

Academic Motivation Maintenance for Students While Solving Mathematical Problems in the Middle School

M. Rodionov, Z. Dedovets

Abstract—The level and type of student academic motivation are the key factors in their development and determine the effectiveness of their education. Improving motivation is very important with regard to courses on middle school mathematics.

This article examines the general position regarding the practice of academic motivation. It also examines the particular features of mathematical problem solving in a school setting.

Keywords—Teaching strategy, mathematics, motivation, student.

I. INTRODUCTION

THE study of student motivation in general and motivation to learn in particular because it helps to understand the driving forces behind students' learning behaviour. Whilst there is no universal consensus on the nature of such motivation, it nevertheless, a variety of Russian and other research in the fields of psychology and pedagogy [1], [3], [4], [6], show teachers have a general understanding of how important it is to consider students' needs and motivations in academic activity to positively drive their work and behaviour. It is believed that a combination of problem focused, person-oriented, and developmental teaching approaches all help to form student motivation. Problem-based learning concentrates on the analytical and deductive activity of students while thinking and arguing. This is a heuristic or research-oriented type of learning, and it has a great potential. Education refers to the teacher's concentration on student's personality - on the development of his or her sense of responsibility, spirituality and emotional, aesthetic and creative potential. Developmental teaching aims improve cognition, moral and physical features of students by activating their potential abilities, as well as by stimulating their academic activity. This approach helps students to acquire new skills, and be able their management skills all of which will be useful for their future education and subsequent career.

Yet despite the evidence of effectiveness of how to motivate students, it is the authors' belief that most teachers do not act on this evidence, for a variety of reasons, both objective and subjective. One of the factors is insufficient

analysis of the unique characteristics of different subjects in school courses [4].

With regards to school mathematics, these characteristics include the relatively high level of conceptual construct abstraction and the complicated logical patterns of many definitions and theorem statements. The challenge of producing logical and convincing proof consists in using a variety of methodologies such as geometric configuration, and other formal mathematical constructions. The goal is to develop an ability to "speak" and switch between various mathematical "languages" and develop an understanding of the underlying elegance and beauty of mathematics.

II. MOTIVATION HIERARCHY WHILE STUDYING MATHEMATICS

Developing mathematical understanding is determined by a set of relevant needs and motivational factors. This can be envisaged as a triangle of needs which demonstrate linked hierarchic requirements (Fig. 1).

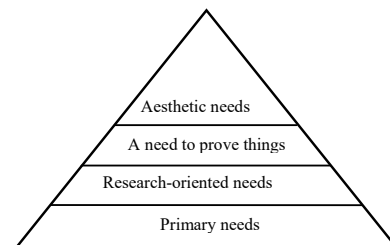


Fig. 1 Motivation hierarchy while studying mathematics

Base of the triangle is our primary need to solve daily tasks. In general, solutions for such tasks require an ability to summarize data gathered during practical activities. Such thing is impossible without a breakthrough in the development of human thinking what marks initiation of research-oriented needs so important for discovering new facts and regularities. Further enhancement of the instrumental mathematical potential was performed, in particular, by minimization of prime postulates for different mathematical theories, as well as such their interpretation with which the conclusion of new patterns would have been the most easy, efficient, and natural.

In real time problem solving by mathematicians, the different factors appear simultaneously. This is especially typical for the creative needs implemented both at the stage of the problem statement and at the stage of a solution choosing

M. Rodionov is with the Department of Mathematics and Computer Science Teaching, Penza State University, Penza, Russia (e-mail: do7tor@gmail.com).

Z. Dedovets is with the Department of School of Education, The University of West Indies, Trinidad and Tobago (e-mail: Zhanna.Dedovets@sta.uwi.edu).

and demonstration argument. The requirement to prove things is being externalized at the stage of solution argument, the need of effective linguistic means—at the stage of characterization of the original problem. Actual needs are especially prominent at the stage of problem choosing and at the stage of the correlation of results with the limitations and restrictions provided by the particular features of the problem. Finally, aesthetic needs—the all-encompassing form of all other needs—can be potentially maintained at all of the mentioned stages.

III. MOTIVATION OF RESEARCH

Research-oriented needs are at the centre of the motivation hierarchy. Their fundamental characteristics are the recognition and acceptance of a problem and the anticipation of an outcome following its solution. For any individual student, acceptance of is an interplay the objective nature and complexity of a problem and that student's personal values and attitude. This conformity is achieved by providing relative freedom both to choose a problem and a method to solve it. A lack of such freedom or its limitation with teacher's instructions can mean rejection of any kind of research activity.

The degree of engagement by a student is also directly related to the challenges involved in producing a solution, and how each student can be helped to engage with and relish such a challenge. "The more complicated is the task, the more valuable becomes the method that leads to the successful solving, the more solid will be its consolidation" [2, p. 96]. This understanding is related to understanding complexity and methodological diversity and limitations. Clearly, the difficulty inherent in any given problem should become an insurmountable barrier where a student becomes trapped in the "psychological vacuum", which causes angst rather than interest [5, p. 81]. Different solutions may be needed to overcome this. For example in the case of Mr. J. Littlewood, he was not able to solve a problem until he found out that his not so advanced colleague had already solved it [7, p. 77, 78].

The more a student anticipates success, the more likely success will be achieved. Students are more likely to accept the "burden" of taking personal responsibility in making decisions and finding solutions [2]-[5]. If they feel their chance to succeed is relatively small, they will fail to be motivated. This can have negative knock on consequences for future problem solving.

Mr. G. Polya has identified several key factors positively identified with a successful approach to methodology and successful research outcomes. These include confirmation of consequences, parallels to the known facts, analysis of contradictory assumptions, credibility of provisional results and inductive control [8].

All of these motivation factors are shown in the next chart (Fig. 2), formed as concentric circles divided into several fields.

The smallest circle of the chart contains the basic mechanisms of motivation for research. The middle circle lists the basic requirements for these mechanisms to function. The

outer, third circle outlines the basic features of such functioning.

All of the marked components are closely linked to each other. However, the relationship between opposite "sectors", reflected in the chart as axes of self-expression and creative work, are the ones of major importance. In particular, the axis of self-expression addresses cooperation between two systems to evaluate the approach of a person to a problem, the evaluation of problem's appeal and personal abilities in terms of its solving. Even if just one of the evaluation features is negative, students will on it be able to perform research only under the external pressure.

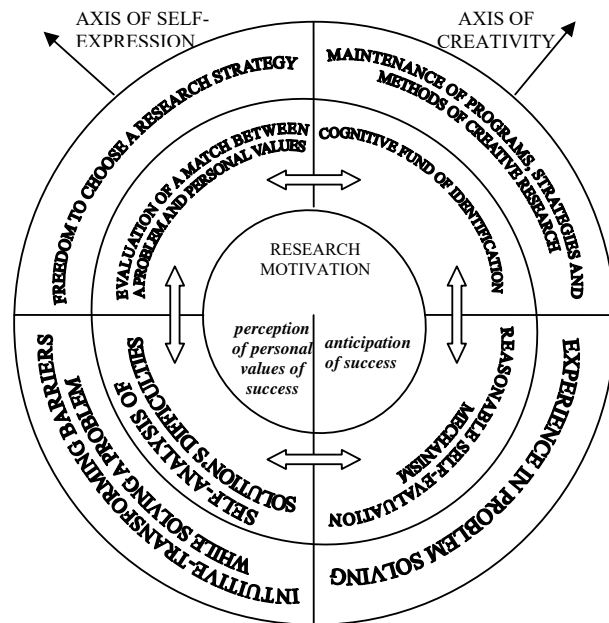


Fig. 2 Motivation factors

Cooperation within the axis of creativity is about constant self-analysis when it comes to different difficulties, barriers, uncertainties in the solution accompanied by the search for means to overcome them. Malfunction of the first factor—that is, students do not recognize the emerging difficulties as a problem—will mean an inability to progress towards a solution without additional intervention. The best case scenario, in the absence of coherent educational strategies and creative work methods is occasional almost happenstance success. The Motivation factors model (Fig. 2) helps the production of effective research-oriented motivation. The two axes of self-expression and creative work are the key strategies.

First strategy will slowly introduce students to their right to choose their research strategy. When students have a recognized "right of the free choice", a chosen object and a method of operating with this object become "personal", which is directly associated with the system of values of students [10]. Successful outcomes are more likely, and will be internalized as personal intellectual values a virtuous circle.

Another strategy to support student research motivation is encouraging an appropriately independent approach to problem solving [5]. By overcoming difficulties, students feel excited by their discoveries, which improve their motivation for further research—again, a virtuous circle. Therefore, the key factor for students to understand the character of difficulties in mathematical tasks and their successful solution is a result-oriented engagement of heuristic procedures to problem solving.

In general, these strategies are implemented together, since the choice itself is a heuristic procedure. However, all creative activity can enhance a general ability to think independently and flexibly, and to transcend boundaries.

IV. RESEARCH MOTIVATION MAINTENANCE AND DEVELOPMENT WHILE SOLVING A MATHEMATICAL PROBLEM

During the initial encounter with a problem, one of the key factors ensuring its acceptance is the student's active role in the inclusion of this problem into his or her system of meaning. This can be facilitated by cooperation with the teacher in shaping the problem, based on the empirical data connected with the previously received experience, or by reference to an existing mechanism of personal adaptation.

The mechanism of personal adaptation can be used, for example, while setting and solving problems of a historical type, where the mechanism is supported by the authors' influence (Pythagorean Theorem, Newton's problems, etc.). Personal interest in solving problems can be achieved directly by the teacher. In this case, the problem has an external nature ("such problems will be included to the test" or "these problems can be solved only by the smartest students", etc.).

Transition to the more complicated levels of research requires a qualitative upgrade of the means to engage students into initial setting of a problem. These primarily include the relatively independent compilation and selection of problems by students based on several initial references (setting a problem based on a chart or a layout; setting a problem associated with the given problem; selection of tasks that are solved in an already known way, etc.).

Let us consider the following example:

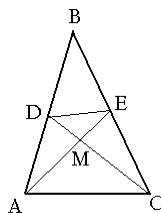


Fig. 3 The triangle ABC

Viewing triangle's configuration in class given together with its midline (Fig. 3), it is necessary to ask students to mark all the possible patterns for this triangle. As a result, based on our personal experience, we might get a lineup of statements, such as:

1. Triangle's midline divides it into a triangle and a

trapezoid.

2. Diagonal lines of the trapezoid are the medians of the initial triangle.
3. Each median divides the triangle into two identically equal triangles.
4. Triangles DMA and EMC are identically equal.
5. Triangles AMC and quadrangle DBEM are identically equal. Etc.

Such analysis of statements, based on an imaginary experiment, is usually more motivating than solving problems from textbooks.

When a problem is introduced to students the primary motivational goal is to understand the complexities and possible contradictions in a statement of any problem, which may initially manifest itself as something uncertain or blurry [4], [7].

For example, many studies show a steady predisposition of some students to thinking with formulas, while others always lean on geometric images. Focusing on re-stating the conditions allows us to choose the most preferable variant, as well as increasing chances of successful solving. For instance, while answering a question "How many solutions does the system of equations (1) have?"

$$\begin{cases} x^2 + y^2 = 4 \\ y = x^3 \end{cases} \quad (1)$$

Students of ninth grade struggle while using the more familiar analytical method. The immediate translation of the problem's statement into the language of geometry makes the solution much clearer.

When moving to the next stage of the problem's solution — search for the solution approaches — there are two possible cases. If the solution has a sufficiently standardized character, the main motivation factor at this stage — the mechanism of the result's anticipation (as well as mechanism of achieving the result) — depends proportionally on the level of person's ability to solve the main contradiction of the problem. When a solution of this contradiction is impossible without an "intuitive impetus", already learnt methods of may interrupt further research activity. To go further, it is necessary to bring in particular heuristic procedures that allow students to link their problem with the given system of familiar ideas and images, but in a new way [2], [8], [9].

One of these procedures is: generalization, which requires a wider review of the situation than the initial one, and a generalization of the content (explicit or implicit) based on extension of terms and links between them. Others include identification — revealing the background of a problem, alignment—the search for links and consistencies between facts not generally connected to each other. Then there are parameterization (and deparameterization) — namely the direct and reversed transition from actual statements to suggestion with a variable and introduction—the involvement of new links that help to reveal hidden connections between elements of a problem. There is also inversion — direct transfer of a problem from one section of mathematics into others,

followed by re-defining of its content in terms of different mathematical language and then specification — the selection of a critical or "confluent" case to demonstrate factors crucial for the solution of a problem. In addition there is decentration — the redefining of subject elements of a problem in the context of alternative configuration, reduction — the simplification of the initial problem into the easier sub-problems. Further, there is alteration — the partial change (upgrade) of some parameters to change the given situation into something familiar and reconstruction — the restoration of an initial configuration into something integral which allows the use of multiple devices. These procedures can be applied separately or in various combinations, depending on the nature of the problem and the level of development of each student. This is the very essence of creative mathematical problem solving [9], [10].

The challenge is to find the right approach, or combination of approaches to develop a repertoire of approaches and a generalized and positive ability to learn. However, relation of activity subjects to such procedures differs at multiple stages. If implementation of any method at the initial level is considered by students as an insight, by moving to a higher stage (if subject heuristics are being formed), its implementation becomes more and more deliberate — it is being actualized because of a conscious choice out of a variety of conceptual choices. Nonetheless, the bigger is the set of procedures available to students and the more generalized these procedures are, the more effective will be the choice.

V. CONCLUSION

Successful mathematical problem solving requires an understanding of its special features and of relevant needs and motivation factors. An analysis of these factors enables an understanding of the hierarchic links between these factors, shown as a triangle of needs. The key part of this triangle is the research-oriented motivation, defined as an aspiration for self-expression when solving creative problems.

The main strategy for its maintenance and development include the phased introduction of students into the free choice of search directions. When students have a recognized "right of a free choice", a chosen object and a method of operating with this object become "personal" and internalized, directly associated with the student's system of values. Another strategy to maintain the research motivation of students is the development of an appropriately independent overcoming of difficulties inherent in the contradiction between the research and the given system of the knowledge and methods. By overcoming this contradiction based on the direct involvement of the heuristic procedures to the analysis, students feel positive emotions which positively reinforce motivation. These statements were approved in some of the schools. The results supported availability and efficiency of the offered approaches.

REFERENCES

- [1] Kovalev, V. I. (1988). Behavior and activity patterns. Moscow, Science.
- [2] Kulkyutkin, Yu.N. (1970). Heuristic methods in the problem solving structure. Moscow, Pedagogics.
- [3] Leontyev, A. N., (1975). Activity. Mind. Personality. Moscow, Politizdat.
- [4] Rodionov, M. A. (2012). Motivation in mathematics. From theory to practice. Saarbrücken, Germany, Palmarium Academic Publishing.
- [5] Shakurov, R. Kh. (1999). Psychology of overcoming: Shape of the new theory. Izvestiya RAO, Moscow, Magistr, pp. 77-88.
- [6] Heckhausen, H. (1980). Motivation und Handeln. Springer-Verlag, Berlin Heidelberg.
- [7] Littlewood, J. E. (1953). A Mathematician's Miscellany. London, Methuen and Co Ltd.
- [8] Polya, G. (1957). How to Solve It. Princeton University Press, 2nd ed.
- [9] Rodionov, M., and Velnisova S. Construction of Mathematical Problems by Students Themselves / M. Rodionov, S. Velmisova // Proceedings of the 34th Conference on Applications of Mathematics in Engineering and Economics (AMEE '08); American Institute of Physics (Melville, NY 11747 USA) DOI: 10.1063/1.3030789 AIP Conf. Proc. --Issue Date: October 30, 2008-- Volume 1067, pp. 221-228. <http://proceedings.aip.org/proceedings/>.
- [10] Rogers, C. R. (1990). Freedom to Learn for the 80's. Macmillan Publishing Company, New York Rothstein P R.