$$

ISE is more suitable to minimize initial large amount of errors. The squared error is mathematically more convenient for analytical and computational purposes.

Another readily instrumented performance criterion is the integral of the absolute magnitude of the error, IAE, which is written as:

$$IAE = \int_0^T |e(t)| dt$$

This index is particularly useful for computer simulation studies. To reduce the contribution of the large initial error to the value of the performance integral, as well as to emphasize errors occurring later in response, the integral of time multiplied by absolute error, ITAE has been proposed, which is defined as:

$$ITAE = \int_0^T t|e(t)| dt$$

Other performance criteria include evaluation of rise time, settling-time and peak overshoot [12].

## V. SIMULATION RESULTS

#### A. MIT Rule

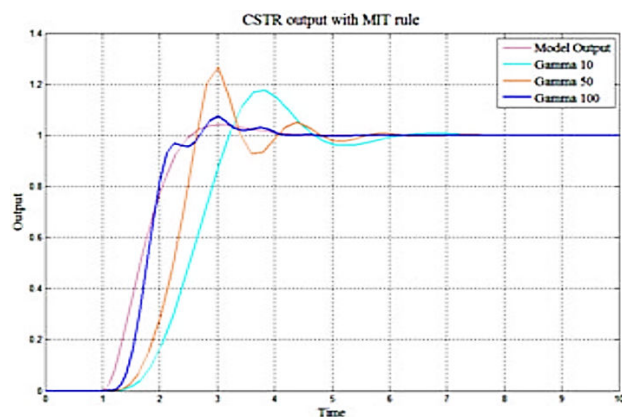


Fig. 6 CSTR output with MIT' rule for different values of adaption Gain

B. Lyapunov's Rule

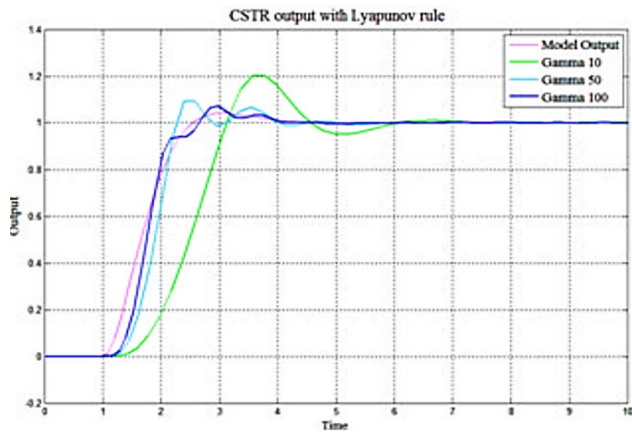


Fig. 7 CSTR output with Lyapunov's for different values of adaption Gain

C. Particle Swarm Optimization

1. PSO Parameters

- Weight / Inertia of the system - 0.5.
- Acceleration constants,  $C_1$  and  $C_2$  - 1.5.
- Swarm population - 100.
- Dimension of the search-space - 3 ( $K_p, K_i, K_d$ )

2. Calculation of Fitness Function

A set of good control parameters  $P, I$  and  $D$  can yield a good step response that will result in performance criteria minimization in the time domain. These performance criteria in the time domain include the overshoot, rise time, settling time, and steady-state error. Therefore, the performance criterion is defined as:

$$W(K) = (1 - e^{-\beta})(M_p + E_{ss}) + e^{-\beta}(T_s - T_r) \quad (9)$$

where  $K$  is  $[P, I, D]$ , and  $\beta$  is the weighting factor. The performance criterion  $W(K)$  can satisfy the designer requirement using the weighting factor  $\beta$  value weighting factor is chosen as 1 in this application.

The fitness function is reciprocal of the performance criterion. It can be written as:

$$F = \frac{1}{W(K)}$$

3. Robustness of PSO Algorithm

To check the robustness of PSO-PID controller, values of PID controller is calculated for different iterations and then calculate the best fitness function [12], [16].

TABLE II  
OPTIMIZATION OF PID TUNING PARAMETERS OF PSO

$K_p$	$K_i$	$K_d$
.0978	.9075	-.1375

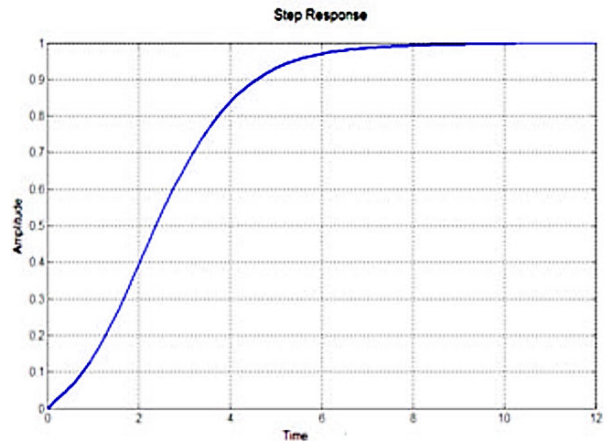


Fig. 8 CSTR output with PSO-PID controller

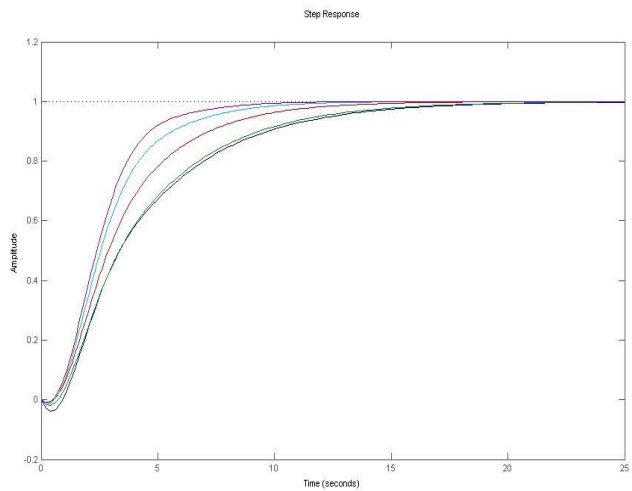


Fig. 9 Plot of PSO parameters for different iterations

TABLE III  
COMPARISON OF PERFORMANCE INDICES

Performance specification	MIT Rule			Lyapunov Rule			PSO
	$\gamma = 10$	$\gamma = 50$	$\gamma = 100$	$\gamma = 10$	$\gamma = 50$	$\gamma = 100$	
Rise time (sec.)	3.27	2.65	2.78	3.1	2.3	2.6	4.47
Peak time (sec.)	3.83	3.0	3.01	3.8	2.4	3	17.00
Maximum Overshoot (%)	17.83	26.7	7.5	20.3	9.6	7.3	0
Settling time (sec.)	7.0	6.2	4.5	6.4	4	3.9	8.65
ISE	.405	.21	.02	.37	.04	.014	.003

VI. CONCLUSION

The proposed adaptive and PSO-PID controller is tested by using Matlab Simulink program and their performance is compared. It is clear from Table II that a PSO-PID controller is best implemented. Also, the PID controller parameters obtained from PSO algorithm gives better tuning result as compared to MIT rule and Lyapunov's rule of adaptive controller. The major impact of PSO is on integral square error and peak overshooting. Both are minimized by PSO-PID controller. PSO-PID is a very simple concept, and paradigms can be implemented in a few lines of computer code. It

requires only primitive mathematical operators, and is computationally inexpensive in terms of both memory requirements and speed. In future the same problem can be solved by adopting other evolutionary algorithms like ant colony algorithm, bacteria foraging algorithm etc.

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