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A Straightforward Approach for Determining the Weights of Decision Makers Based on Angle Cosine and Projection Method

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***Abstract*—**Group decision making with multiple attribute has attracted intensive concern in the decision analysis area. This paper assumes that the contributions of all the decision makers (DMs) are not equal to the decision process based on different knowledge and experience in group setting. The aim of this paper is to develop a novel approach to determine weights of DMs in the group decision making problems. In this paper, the weights of DMs are determined in the group decision environment via angle cosine and projection method. First of all, the average decision of all individual decisions is defined as the ideal decision. After that, we define the weight of each decision maker (DM) by aggregating the angle cosine and projection between individual decision and ideal decision with associated direction indicator μ. By using the weights of DMs, all individual decisions are aggregated into a collective decision. Further, the preference order of alternatives is ranked in accordance with the overall row value of collective decision. Finally, an example in a chemical company is provided to illustrate the developed approach.

***Keywords*—**Angel cosine, ideal decision, projection method, weights of decision makers.

I. Introduction

In Multiple Attribute Decision Making (MADM) problems, the Decision Maker (DM) needs to choose the most appropriate alternative from a set of feasible alternatives, which are presented by multiple attributes, DM's subjective preferences and judgments [1], [2]. However, because of the drastic development of society and the economy, it seems to be very difficult or unrealistic for a single DM to take all relevant aspects of a complex problem into account [3]. The possible reason for using the opinions of a group of DMs when solving a problem is that a group approach may come up with better solutions to complex problems [4]. As a result, many decision-making processes in the real world take place in group settings. Over the past few decades, Multi-Attribute Group Decision Making (MAGDM) has been receiving more and more attention from researchers.

The aim of a MAGDM problem is to obtain the collective order of alternatives or select an optimal alternative based on the decision information of each DM [5], [6]. As the group members usually have different backgrounds and levels of knowledge, each DM has his/her own preference and only partially shares the goals of other DMs. Since a diversity of opinions commonly exists, it is of great importance to obtain the collective opinion of a group based on determining the weights of DMs.

Our literature search revealed that a limited number of works has been done in terms of determination of DMs' weights. Bodily [7] developed a delegation process to setting the members' weights, which is obtained using the theory of Markov chains. Mirkin and Fishburn [8] presented two approaches which use the eigenvectors method to determine the relative importance of the group's members. Ramanathan [9] put forward an AHP method to obtain members' weights, and aggregated group preferences. Hsi-Mei Hsu and Chen-Tung Chen [10] presented a method to define the index of consensus of each DM to the other DMs by using a similarity measure, in MCDM with group decision-making. Martel and Ben Khelifa [11] proposed a method to determine the relative importance of group members by using individual outranking indexes. Van den Honert [12] used multiplicative AHP and SMART method to derive group members' influence weights. Beynon [13] combined the Dempster-Shafer theory of evidence and AHP to aggregate the evidence from members of a decision-making group, and defined a discount rate value for each DM based on the perceived individual levels of importance. In addition, Xu [14] put forward some straightforward formulas to determine the weights of DMs by using the deviation measures between additive linguistic preference relations. Fu and Yang [15] suggested a group consensus based evidential reasoning approach for multiple attributive group decision analysis. Xu and Wu [16] presented a discrete model to support the consensus reaching process for MAGDM problems, in which, the weights of DMs is pre-defined. Besides, Zhou et al. [17] developed the generalized logarithm chi-square method to determine the generalized ordered weighted logarithm averaging operator weights for aggregating information in group decision-making. Zhang [18] developed a series of generalized Atanassov's intuitionistic fuzzy power geometric operators to aggregate input arguments. Moreover, Tsabadze [19] proposed a new approach to determine DMs' degrees of importance, which depends on how close DMs' estimates are to a representative.

The methods mentioned above have made important contributions to resolve the problem of DMs' weights in group

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decision-making. However, most of the existing approaches utilize Saaty's multiplicative preference relation in group decision making [20]. As a result, the subjectivity of DMs are too strong and the procedure determining the weights of DMs is very complicated in these approaches.

In this study, we propose a straightforward and comprehensive method to derive the weights of DMs and ranking the preference order of alternatives based on angle cosine and projection method. Angle cosine is used to determine the weights of DMs in direction, while projection method is used to rank the importance ratings of DMs in magnitude. Via angle cosine and projection method, we can obtain the weights of DMs in an objective and rational way.

The remaining paper is organized as follows. In Section II, the concepts of angle cosine and projection are presented and discussed. Based on the concepts of Section II, the proposed method for determining the weights of DMs using the angle cosine and projection between matrices is shown in Section III. Section IV compares the developed method with other existing methods. Section V demonstrates an illustrative example. The final section concludes.

II. Angle Cosine and Projection Method

In Section II, we shall introduce some concepts of angle cosine and projection method.

Definition 1. Let $\alpha=(\alpha_1,\alpha_2,...,\alpha_n)$ be a vector, then;

$$|\alpha| = \sqrt{\sum_{j=1}^{n}\alpha_j^2} \tag{1}$$

is called the module of vector α [21].

Definition 2. Let $\alpha=(\alpha_1,\alpha_2,...,\alpha_n)$ and $\beta=(\beta_1,\beta_2,...,\beta_n)$ be two vectors, then [21];

$$\alpha\beta = \sum_{j=1}^{n}\alpha_j\beta_j \tag{2}$$

is called the inner product between α and β. Besides this,

$$\cos(\alpha,\beta) = \cos\theta = \frac{\alpha\beta}{|\alpha||\beta|} \tag{3}$$

is called angle cosine between α and β, $0 \le \cos\theta \le 1$. The angle θ is shown in Fig. 1.

In general, the bigger the value of $\cos\theta$, the more the degree of the vector α approaching to the vector β in direction.

Through a combination of (1)-(3), we have the concept of projection between two vectors as follows:

Definition 3. Let $\alpha=(\alpha_1,\alpha_2,...,\alpha_n)$ and $\beta=(\beta_1,\beta_2,...,\beta_n)$ be two vectors and there is no loss of generality in assuming that $|\alpha|>0$ and $|\beta|>0$, then [21], [22];

$$Prj_\beta(\alpha) = |\alpha|\cos(\alpha,\beta) = |\alpha|\frac{\alpha\beta}{|\alpha||\beta|} = \frac{\alpha\beta}{|\beta|} \tag{4}$$

is called the projection of the vector α on the vector β. The projection can be illustrated in Fig. 1.

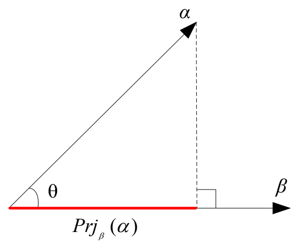

Fig. 1 Angle θ and projection of vector α on β

In general, the bigger the value of $Prj_\beta(\alpha)$, the more the degree of the vector α approaching to the vector β in magnitude.

From Yue's point of view [23], it is enough to determine the approaching degree between two vectors by projection method. However, the projection can just describe the closeness between two vectors in magnitude. For example, the projection of the vector α on the vector β equals the projection of the vector γ on the vector β in Fig. 2. That means the degree of the vector α approaching to the vector β equals the degree of the vector γ approaching to the vector β in magnitude. As the angle θ between α and β is greater than the angle δ between γ and β, the degree of the vector α approaching to the vector β is lower than the degree of the vector γ approaching to the vector β in fact. Thus, it is more reasonable to define the closeness degree between two vectors from the two aspects of direction and magnitude.

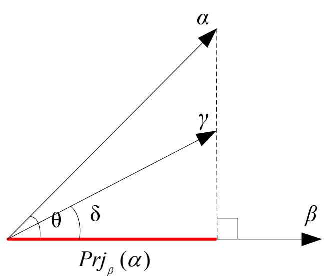

Fig. 2 Angle θ, angle δ and projection of vector α on β

Similarly to the angle cosine and projection between vectors, in the following, we introduce the angle cosine and projection between matrices.

Definition 4. Let $A=(a_{ij})_{m\times n}$ and $B=(b_{ij})_{m\times n}$ be two matrices, and there is no loss of generality in assuming that A and B are non-zero matrices, then;

$$\cos(A,B)=\frac{\sum_{i=1}^{m}\sum_{j=1}^{n}a_{ij}b_{ij}}{\sqrt{\sum_{i=1}^{m}\sum_{j=1}^{n}a_{ij}^2}\sqrt{\sum_{i=1}^{m}\sum_{j=1}^{n}b_{ij}^2}} \tag{5}$$

is called the angle cosine between the matrix A and the matrix B.

$$Prj_B(A)=\frac{\sum_{i=1}^{m}\sum_{j=1}^{n}a_{ij}b_{ij}}{\sqrt{\sum_{i=1}^{m}\sum_{j=1}^{n}b_{ij}^2}} \tag{6}$$

is called the projection of the matrix Aon the matrix B.

Similarly, the bigger the value of $\cos\theta$, the more the degree of the matrix A approaching to the matrix B in direction; the bigger the value of $Prj_\beta(\alpha)$, the more the degree of the matrix A approaching to the matrix B in magnitude.

III. The Presented Approach

In Section III, we will describe the MAGDM problems by using the angle cosine and projection between matrices.

For convenience, we let $M=\{1,2,...,m\}$, $N=\{1,2,...,n\}$ and $T=\{1,2,...,t\}$ be three sets of indicators; $i\in M$, $j\in N$, $k\in T$. Let $A=\{A_1,A_2,...,A_m\}$ ($m\geq 2$) be a discrete set of m feasible alternatives, $U=\{u_1,u_2,...,u_n\}$ be a finite set of attributes, $w=\{w_1,w_2,...,w_n\}$ be the weight vector of attributes, which satisfies $0\leq w_j\leq 1$ and $\sum_{j=1}^{n}w_j=1$. Let $D=\{d_1,d_2,...,d_t\}$ be a finite set of DMs, $\lambda=\{\lambda_1,\lambda_2,...,\lambda_t\}$ be the weight vector of DMs, which satisfies $\lambda_k\geq 0$ and $\sum_{k=1}^{t}\lambda_k=1$.

A. Standardization of Decision Matrix

Firstly, we invite DMs to give the performance judgements over m feasible alternatives under n attributes. The decision matrix of kth DM is as follows:

$$X_k=\left(x_{ij}^k\right)_{m\times n}=\begin{array}{c} \\ A_1\\ A_2\\ \vdots\\ A_m\end{array}\begin{bmatrix} u_1 & u_2 & \cdots & u_n\\ x_{11}^k & x_{12}^k & \cdots & x_{1n}^k\\ x_{21}^k & x_{22}^k & \cdots & x_{22}^k\\ \vdots & \vdots & \vdots & \vdots\\ x_{m1}^k & x_{m2}^k & \cdots & x_{mn}^k\end{bmatrix} \tag{7}$$

In general, MAGDM problems have benefit attributes (the larger the value, the better the decision) and cost attributes (the smaller the value, the better the decision). In order to acquire the dimensionless of attributes, it is necessary to normalize each attribute value x_{ij}^k in decision matrix X_k into a corresponding element y_{ij}^k in normalized decision matrix Y_k by (9) and (10):

$$Y_k=\left(y_{ij}^k\right)_{m\times n}=\begin{array}{c} \\ A_1\\ A_2\\ \vdots\\ A_m\end{array}\begin{bmatrix} u_1 & u_2 & \cdots & u_n\\ y_{11}^k & y_{12}^k & \cdots & y_{1n}^k\\ y_{21}^k & y_{22}^k & \cdots & y_{22}^k\\ \vdots & \vdots & \vdots & \vdots\\ y_{m1}^k & y_{m2}^k & \cdots & y_{mn}^k\end{bmatrix} \tag{8}$$

where

$$y_{ij}^k=\frac{x_{ij}^k}{\sqrt{\sum_{i=1}^{m}(x_{ij}^k)^2}},\ \text{for benefit attribute } x_{ij}^k \tag{9}$$

and

$$y_{ij}^k=1-\frac{x_{ij}^k}{\sqrt{\sum_{i=1}^{m}(x_{ij}^k)^2}},\ \text{for cost attribute } x_{ij}^k \tag{10}$$

As mentioned before, $w=\{w_1,w_2,...,w_n\}$ is the weight vector of attributes. Assuming the attributes' weight vector $\{w_1^k,w_2^k,...,w_n^k\}$ is given by kth DM, we can construct the weighted normalized decision matrix as;

$$V_k=\left(v_{ij}^k\right)_{m\times n}=\left(w_j^k y_{ij}^k\right)_{m\times n}=\begin{array}{c} \\ A_1\\ A_2\\ \vdots\\ A_m\end{array}\begin{bmatrix} u_1 & u_2 & \cdots & u_n\\ v_{11}^k & v_{12}^k & \cdots & v_{1n}^k\\ v_{21}^k & v_{22}^k & \cdots & v_{22}^k\\ \vdots & \vdots & \vdots & \vdots\\ v_{m1}^k & v_{m2}^k & \cdots & v_{mn}^k\end{bmatrix} \tag{11}$$

B. Definition of DMs' Weights

Then, we suppose that a MAGDM problem needs t DMs, each DM shall provide his/her preferences over alternatives with respect to attributes, all provided preference values can be expressed by a matrix, and V_1, V_2, …, V_t are the decision matrices of t DM, and V^* is the ideal decision of V_1, V_2, …, V_t. The basic idea of this approach is that the more the degree of the decision matrix V_k approaching to the ideal decision V^* in direction and magnitude, the bigger the weight of kth DM. That is to say, the larger the value of the angle cosine and the value of the projection between the decision matrix V_k and the ideal decision V^*, the bigger the weight of kth DM. The problem is how to define the ideal solution from all individual decision matrices?

According to the individual decision $V_k=\left(v_{ij}^k\right)_{m\times n}$ in (11), we can get the average decision of V_k as:

$$V^* = \left(v_{ij}^*\right)_{m\times n} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{bmatrix} u_1 & u_2 & \cdots & u_n \\ v_{11}^* & v_{12}^* & \cdots & v_{1n}^* \\ v_{21}^* & v_{22}^* & \cdots & v_{22}^* \\ \vdots & \vdots & \vdots & \vdots \\ v_{m1}^* & v_{m2}^* & \cdots & v_{mn}^* \end{bmatrix} \quad (12)$$

where $V^* = \frac{1}{t}\sum_{k=1}^{t} V_k$, and $v_{ij}^* = \frac{1}{t}\sum_{k=1}^{t} y_{ij}^k$.

In comprise sense, we define $V^* = \left(v_{ij}^*\right)_{m\times n}$ as the ideal decision of all individual decision V_k in (11). In this sense, the more the degree that V_k is closer to the V^*, the better the decision V_k.

In order to measure the decision level of each DM, we can calculate the angle cosine between each individual decision matrix V_k and ideal decision V^*. By (5), the angle cosine can be given as;

$$\cos(V_k, V^*) = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} v_{ij}^k v_{ij}^*}{\sqrt{\sum_{i=1}^{m}\sum_{j=1}^{n}\left(v_{ij}^k\right)^2}\sqrt{\sum_{i=1}^{m}\sum_{j=1}^{n}\left(v_{ij}^*\right)^2}} \quad (13)$$

Clearly, $\cos(V_k, V^*)$ represents the closeness between each individual decision V_k and ideal decision V^* in direction.

Then we can calculate the projection of each individual decision matrix V_k on ideal decision V^*. By (6), the projection can be given as;

$$Prj_{V^*}(V_k) = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} v_{ij}^k v_{ij}^*}{\sqrt{\sum_{i=1}^{m}\sum_{j=1}^{n}\left(v_{ij}^*\right)^2}} \quad (14)$$

Clearly, $Prj_{V^*}(V_k)$ represents the closeness between each individual decision V_k and ideal decision V^* in magnitude.

In order to get the weights of DMs by these angle cosines and projections, we define direction closeness C_k and magnitude closeness P_k, respectively.

$$C_k = \frac{\cos(V_k, V^*)}{\sum_{k=1}^{t}\cos(V_k, V^*)} \quad (15)$$

where $C_k \geq 0$, $\sum_{k=1}^{t} C_k = 1$.

$$P_k = \frac{Prj_{V^*}(V_k)}{\sum_{k=1}^{t} Prj_{V^*}(V_k)} \quad (16)$$

where $P_k \geq 0$, $\sum_{k=1}^{t} P_k = 1$.

To covert direction closeness and magnitude closeness into relative closeness, here we introduce the direction indicator μ $\left(0 \leq \mu \leq 1\right)$ to transform direction closeness C_k and magnitude closeness P_k into relative closeness λ_k. If DMs pay more attention to the direction, then μ can select a bigger value $\left(\mu > 0.5\right)$. If DMs pay more attention to the magnitude, then μ can select a smaller value $\left(\mu < 0.5\right)$. If DMs keep a moderate attitude, in other words, neither more attention to the direction nor more attention to the magnitude, μ selects a certain value 0.5. The transformation calculation is as follows:

$$\lambda_k = \mu C_k + (1-\mu) P_k \quad (17)$$

where $\lambda_k \geq 0$, $\sum_{k=1}^{t}\lambda_k = \mu\sum_{k=1}^{t} C_k + \left(1-\mu\right)\sum_{k=1}^{t} P_k = 1$.

C. Priority Order of Alternatives

In this section, we can obtain a group decision matrix Y by using the following formula with the weight of each DM:

$$Y = \sum_{k=1}^{t}\lambda_k Y_k = (y_{ij})_{m\times n} \quad (18)$$

Then, use the aggregation operator

$$y_i = \sum_{j=1}^{n} y_{ij},\ i \in M \quad (19)$$

to aggregate all the elements in the *i*th row of Y , and get the overall attribute value y_i of the alternative A_i .

According to the overall attribute value y_i, we can rank the priority order of all feasible alternatives and choose the best alternative.

D. The Presented Approach

In summary, an approach for determining the DMs' weights in MAGDM, based on angle cosine and projection method, can be presented as follows.

Step1. Utilize the (9) and/or (10) to normalize X_k into Y_k in (8).

Step2. Calculate the weighted normalized decision matrix V_k by multiplying $\left\{w_1^k, w_2^k, \ldots, w_n^k\right\}$ and Y_k in (11).

Step3. Define ideal decision V^* for all individual decisions in (12).

Step4. Calculate the angle cosine between each individual decision V_k and ideal decision V^* in (13)

Step5. Calculate the projection of each individual decision V_k on ideal decision V^* in (14)

Step6. Define the direction closeness C_k and magnitude closeness P_k based on the angle cosines and the projections by (15), (16), respectively.

Step7. Calculate the relative closeness λ_k in (17) by combining associated C_k and P_k with direction indicator μ.

Step8. Calculate the collective decision matrix by (18), with the obtained weight vector $\lambda = (\lambda_1, \lambda_2, ...\lambda_t)^T$ of DMs under a certain direction indicator μ.

Step9. Sum all elements in each line of the collective decision matrix in (19) and obtain overall assessment value for each alternative.

Step10. Rank the preference order of alternatives in accordance with their overall assessment values.

IV. Illustrative Example

In Section IV, a human resources selection (adapted from [24]) is taken as an example to illustrate the application of the proposed approach.

A local chemical company tries to recruit an on-line manager. The company's human resources department provides some relevant selection tests as the benefit attributes to be evaluated. These objective test include knowledge tests (language test, professional test and safety rule test), skill tests (professional skills and computer skills). After these objective tests, there are 17 qualified candidates (as alternatives marked by $A_1, A_2,\ldots, A_{17}$, or briefly marked by 1,2,…,17) on the list for the selection. Then four DMs (marked by d_1, d_2, d_3, d_4) are responsible for the selection based on subjective tests. The basic data of subjective attributes, including panel interview and 1-on-1 interview tests (only quantitative information here) for the decision are list in Table I.

Following the suggested steps in Section III, each DM will construct a normalized decision matrix for group decision making. Since all listed attributes are benefit attributes, by (9), we first normalize Table I according to Step 1.

In addition, the weights of attributes, elicited by DMs, are shown in Table II.

Then, the weighted normalized decision matrix can be obtained by Step 2.

The ideal decision V^*, by Step 3, is shown in Table III.

By Step 4, we can obtain the angle cosine between each individual decision and ideal decision.

By Step 5, we can obtain the projection of each individual decision on ideal decision. Then, we can calculate the direction closeness C_k and magnitude closeness P_k based on the angle cosines and the projections by Step 6, respectively. The angle cosines, projections, C_k and P_k are shown in Table IV.

TABLE I
Decision Matrixes of Example-Subjective Attributes

No	X_1		X_2		X_3		X_4	
	Panel	1-on-1	Panel	1-on-1	Panel	1-on-1	Panel	1-on-1
1	80	75	85	80	75	70	90	85
2	65	75	60	70	70	77	60	70
3	90	85	80	85	80	90	90	95
4	65	70	55	60	68	72	62	72
5	75	80	75	80	50	55	70	75
6	80	80	75	85	77	82	75	75
7	65	70	70	60	65	72	67	75
8	70	60	75	65	75	67	82	85
9	80	85	95	85	90	85	90	92
10	70	75	75	80	68	78	65	70
11	50	60	62	65	60	65	65	70
12	60	65	65	75	50	60	45	50
13	75	75	80	80	65	75	70	75
14	80	70	75	72	80	70	75	75
15	70	65	75	70	65	70	60	65
16	90	95	92	90	85	80	88	90
17	80	85	70	75	75	80	70	75

Note: (1) There are four DMs selected for the evaluation. (2) There are a total of 17 candidates for evaluation. (3) All listed attributes are benefit attributes.

TABLE II
Weights on Attributes of Example

No.	Attributes	The weights of the group			
		d_1	d_2	d_3	d_4
1	Panel	0.5243	0.4574	0.4160	0.4503
2	1-on-1	0.4757	0.5426	0.5840	0.5497

TABLE III
Ideal Decision of All Individual Decisions

No. of candidates	Panel interview	1-on-1 interview
1	0.1260	0.1339
2	0.0973	0.1264
3	0.1302	0.1539
4	0.0955	0.1186
5	0.1038	0.1243
6	0.1174	0.1394
7	0.1019	0.1199
8	0.1150	0.1202
9	0.1350	0.1502
10	0.1062	0.1313
11	0.0899	0.1128
12	0.0843	0.1080
13	0.1110	0.1320
14	0.1185	0.1242
15	0.1032	0.1171
16	0.1357	0.1529
17	0.1130	0.1360

Further, we can respectively calculate the relative closeness and DMs' ranking under different direction indicators (here we introduce μ=0, 0.5 and 1) by Step 7, which are organized in Table V.

As DMs are not willing or able to express their preferences on direction or projection explicitly, we select the direction indicator μ=0.5. By Step 8 - 10, the (18) is used to aggregate all the individual decisions into the collective decisions in the columns 2 and 3 of Table VI. Then, summing all elements in each line of columns 2 and 3 of Table VI, the integrated evaluation of 17 candidates are shown in column 4 of Table VI. The ranking of 17 candidates are also shown in last column of Table VI. Obviously, we can find that the 16th candidate is ranked first, and the 12th candidate is ranked last.

TABLE IV
ANGLE COSINES, PROJECTIONS, C_k AND P_k

DMs	Angle cosines	Projections	C_k	P_k
d_1	0.4959	0.7016	0.2490	0.2478
d_2	0.4994	0.7082	0.2508	0.2502
d_3	0.4973	0.7125	0.2497	0.2517
d_4	0.4988	0.7087	0.2505	0.2503

TABLE V
WEIGHTS AND RANKING OF DMS UNDER DIFFERENT DIRECTION INDICATORS

DMs	μ=0		μ=0.5		μ=1	
	λ_k	Ranking	λ_k	Ranking	λ_k	Ranking
d_1	0.2478	4	0.2484	4	0.2490	4
d_2	0.2502	3	0.2505	2	0.2508	1
d_3	0.2517	1	0.2507	1	0.2497	3
d_4	0.2503	2	0.2504	3	0.2505	2

TABLE VI
INTEGRATED ASSESSMENT OF 17 CANDIDATES

No.	Panel interview	1-on-1 interview	Sum	Ranking
1	0.1260	0.1340	0.2599	4
2	0.0973	0.1265	0.2237	12
3	0.1302	0.1539	0.2841	3
4	0.0955	0.1187	0.2141	15
5	0.1038	0.1243	0.2281	11
6	0.1174	0.1395	0.2568	5
7	0.1018	0.1200	0.2218	13
8	0.1150	0.1203	0.2353	10
9	0.1350	0.1502	0.2852	2
10	0.1061	0.1313	0.2374	9
11	0.0899	0.1128	0.2027	16
12(#)	0.0843	0.1080	<u>0.1923</u>	<u>17</u>
13	0.1109	0.1320	0.2430	7
14	0.1184	0.1242	0.2427	8
15	0.1032	0.1171	0.2203	14
16(*)	0.1356	0.1529	<u>0.2885</u>	<u>1</u>
17	0.1129	0.1361	0.2490	6

Note: "*" and "#" mark the first and the last candidate, respectively.

V. CONCLUSION

The determination of DMs' weights which refers to obtain a relative contribution degree to the group decision is an important aspect in MAGDM problems. In order to achieve weights of DMs in group decision making, we have developed a novel approach for determining weights of DMs in a group decision environment based on the angle cosine and projection method. Compared to the existing MAGDM approaches, the method presented in this paper has certain distinguishing characteristics. By contrast, the proposed method can describe DMs' preferences on direction and magnitude with respect to the ideal solution at the same time by utilizing direction indicator μ to take direction closeness and magnitude closeness into consideration. Further, the developed approach is applicable not only ranking DMs, but also aggregating individual decision into a collective decision, then ranking alternative according to the collective decision. Due to the amount of information, it will be easier and faster to solve these problems with software MATLAB. Although the method in this paper provides a simple and effective mechanism for weights of DMs in group decision setting, it is only useful for real number form of attributes. Therefore, the proposed method would be extended to support situations in which the information of attributes is in other forms, e.g., linguistic variables or fuzzy numbers in the near future.

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