A Simulation-Optimization Approach to Control Production, Subcontracting and Maintenance Decisions for a Deteriorating Production System

Héctor Rivera-Gómez, Eva Selene Hernández-Gress, Oscar Montaño-Arango, Jose Ramon Corona-Armenta

Abstract—This research studies the joint production, maintenance and subcontracting control policy for an unreliable deteriorating manufacturing system. Production activities are controlled by a derivation of the Hedging Point Policy, and given that the system is subject to deterioration, it reduces progressively its capacity to satisfy product demand. Multiple deterioration effects are considered, reflected mainly in the quality of the parts produced and the reliability of the machine. Subcontracting is available as support to satisfy product demand; also, overhaul maintenance can be conducted to reduce the effects of deterioration. The main objective of the research is to determine simultaneously the production, maintenance and subcontracting rate, which minimize the total, incurred cost. A stochastic dynamic programming model is developed and solved through a simulation-based approach composed of statistical analysis and optimization with the response surface methodology. The obtained results highlight the strong interactions between production, deterioration and quality, which justify the development of an integrated model. A numerical example and a sensitivity analysis are presented to validate our results.

Keywords—Deterioration, simulation, subcontracting, production planning.

I. INTRODUCTION

IN many industries, firms are faced to a number of factors (i.e. delays, defectives, failures, wear, etc.) that limit progressively its capacity to satisfy product demand. Furthermore; customers require that production systems respond quickly to their demand, thus, subcontractors can be an attractive option. Certainly, the higher costs related with subcontracting can be justified by the reduction in the inventory, and in the risk of lost sales. Also in real production, maintenance strategies could be available to mitigate the effects of deterioration. In this context, we investigate the joint production, subcontracting and maintenance strategies for an unreliable manufacturing system, where the effects of its deterioration are reflected in several performance indices, reducing progressively its capacity to satisfy product demand.

The production control problem for a failure prone machine was introduced by [1], who derived the optimality conditions that led to the control policy of a small production system. Subsequent papers refined and extended the HPP in different

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directions, as in [2], who described the connection between the HPP and other scheduling policies for manufacturing systems. Some papers have highlighted the importance of the relationship of quality issues and production. For instance [3], [4] introduced analytical and computational methods to evaluate the performance of small and long manufacturing lines with defective production. A recent area of research, integrates the influence of quality issues on the production control policy, as in [5], where they presented a joint strategy of lot production and preventive maintenance, where the rate of nonconforming units is used to determine the conduction of preventive maintenance. Reference [6] dealt with the joint optimization of the batch size, the production threshold and the sample size for inspection, of an unreliable machine. As we can notice since the manufacturing system can be subject to progressive deterioration, the relationship between quality and production can be studied from a different angle. Thus, the concept of deterioration may provide useful insight to our research.

In most existing deterioration models, uses the age of the machine or the number of repairs to denote the level of deterioration of the machine, as in [7], where the failure intensity can be partially reset with a repair or it can be completely refresh with a major maintenance. A model for the performance evaluation of a multi-state degraded system is provided in [8], where if inspection finds that the system is in an unacceptable state, where it cannot satisfy the demand, preventive maintenance is performed to restore partially the machine. Reference [9] addressed the case of a deteriorating system, where preventive maintenance causes an age reduction and a decrease in the failure rate. In [10] it is proposed a general theory to analyze the production rate of conforming units in production systems with progressively deteriorating machines and preventive maintenance. From the presented papers, we observe that deterioration can have multiple severe effects. However, it is needed more research to fully understand such effects on the production control policy.

Operation planning is critical for the proper profitability of a firm, and in the context of limited capacity, subcontracting can be a viable solution as indicated in [11], who derived a feedback policy to determine the production rate and the rate at which subcontractors are required for the case of backlog-dependent demand. In [12] it is studied the joint maintenance and production management for a production system which satisfies a principal customer, but that also performs subcontracting tasks. We find also in [13], a production

control policy of an unreliable machine, where a reserve machine is called upon in support to satisfy product demand. From a practical point of view, [14] treated the case of a pharmaceutical industry with multiple facilities producing two meditations (brand name and generic), where subcontracting was applied to compensate for insufficient production capacity. Indeed, in these papers, subcontracting is not analyzed in the case where the machine progressively reduces its capacity by deterioration. In real production, a degradation process may have multiple effects, reflected mainly in the reduction of quality and reliability of the system.

The main purpose of this study is to analyze a manufacturing system subject to deterioration, which increases the rate of defectives, and the failure intensity. The problem is formulated as a stochastic dynamic programming model, and given the difficulty to obtain analytical solutions, a simulation-based approach is proposed, to jointly optimize the related control parameters. We seek to minimize the total incurred cost, and an extensive sensitivity analysis is conducted to provide a better insight about the production system behavior.

The rest of the paper is organized as follow: Section I presents an Introduction for the addressed problem; the notations are introduced in Section II. The problem's motivation is explained in Section III. The formulation of the model is described in Section IV. In Section V, we detailed the structure of the control policy. Section VI describes the simulation-based approach applied. A numerical example is illustrated in Section VII. In Section VIII we present a sensitivity analysis. Finally, Section IX concludes de paper.

II. NOTATION

The following notations are used in the formulation of the production control model:

$x(t)$	Inventory level at time t
$a(t)$	Age of the machine at time t
d	Constant demand rate of products
$\xi(t)$	Stochastic process
$Q(t)$	Transition rate matrix of the stochastic process
$u_0(\cdot)$	Production rate of the manufacturing system
$u_1(\cdot)$	Subcontracting rate
$\bar{u}_o$	Maximum production rate of the producer
$\bar{u}_1$	Maximum production rate for subcontracting
$\beta(\cdot)$	Rate of defectives
ρ	Discount rate
π_i	Limiting probability at mode i
$q_{\alpha\alpha'}(\cdot)$	Transition rate from mode α to mode α'
$\gamma^{\xi(t)}(\cdot)$	Cost rate function associated with the process
$\xi(t) J(\cdot)$	Expected discounted cost function
$v(\cdot)$	Value function
$\omega(\cdot)$	Control variable for the overhaul activity
$\bar{\omega}$	Maximum overhaul rate
$\underline{\omega}$	Minimum overhaul rate
τ	Jump time of $\xi(t)$
c^+	Inventory holding cost / units / time unit
c^-	Backlog cost / units / time unit
c_r	Repair cost
c_o	Overhaul cost
c_d	Cost of defectives
C_{M1}	Cost of production of the manufacturing system per unit of produced parts
C_{M2}	Cost of production of subcontracting per unit of produced parts
θ_d	Adjustment parameter for the rate of defectives
θ_f	Adjustment parameter for the failure rate

III. MOTIVATION

Our paper is mainly motivated by [15], who studied a real production case of a multiple unreliable manufacturing facilities with limited production capacity, which produce two chemicals that are difficult and expensive to store. They determined production and subcontracting policies, and in order to simplify their model, these authors formulated a continuous-time stochastic optimal control problem, assuming a constant failure rate. Nevertheless, the actual failure rate of such system is not constant; in fact, it increases very slightly as the time since the last repair increases. Hence, this condition implies a deterioration process. However, the effect of deterioration has been disregarded in that paper. In practice, deterioration may have a severe influence not only in the failure rate, but also in other performance indices of the system such as quality, safety, throughput, etc. that may reduce progressively its production capacity, as presented in [16], [17]. In this context, subcontracting can be an attractive option to provide additional capacity to achieve high levels of customer service. The study presented in our paper has many applications in several production sectors, such as the textile, apparel, retail channel, pharmaceutic, semiconductor, etc., normally in situations where the production system has a limited capacity and subcontracting is used to achieve capacity flexibility to gain and maintain a competitive advantage.

IV. CONTROL MODEL FORMULATION

Without loss of generality, we focus on the case of one machine subject to a double deterioration in order to gain insight about the behavior of the production system. Our formulation begins with a stochastic process $\{\xi(t), t \geq 0\}$, with values in $\Omega = \{1,2,3\}$. The machine is operational when $\{\xi(t) = 1\}$, and unavailable when under repair $\{\xi(t) = 2\}$ or under overhaul $\{\xi(t) = 3\}$. The repair refers to a minimal maintenance that leaves the machine to as-bad-as-old conditions (ABAO). Meanwhile, overhaul is a perfect maintenance that restores the machine to as-good-as-new conditions (AGAN).

At any instant of time, the production rate must satisfy the capacity constraint: $0 \leq u_0(\cdot) \leq \bar{u}_o$, where $\bar{u}_o$ is the maximum production rate of the machine. Since deterioration increases progressively the rate of defectives, in our model the dynamics of the inventory/backlog of products $x(t)$ evolves according with the following differential equation:

$$\dot{x}(t) = u_0(t,\alpha) \cdot [1 - \beta(a)] + u_1(t) - d, x(0) = x_0 \quad (1)$$

where x_0 refers to the initial inventory level, a is the age of the manufacturing system, and $\beta(a)$ represents the rate of

defectives as a function of the age. We define the age of the machine at time t as a function of its production rate since the last restart, as:

$$\dot{a}(t) = k_1[u_0(t)], \quad a(T)=0 \qquad (2)$$

where the constant k_1 denotes a positive constant and T is the last restart time of the machine following overhaul activities. The quality deterioration phenomenon is modeled with an increasing function, given that the wear of the machine can be used to model different quality yields, as suggested in [4]. Therefore, we have:

$$\beta(a) = b_1 + b_2(1 - e^{-k_2 \cdot \theta_d \cdot [a(t)^3]}) \qquad (3)$$

where θ_d is an adjustment parameter for the pace of deterioration of the rate of defectives, and it varies in the interval $0 \le \theta_d \le 1$, b_1 is the initial value of the rate of defectives, b_2 represents the upper limit and k_2 indicates a

given constant. Fig. 1 (a) illustrates the quality deterioration of the rate of defectives for different values of θ_d (with $b_1 = 0.01$, $b_2 = 0.99$ and $k_2 = 15x10^{-6}$).

In our model deterioration also has an influence on the reliability of the machine, principally reflected on its failure intensity as presented in [7] and [8]. Therefore, we propose an increasing function for the failure rate as,

$$q_{12}(a) = q_1 + q_2(1 - e^{-k_3 \cdot \theta_f \cdot [a(t)^3]}) \qquad (4)$$

with $0 \le \theta_f \le 1$, where the parameter θ_f is used to adjust the trend of the failure rate as presented in Fig. 1 (b) (with $q_1 = 0.1$, $q_2 = 0.4$ and $k_3 = 15x10^{-6}$), q_1 is the transition q_{12} at initial conditions, q_2 is the upper limit considered for the deterioration of the transition q_{12}, and k_3 is a given constant. The adjustment parameters θ_d and θ_f, serves us to emphasize whether the deterioration has a stronger effect on the quality part or reliability.

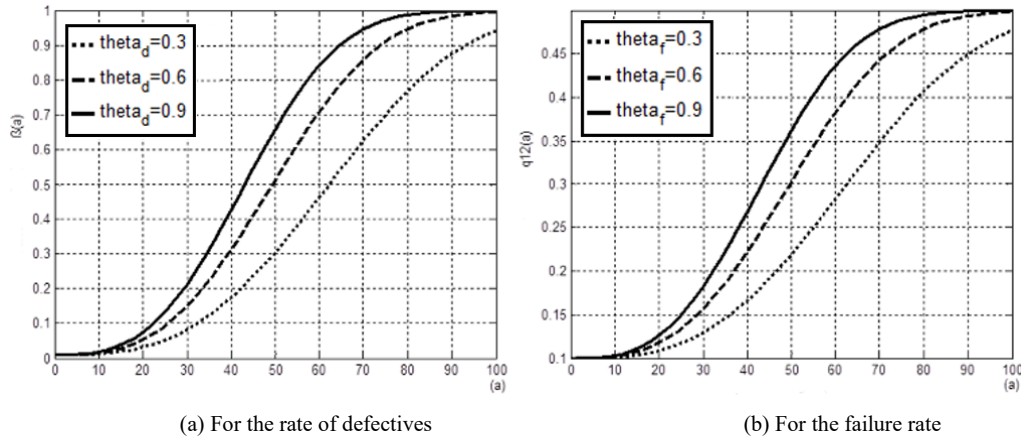

(a) For the rate of defectives
(b) For the failure rate

Fig. 1 Trend of the deteriorating process

We assume Markov dynamics for $\xi(t)$, with exponential switching times for the transition rates $q_{\alpha\alpha'}$ from mode α to mode α'. The manufacturing system will only fulfill the product demand if $\bar{u}_o(\cdot) \cdot [1 - \beta(a)] \cdot \pi_1 \ge d$, where π_1 denotes the steady state availability of the manufacturing system, given by, $\pi_1(a) = 1/\left(1 + \frac{q_{12}(a)}{q_{21}}\right)$. However, due to the twofold effect of deterioration on the machine, eventually we need more subcontracting to fulfill the product demand. Hence, the whole capacity constraint of the systems is:

$$\bar{u}_o(\cdot) \cdot [1 - \beta(a)] \cdot \pi_1(a) + \bar{u}_1(\cdot) \ge d \qquad (5)$$

where $\bar{u}_1(\cdot)$ is the maximum allowable subcontracting rate. Our decision variables are then, the production rate of the machine $u_0(t)$, the subcontracting rate $u_1(\cdot)$ and the overhaul rate $\omega(\cdot)$. We define $\omega(\cdot)$ equal to the transition q_{13}, where the reciprocal of $\omega(\cdot)$ denotes the delay to send the machine to overhaul activities, with $\underline{\omega} \le \omega(\cdot) \le \bar{\omega}$, where $\underline{\omega}$ and $\bar{\omega}$, denote the minimum and maximum overhaul rate. Let $\Gamma(\alpha)$ be the set

of admissible decisions, including the decision variables (u_o, u_1, ω), we have:

$$\Gamma(\alpha) = \{(u_0(a,\cdot), u_1(a,\cdot), \omega(a,\cdot)) \in R^3,$$
$$0 \le u_0(a,\cdot) \le \bar{u}_o, 0 \le u_1(a,\cdot) \le \bar{u}_1, \underline{\omega} \le \omega(a,\cdot) \le \bar{\omega}\} \qquad (6)$$

The key performance measure for this system is denoted by the instantaneous cost function of the model at mode $\alpha \in \Omega$, which is defined as:

$$\varphi^\alpha(\alpha, x, a, u_0, u_1, \omega) = c^+ x^+ + c^- x^- + C_{M1} \cdot u_0(t) + C_{M2} \cdot$$
$$u_1(t) + c_d \cdot \beta(a) \cdot u_0(t) + c^\alpha, \forall \alpha \in \Omega \qquad (7)$$

with:

$$x^+ = max(0, x)$$
$$x^- = max(-x, 0)$$
$$c^\alpha = C_r \cdot Ind\{\theta(t) = 2\} + C_o \cdot Ind\{\theta(t) = 3\}$$
$$Ind\{\theta(t) = \alpha\} = \begin{cases} 1 \; if \; \theta(t) = \alpha \\ 0 \; otherwise \end{cases}$$

where c^+ and c^- are the inventory and backlog cost, respectively; C_{M1} is the production cost of the manufacturing

system; C_{M2} refers to subcontracting cost, and c_d denotes the cost due to the additional inspection and disposal of defective parts. In addition, C_r defines the repair cost, and C_o denotes the overhaul cost. It is assumed that the cost of subcontracting C_{M2}, is much higher than the production cost C_{M1}, and so $0 < C_{M1} \ll C_{M2}$. To make it more concrete, our objective then is to determine in $\Gamma(\alpha)$ an optimal control policy (u_0^*, u_1^*, ω^*) which minimizes the integral of the following expected discounted cost:

$$v(\alpha, x, a, u_0, u_1, \omega) = \inf_{(u_o, u_1, \omega) \in \Gamma(\alpha)} E\left[\int_0^\infty e^{-\rho t} \varphi^\alpha(\cdot) dt \mid \xi(0) = \alpha, x(0) = x_0, a(0) = a_0\right], \forall (u_0, u_1, \omega) \in \Gamma(\alpha) \,(8)$$

where ρ is the discount rate and $E[\cdot \mid \alpha, x_0, a_0]$ symbolize the conditional expectation operator, and $v(\cdot)$ is the value function. Problem (1)-(8) is a stochastic dynamic programming problem. Hence, we use the HJB equation to determine the structure of the solution. However, since analytical solutions for this kind of models are troublesome to obtain, we adopted a numerical approach to approximate a solution.

V. STRUCTURE OF THE OPTIMAL CONTROL POLICY

The numerical method used to determine the structure of the optimal control policy is based on the Kushner's approach. The main idea of this numerical method is to apply an approximation scheme for the gradient of the value function, where a discrete function is used to approximate iteratively the continuous value function and its derivatives. For more information about the Kushner's approach refer to [18], [19]. We present in Table I the parameters used in the numerical example.

TABLE I
PARAMETERS FOR THE NUMERICAL EXAMPLE

Parameter:	$c+$	$c-$	c_r	c_o	C_{M1}
Value:	1	250	5	10	4
Parameter:	C_{M2}	c_d	h_x	h_a	ρ
Value:	40	10	1	1	0.9
Parameter:	$\bar{u}_o$	$\bar{u}_1$	$\underline{\omega}$	$\bar{\omega}$	d
Value:	9	20	10^{-6}	20	4
Parameter:	q_{21}	q_{31}	q_1	q_2	θ_f
Value:	5	4	0.55	1	0.6
Parameter:	θ_d	k_1	k_2	k_3	b_1
Value:	0.7	0.1	15×10^{-6}	15×10^{-6}	0.01
Parameter:	b_2				
Value:	0.99				

The solution of the Kushner's approach divides the plan (x, a) into three regions, where the production rate is set to $\bar{u}_o, d/[1 - \beta(a)]$ and 0, respectively, as presented in Fig. 3 (a). Moreover, the subcontracting policy divides the plan (x, n) into three regions, where the subcontracting rate is set to 0, then in the second region at the difference $(d - \bar{u}_o \cdot [1 - \beta(\cdot)])$ or at the demand rate, and in the third region at the maximum subcontracting rate $\bar{u}_1$, as illustrated in Fig. 2 (b).

Given the effects of deterioration, the production policy leads to an extension of the Hedging Point Policy which needs to be determined in three intervals: $a \leq A$, $A < a < B$ and $a \geq B$, where point A indicates the age when $\bar{u}_0 = d/[(1 - \beta(a)) \cdot \pi_1(a)]$ and at which subcontracting is required (around age $a = 42$ in Fig. 2 (b)), and point B indicates the age when the machine is so deteriorated that is must be stopped (around age $a = 60$ in Fig. 2 (a)). The *production policy* is thus defined as follows:

i. For the case where $a \leq A$, we have $\bar{u}_o \geq d/[(1 - \beta(a)) \cdot \pi_1(a)]$, hence the production policy is:

$$u_0^*(1, x, a) = \begin{cases} \bar{u}_o & if \ x < Z_0 \\ d/[1 - \beta(a)] & if \ x = Z_0 \\ 0 & if \ x > Z_0 \end{cases} \quad (9)$$

For the case where $A < a < B$, the machine is no longer capable of satisfying the product demand on its own because $\bar{u}_o < d/[(1 - \beta(a)) \cdot \pi_1(a)]$ and so:

$$u_0^*(1, x, a) = \begin{cases} \bar{u}_o & if \ x \leq Z_0 \\ 0 & otherwise \end{cases} \quad (10)$$

ii. For the case where $a \geq B$, it is not profitable to continue operating the manufacturing system, then:

$$u_0^*(1, x, a) = \{0 \quad in \ any \ case \quad (11)$$

where Z_0 represents the stock level that delimits the optimal production threshold at the operational mode

With respect to the *subcontracting policy* we observe that it is triggered, as indicated in the following two intervals:

i. For the case where $a \leq A$, we must recall that $\bar{u}_0 \geq d/[(1 - \beta(a)) \cdot \pi_1(a)]$, and hence the subcontracting policy is:

$$u_1^*(1, x, a) = \{0 \quad in \ any \ case \quad (12)$$

ii. For the case where $A < a$, we have $\bar{u}_0 < d/[(1 - \beta(a)) \cdot \pi_1(a)]$, and subcontracting is required, as follows:

$$u_1^*(1, x, a) = \begin{cases} \bar{u}_1 & if \ x < 0 \\ d - \bar{u}_o \cdot [1 - \beta(\cdot)] & if \ x = 0 \ and \ A < a < B \\ d & if \ x = 0 \ and \ B \leq a \\ 0 & otherwise \end{cases} \quad (13)$$

As indicated in Fig. 2 (b), we note that once subcontracting is available, first, it is conducted when the stock level is negative to avoid backlog, and then when the machine has stopped, subcontracting satisfies all the product demand.

The *overhaul policy* is presented in Fig. 3 (a), which serves us to identify two zones:

- Zone A_o: The recommendation is to perform overhaul activities, because the amount of deterioration justifies the cost for this type of maintenance.
- Zone B_o: In this zone of the plane, an overhaul is not recommended, and it is more profitable to continue operating the manufacturing system.

At this juncture, to define de overhaul policy, we must consider simultaneously the production and the overhaul boundaries as presented in Fig. 3 (b). Since the stock level is limited by the production threshold, and only a part of the overhaul zone A is active, defining then the feasible overhaul Zone A_o'.

Thus, the logic behind the obtained results indicates that overhaul activities are conducted at a rate $\omega^*(\cdot)$ given by the following equation:

$$\omega^*(1, x, a) = \begin{cases} \bar{\omega} \ if \ [a(t) > SA \ and \ x(t) \geq 0] \ or \ [a(t) \geq B] \\ \underline{\omega} \ otherwhise \end{cases} \quad (14)$$

When the manufacturing system has stopped its operation (after point B), it is sent automatically to overhaul maintenance, to restore the machine to initial conditions.

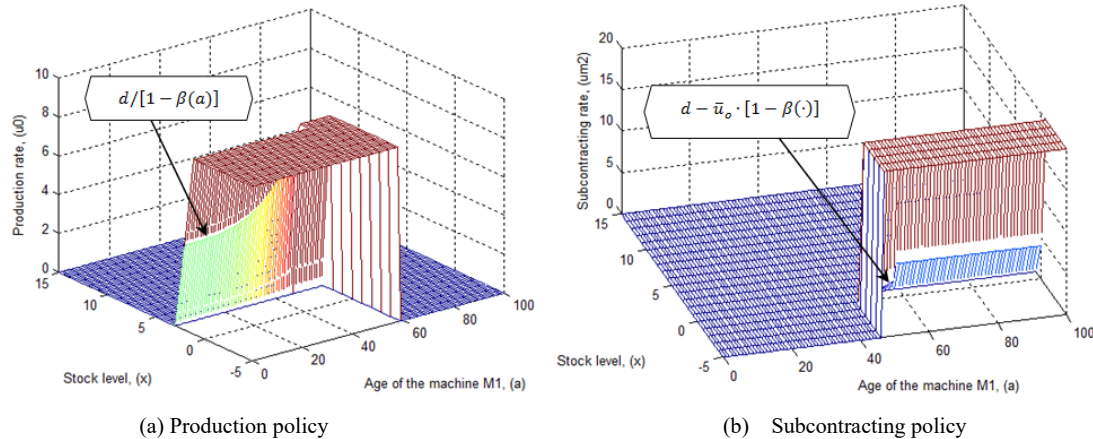(a) Production policy   (b) Subcontracting policy

Fig. 2 Control policies

The practical disadvantage of the control policy obtained by numerical methods stems from the standpoint that an approximation of the control parameters will be too time-consuming to be applicable at the operational level. Therefore, we propose a simulation control approach to approximate the optimal control parameters, $Z_0^*(\cdot)$ and points A and B for the production and subcontracting rates, and point SA for the overhaul policy. A significant advantage of the simulation approach is that it is more flexible and it allows us to examine the control policy in a wide range of time and different cost variations in reasonable times.
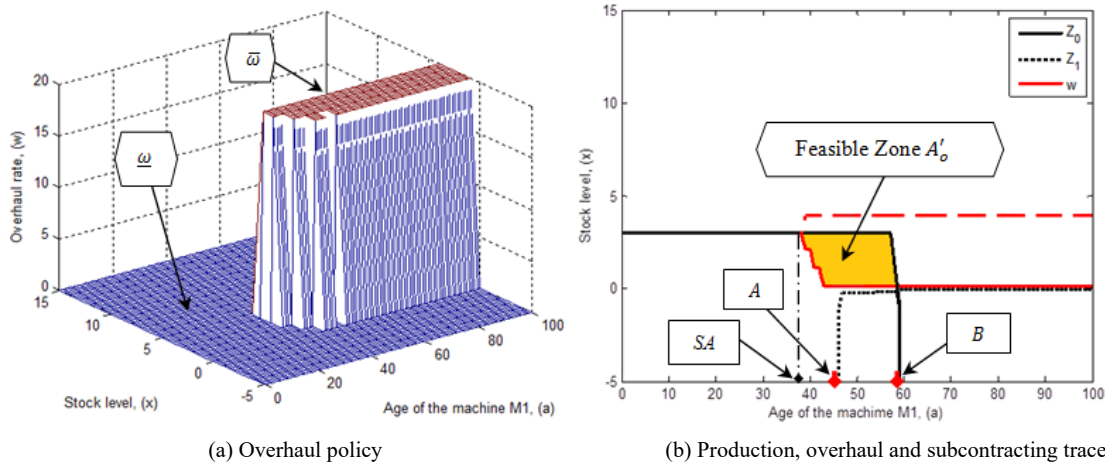

(a) Overhaul policy   (b) Production, overhaul and subcontracting trace

Fig. 3 Joint control policy

VI. SIMULATION OPTIMIZATION APPROACH

In order to approximate the control parameters (Z_0, SA, A, B) and the overall expected cost, we proposed a simulation-based approach, which combines a simulation model with statistical analysis, comprising design of experiments and parameter optimization with the responses surface methodology. This approach has been successfully applied in several control problems as presented in [20], and it consists of the following sequential steps:

Step 1. *Control problem formulation:* this step aims to formulate analytically a stochastic dynamic programing model of the manufacturing system under analysis, as presented previously in Section IV. Here we specify the dynamics, states, calculation of the expected average cost.

Step 2. *Structure of the optimal control policy:* Numerical methods are applied, as detailed in Section V, to approximate the structure of the control policy; with this, we identified the control parameters (Z_0, SA, A, B).

Step 3. *Simulation model:* it is developed a simulation model to reproduce accurately the stochastic behavior and all the set of characteristics considered in our manufacturing system. The simulation model uses as inputs the control parameters (Z_0, SA, A, B), and provides a measure of its performance, with the incurred total cost, which is defined as the output of the model. More details about the simulation model are presented in the next section.

Step 4. *Statistical analysis:* through a minimal set of simulation runs, a design of experiments is conducted to define how the variation of the control factors (Z_0, SA, A, B) can be used to define the significant main factors and their interactions with respect to the total incurred cost.

Step 5. *Parameters optimization:* once identified the significant main factors and interactions, it is applied the response surface methodology to define their relationship with the total incurred cost. The expression obtained is then optimized to determine the optimal values of the control parameters (Z_0, SA, A, B) and their total incurred cost.

A. Simulation Model

The logic associated with the combined discrete-continuous simulation model developed is examined in this section. We used the simulation software Arena with C++ subroutines, the model consists of a number of networks, which describes a specific activity or event in the system. The simulation model can be described with the following networks, presented in the block diagram of Fig. 4, as follows:

1. *The INITIALIZATION:* this block sets the value of different parameters, such as the control parameters (Z_0, SA, A, B), system's parameters such as $(\bar{u}_o, d, \bar{u}_1)$, and the transitions values for the different modes. Further, it defines the simulation run-time and the length of the warm-up period to ensure that steady-state conditions are reached.

2. *The FAILURES AND REPAIRS:* it models the Markov dynamics inherent of the failure and repair events, it uses the transition values defined in the previous network. It communicates with the *state equation block* to define the operational and failure state.

3. *The PRODUCTION CONTROL POLICY:* it uses (9)-(11) to define the proper production rate in function of the stock level and the age of the machine. A set of flags are required as a detection mechanisms, to indicate when the current stock level or machine's age crosses certain thresholds, and adjust the production rate applied in the differential equations (1) and (2).

4. *The SUBCONTRACTING POLICY:* based on (12)-(13) trough detections mechanisms, the stock level and the age of the machine are permanently monitored to set adequately the subcontracting rate.

5. *The OVERHAUL POLICY:* is implemented with the use of (14). It communicates with the *Failures and Repairs block* to synchronize the random events (failures, repairs and overhaul). Also it interacts with the *State Equation block* to restore the rate of defectives and the failure intensity once an overhaul has been conducted.

6. *The DEGRADATION block:* receives information from the failures & repairs block, and the production, subcontracting and overhaul policy to update properly the rate of defectives and the failure intensity as described in (3) and (4). The effects of deterioration are restored to initial conditions when an overhaul is conducted.

7. The *STATE EQUATION*: it is a very important component of the model, since it describes the differential equations (1) and (2), embedded in a C++ language insert, defining the evolution of the stock level and the age of the machine. For its correct operation, it requires the production rate, the rate of defectives, the state of the machine, and the overhaul and subcontracting rate, then it involves an interaction with several blocks of the model.

8. The *ADVANCE TIME*: with a combination of discrete events scheduling (i.e., failures, repairs and overhaul), continuous variables threshold crossing events (i.e., stock level and age of the machine) and time step specifications, the current time is updated.

9. The *UPDATE STOCK AND AGE LEVEL:* this block tracks any variations of the stock level and the age of the machine for the specified time step, and it integrates the cumulative variables using the Runge-Kutta-Fehlberg algorithm.

10. *The OUTPUT:* once the current time T_{now} has reached the predefined simulation run-time T_{sim}, the simulation model is stopped, providing time persistent statistics of the positive, negative stock, the number of overhauls conducted, age of the machine, and rate of defectives.

At the end of the simulation run, the total expected cost $J(\cdot)$ is calculated based on (7). The length of the simulation period T_{sim} was defined as 100,000 time units to ensure steady-state conditions.

VII. NUMERICAL EXAMPLE

The true usefulness of our simulation control approach is illustrated with a numerical example. We combine the simulation model with statistical analysis based on designs of experiments and parameter optimization with the response surface methodology. To determine the control parameters, we propose an approximation as presented in Fig. 5.
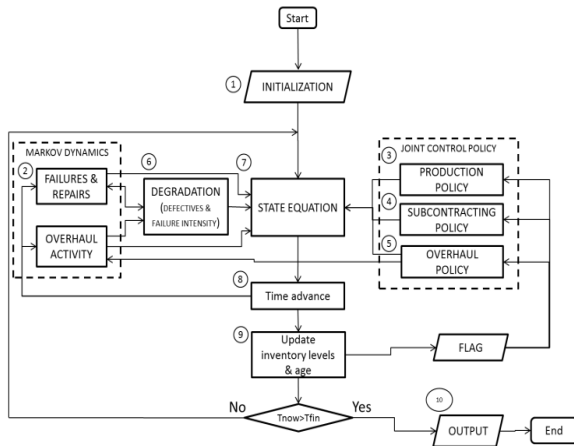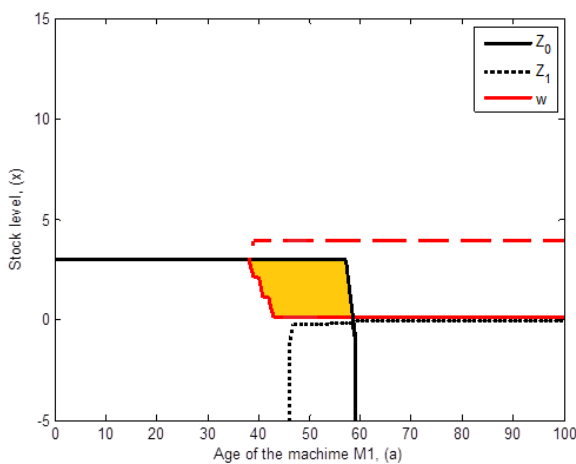
Fig. 4 Simulation model block diagram

Fig. 5 Control parameters approximation

To facilitate the determination of the control parameters, we define in Fig. 5. The stock level Z_0 and point B to characterize the production policy, also we define point SA for the overhaul policy, and regarding the subcontracting policy, we can calculate point A from condition (5), given that we can know the trend of deterioration at setting the constants in (3) and (4). With this parameter approximation, we will be able to control the manufacturing system and determine the incurred total cost. The statistical analysis of the simulation data implies a multifactor analysis of variance (ANOVA), where we identify one dependent variable denoted by the total incurred cost and three independent variables (Z_0, SA, B). Simulation runs are conducted according to a complete factorial design 3^3 with four replications, to fit the cost function. This requires $(3^3 x4) = 108$ simulation runs. To ensure steady states conditions the duration of each simulation run is set to 100,000 time units. Table II reports the cost values applied in the statistical analysis, the rest of parameters are defined as indicated in Table I.

The ANOVA is conducted on the 3^3 desing using the statistical software STATGRAPHICS. Based on off-line simulations, we select the minimum and maximum values of the factors (Z_0, SA, B) as presented in Table III, also we use the common random numbers technique to reduce the variability in the simulations results.

TABLE II
COST PARAMETERS FOR THE STATISTICAL ANALYSIS

Parameter:	c+	c-	C_{M1}	C_{M2}	c_o	c_d
Value:	1	350	4	40	4000	10

TABLE III
FACTOR LEVELS FOR THE STATISTICAL ANALYSIS

Factor	Low level	Center	High level	Description
Z_0	3	6	9	Production threshold of the machine
SA	20	35	50	Age of the machine required to conduct overhaul
B	30	60	90	Age limit to stop the machine

Table IV summarizes the ANOVA of the collected data, we found a R^2 value of 0.9736, this indicates that about of 97.36% of the observed variability in the total incurred cost is explained by the 3^3 desing. From Table IV, we can see that the linear and quadratic effects of the three factors and their interactions are significant for the total cost at a 95% confidence level. To test the homogeneity of the variances and the residual normality a complete residual analysis was conducted, consisting of the residual versus predicted values plot and the normal probability plot of residual. This therefore confirms that the total expected cost could be well determined by the second-order polynomial model.

TABLE IV
ANOVA TABLE FOR THE TOTAL COST

Source	Sum of Squares	Df	Mean Square	F-Ratio	P-Value
A:Z0*	71,92	1	71,92	12,16	0,0007
B:SA*	15069,3	1	15069,3	2548,65	0,0000
C:B*	184,64	1	184,64	31,23	0,0000
AA	48,8491	1	48,8491	8,26	0,0050
AB	147,42	1	147,42	24,93	0,0000
AC	69,9626	1	69,9626	11,83	0,0009
BB	4801,1	1	4801,1	812,00	0,0000
BC	39,9858	1	39,9858	6,76	0,0108
CC	277,168	1	277,168	46,88	0,0000
blocks	7,79682	3	2,59894	0,44	0,7252
Total error	561,705	95	5,91269		
Total (corr.)	21279,9	107			

A response surface methodology was applied with the software Statgraphics, to determine the following quadratic function:

$$\tilde{J}(Z_0, SA, B) = 183.984 + 0.598843 \cdot Z_0 - 5.00939 \cdot SA - 0.355016 \cdot B + 0.158519 \cdot Z_o^2 - 0.0389444 \cdot Z_0 SA - 0.0134144 \cdot Z_0 B + 0.0628611 \cdot SA^2 - 0.00202824 \cdot SA \cdot B + 0.00377593 B^2 + \epsilon \quad (15)$$

The minimum total expected cost is $\tilde{J}(\cdot) = 65.95$, and is defined with $Z_0^* = 6.33$, $SA^* = 42.93$ and $B^* = 69.80$, as indicated in Fig. 6, where the projection of the cost parameters in a two-dimensional plan is presented. Based on the data of Table I, we can define, $A^* = 51$. These values (Z_0^*, SA^*, B^*, A^*) are the

best parameters for our joint control policy, which control simultaneously the production, subcontracting, and overhaul rate.

From 35 extra-replications, we validate the control parameters (Z_0^*, SA^*, B^*, A^*) by verifying that the estimated cost $\tilde{J}(\cdot) = 65.95$ is inside the 95% confidence interval.

$$\bar{X}(y) \pm t_{n-1,1-\left(\frac{\alpha}{2}\right)}\sqrt{\frac{S^2(y)}{y}} = [65.79, 66.04]$$
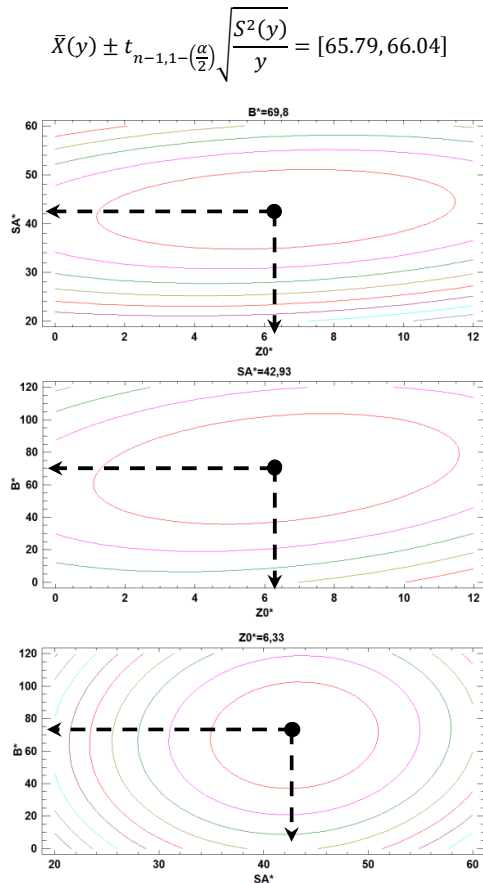

Fig. 6 Contours of the estimated response surface

Since the total cost fall inside the confidence interval, we can state that our simulation-based approach determines the values of the control parameters adequately and that the total cost determined by the second-order model is accurate.

VIII. SENSITIVITY ANALYSIS

We aim in this section to assess the robustness of our simulation control approach; this implies the analysis of the obtained control policy for different cost scenarios. Table IV presents twelve configurations of cost parameters derived from the basic case, we are interested to assess their impact on the control parameters and the total incurred cost. The set of cost values are related with the inventory, backlog, overhaul, defectives, production and subcontracting cost.

The results of the sensitivity analysis are presented in Table VI, and they highlight the relationship between the cost variation and the control parameters (Z_0^*, SA^*, B^*) and their respective incurred cost.

TABLE V
COST PARAMETERS FOR THE SENSITIVITY ANALYSIS

Case	c^+	c^-	c_o	CM_1	CM_2	c_d
Basic case	1	350	4000	4	40	10
Sensitivity of the Inventory cost						
1	**0.5**	350	4000	4	40	10
2	**1.5**	350	4000	4	40	10
Sensitivity of the Backlog cost						
3	1	**250**	4000	4	40	10
4	1	**450**	4000	4	40	10
Sensitivity of the Overhaul cost						
5	1	350	**3000**	4	40	10
6	1	350	**5000**	4	40	10
Sensitivity of the Production cost						
7	1	350	4000	**2**	40	10
8	1	350	4000	**6**	40	10
Sensitivity of the Subcontracting cost						
9	1	350	4000	4	**20**	10
10	1	350	4000	4	**60**	10
Sensitivity of the Defectives cost						
11	1	350	4000	4	40	**5**
12	1	350	4000	4	40	**15**

TABLE VI
SENSITIVITY ANALYSIS FOR COST PARAMETERS

Case	Z_0^*	SA^*	B^*	Optimal cost	Remark
Basic case	6.33	42.93	69.80	65.95	Base for the comparison
Sensitivity of the Inventory cost					
1	8.12	43.50	72.97	62.43	$Z_1 \uparrow, SA \uparrow, B \uparrow, C \downarrow$
2	4.55	42.36	66.66	68.57	$Z_1 \downarrow, SA \downarrow, B \downarrow, C \uparrow$
Sensitivity of the Backlog cost					
3	5.71	42.91	69.10	65.79	$Z_1 \downarrow, SA \leftrightarrow, B \downarrow, C \downarrow$
4	6.71	42.90	70.18	66.03	$Z_1 \uparrow, SA \leftrightarrow, B \uparrow, C \uparrow$
Sensitivity of the Overhaul cost					
5	5.87	41.28	61.72	55.92	$Z_1 \downarrow, SA \downarrow, B \downarrow, C \downarrow$
6	6.58	44.16	74.45	75.52	$Z_1 \uparrow, SA \uparrow, B \uparrow, C \uparrow$
Sensitivity of the Production cost					
7	6.39	43.51	71.67	58.25	$Z_1 \leftrightarrow, SA \uparrow, B \uparrow, C \downarrow$
8	6.27	42.39	67.80	73.59	$Z_1 \leftrightarrow, SA \downarrow, B \downarrow, C \uparrow$
Sensitivity of the Subcontracting cost					
9	5.88	43.54	73.64	65.49	$Z_1 \downarrow, SA \uparrow, B \uparrow, C \downarrow$
10	6.68	42.46	66.92	66.19	$Z_1 \uparrow, SA \downarrow, B \downarrow, C \uparrow$
Sensitivity of the Defectives cost					
11	6.34	44.20	73.95	63.60	$Z_1 \leftrightarrow, SA \uparrow, B \uparrow, C \downarrow$
12	6.27	41.86	65.38	68.01	$Z_1 \leftrightarrow, SA \downarrow, B \downarrow, C \uparrow$

The results obtained in Table VI, make sense, and can be explained as follows:

- *Variation of the Inventory cost, c^+:* At increasing c^+, the three control parameters decreases, because with higher c^+ the stock level is more penalized then Z_0 decreases. Further, this threshold reductions also decreases the overhaul zone, reducing as a consequence SA and B, because the machine operates less time at its maximum rate, thus it deteriorates less. Subcontractors take over all the product demand at an earlier age, then point B increases.

- *Variation of the Backlog cost, c^-:* When we increase c^-, Z_0 increases, because the backlog of product is more penalized, and so we need more stock to palliate shortages. Moreover, B increases considerably leading to conduct more overhaul, since backlog is more penalized with higher c^-, thus the machine remains operational

more time. At increasing B, subcontractors satisfies later all the product demand. From the results of Table VI, we note that the effect of the backlog cost is the opposite of the inventory cost.

- *Variation of the Overhaul cost, c_o:* At increasing c_o, it increases the threshold Z_0 because as overhaul activities are more expensive with higher c_o, we need more stock to conduct the overhaul. With higher c_o is normal to observe that the required age to conduct the overhaul SA increases, point B also increases since as it is more expensive to conduct overhaul activities, the machine remains operational more time to compensate the incurred cost. Moreover, subtracting fulfills all the product demand at a higher age, because given the higher overhaul cost, it is needed a higher age to justify the machine stoppage.

- *Variation of the Production cost, CM_1:* when we increase CM_1, the production with the machine is more expensive, and so this reduces the threshold Z_0, also this cost increment stops earlier the machine, conducting less overhaul, hence SA and point B decreases. Subcontracting is triggered to fulfill all the product demand earlier in time.

- *Variation of the Subcontracting cost, CM_2:* when we increase the subcontracting cost CM_2, we observe that the productions threshold increases, because we use more stock to satisfy the product demand given a higher subcontracting cost. The effect of CM_2 is also reflected in the overhaul policy, since with higher subcontracting cost, we need to restore the machine more rapidly to AGAN conditions in order to avoid further subcontracting cost. Thus, it is logical to decrease SA, this reductions also reduces point B, hence subcontracting satisfies earlier the whole product demand earlier.

- *Variation of the Defectives cost, c_d:* when we increase c_d, the system reacts by promoting earlier conduction of overhaul activities to restore the machine to AGAN conditions. Consequently, SA and point B decreases to avoid the production of defective units, hence subcontracting satisfies earlier the whole product demand.

IX. CONCLUSION

To some extent, our study has contributed to the production-planning problem of an unreliable manufacturing system consisting of a single machine single part type, in three important ways. We model the effect of the relationship between quality issues and the production control rule; we include the twofold effect of a deterioration process in the control policy, observed mainly in the rate of defectives and the failure intensity; also, we model the possibility of subcontractors to satisfy the product demand when the capacity of the manufacturing system is insufficient. The problem was formulated with a stochastic dynamic programing model in which the production, overhaul and subcontracting rate are considered as decision variables. Because of the complexity and the stochastic dynamics of the model, analytical solutions are intractable. Nevertheless, we

proposed a simulation-optimization approach to approximate the parameters of the obtained feedback policy, which is a derivation of the HPP with overhaul and subcontracting activities. The efficiency of the joint control policy was illustrated with a numerical example. The principal advantage of our simulation-based approach is that it allows a realistic representation of the stochastic dynamics of the manufacturing system under analysis using a discrete-continuous simulation model; also, it serves to analyze and optimize the control policy parameters trough design of experiments and the response surface methodology. Finally, a complete sensitivity analysis was performed, which emphasizes the complex interaction between production, subcontracting and deterioration. The final assessment is that our simulation-based approach works efficiently providing accurate results, this enables us to study more complex phenomena encountered in real manufacturing systems.

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