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A Method to Saturation Modeling of Synchronous Machines in d-q Axes

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Abstract**—**This paper discusses the general methods to saturation in the steady-state, two axis (d & q) frame models of synchronous machines. In particular, the important role of the magnetic coupling between the d-q axes (cross-magnetizing phenomenon), is demonstrated. For that purpose, distinct methods of saturation modeling of dumper synchronous machine with cross-saturation are identified, and detailed models synthesis in d-q axes. A number of models are given in the final developed form. The procedure and the novel models are verified by a critical application to prove the validity of the method and the equivalence between all developed models is reported. Advantages of some of the models over the existing ones and their applicability are discussed.

Keywords**—**Cross-magnetizing, models synthesis, synchronous machine, saturated modeling, state-space vectors.

I. Introduction

THE effects of saturation of the main flux path on the performance of electrical machines have been discussed in very many papers in the literature [1], [2], but there are no generalized analytical treatments of these effects. As with the theory of electrical machines before the advent of Kron's generalized theory authors tend to consider the effects of saturation in a specific machine type, and crucially the effects of inter-saturation, sometimes called cross-saturation do not appear to have been included in any general analysis [3], [4]. In the same way the magnetic saturation in orthogonal d-q axis models of synchronous machines is generally modeled by selecting either all the winding currents as state-space variables [5], or all the winding flux linkages as state-space variables [6], [7]. Out of the two, the current state-space model appears to be the more frequent choice. The most of the research in conjunction with this model has been directed towards investigation of the cross-saturation impact on accuracy of the simulation results and towards accurate representation of the q-axis saturation. The trend in the past was to ignore completely cross-saturation and account for the saturation at first in the d-axis only, and then in both d- and q-axes [8], [9]. With or without magnetic saturation, during transient operations, synchronous machines are modeled using mostly either winding currents or linkage fluxes, except for vector control purposes. In this case, two other models are generally solicited where the stator currents are mixed with the stator or rotor fluxes to achieve respectively the so-called stator or rotor flux-oriented control [10], [11]. It has to be noted that, the choice of these latter kinds of models is rather dictated by the requirements of the vector control methods than the will of changing deliberately the state variables. Obviously, when considering the different currents and fluxes as state space variables, the number of synchronous machines models is higher than known. Synthesis of possible models is rarely treated in the literature, except in [12], [13] and recently in [14]. In all cases, the subject is treated separately for synchronous machines. In this context, it has been noticed that there are large discrepancies between the development of models with and without cross saturation of a smooth air-gap synchronous machines [15], [16].

The paper is organized as follows. After writing the fundamental space vector equations of a round rotor synchronous machine with the usual assumptions, a complete and detailed synthesis of d-q models is discussed, in section II. The extension of the procedure introduce the magnetic saturation with and without cross saturation is contained in Section III. Derivation procedure and some of the models with cross-magnetizing are given in Section IV. The determination of static and dynamic saturation coefficients is presented in section V. Section VI discusses the validity of the proposed approaches as well as the equivalence between the existing models and comparisons between the two methods exposed for synchronous machine, while Section VII summarizes conclusions.

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II. Space-Vector Equations and Models Synthesis

A. Space-Vector Equations

With the usual assumptions, transient operations of damper synchronous machines with constant air gap are generally analyzed in a synchronous reference frame by the following electric equations:

$$\overline{v}_s = R_s \overline{i}_s + \frac{d\overline{\lambda}_s}{dt} + j\omega\overline{\lambda}_s \tag{1}$$

$$\overline{v}_r = R_r \overline{i}_r + \frac{d\overline{\lambda}_r}{dt} \tag{2}$$

$$\overline{v}_f = R_f \overline{i}_f + \frac{d\overline{\lambda}_f}{dt} \tag{3}$$

where:

$$\overline{\lambda}_s = l_s \overline{i}_s + \overline{\lambda}_m \tag{4}$$

$$\overline{\lambda}_r = l_r \overline{i}_r + \overline{\lambda}_m \tag{5}$$

$$\bar{\lambda}_f = l_f \bar{i}_f + \bar{\lambda}_m \tag{6}$$

and:

$$\bar{\lambda}_m = L_m \bar{i}_m \tag{7}$$

$$\bar{i}_m = \bar{i}_s + \bar{i}_r + \bar{i}_f \tag{8}$$

All rotor quantities are referred to the stator. Voltages, currents and fluxes are space vectors and expressed with complex quantities. The real and imaginary parts of each space vector are known as d and q components. Because of the absence of a winding dc excitation on the quadrature axis, v_f i_f and λ_f are real elements. If the shaft speed is not uniform, the electric equations (1)-(3) must be completed by a mechanical equation. Due to the absence of the q-axis excitation winding, the number of equations in the d and q axis is not the same. Thus, it is more convenient to separate the space vector equations in d–q ones.

The d-axis equations are:

$$v_{ds} = R_s i_{ds} + \frac{d\lambda_{ds}}{dt} - \omega\lambda_{ds} \tag{9}$$

$$0 = R_r i_{dr} + \frac{d\lambda_{dr}}{dt} \tag{10}$$

$$v_f = R_f i_f + \frac{d\lambda_f}{dt} \tag{11}$$

with:

$$\lambda_{ds} = l_s i_{ds} + \lambda_{dm} \tag{12}$$

$$\lambda_{dr} = l_r i_{dr} + \lambda_{dm} \tag{13}$$

$$\lambda_f = l_f i_f + \lambda_{dm} \tag{14}$$

and:

$$\lambda_{dm} = L_m i_{dm} \tag{15}$$

$$i_{dm} = i_{ds} + i_{dr} + i_f \tag{16}$$

Equations of q-axis are:

$$v_{qs} = R_s i_{qs} + \frac{d\lambda_{qs}}{dt} + w\lambda_{qs} \tag{17}$$

$$0 = R_r i_{qr} + \frac{d\lambda_{qr}}{dt} \tag{18}$$

where:

$$\lambda_{qs} = l_s i_{qs} + \lambda_{qm} \tag{19}$$

$$\lambda_{qr} = l_r i_{qr} + \lambda_{qm} \tag{20}$$

and:

$$\lambda_{qm} = L_m i_{qm} \tag{21}$$

$$i_m = i_{qs} + i_{qr} \tag{22}$$

B. Models Synthesis

Because (3) is real, resolving (1)-(3) for any transient operation requires the selection of two vectors and a d-component of a third one. Hence, determination of possible combinations can be achieved as follows:

Select two vectors among $(\bar{i}_s, \bar{i}_r, \bar{i}_m, \bar{\lambda}_s, \bar{\lambda}_r, \bar{\lambda}_m)$ and complete each pair by i_f or λ_f. Because of the relation (7), the pair $(\bar{i}_m, \bar{\lambda}_m)$ has to be omitted. This group contains $(C_6^2 - 1) \times 2 = 28$ models.

Select three vectors among the six having d and q components, except eight cases that are: $(\bar{i}_m, \bar{\lambda}_m, \bar{\lambda}_s)$, $(\bar{i}_m, \bar{\lambda}_m, \bar{i}_r)$, $(\bar{i}_m, \bar{\lambda}_m, \bar{\lambda}_s)$, $(\bar{i}_m, \bar{\lambda}_m, \bar{\lambda}_r)$ where the pair $(\bar{i}_m, \bar{\lambda}_m)$ exists and $(\bar{i}_s, \bar{\lambda}_s, \bar{i}_m)$, $(\bar{i}_s, \bar{\lambda}_s, \bar{\lambda}_m)$, $(\bar{i}_r, \bar{\lambda}_r, \bar{i}_m)$, $(\bar{i}_r, \bar{\lambda}_r, \bar{\lambda}_m)$ because of the dependency formulated in (4) and (5). Also, it's to be noted that each combination of three vectors generates three possible cases and thus three different models in terms of space vectors. As an example, with $(\bar{i}_s, \bar{i}_r, \bar{\lambda}_s)$ we can form $(\bar{i}_s, \bar{i}_r, \lambda_{ds})$, $(\bar{i}_s, i_{dr}, \bar{\lambda}_s)$ and $(i_{ds}, \bar{i}_r, \bar{\lambda}_s)$. This second group encompasses $(C_6^3 - 8) \times 3 = 36$ models.

The total combinations of possible state space variables for damped synchronous machines, in terms of space vectors, are sixty four.

They are classified into three families: current, flux and mixed models.

Current models: $(\bar{i}_s, \bar{i}_r, i_f)$, $(\bar{i}_s, \bar{i}_m, i_f)$, $(\bar{i}_r, \bar{i}_m, i_f)$, $(\bar{i}_s, \bar{i}_r, i_{dm})$, $(\bar{i}_s, i_{dr}, \bar{i}_m)$, $(i_{ds}, \bar{i}_r, \bar{i}_m)$

Flux models: $(\bar{\lambda}_s, \bar{\lambda}_r, \lambda_f)$, $(\bar{\lambda}_s, \bar{\lambda}_m, \lambda_f)$, $(\bar{\lambda}_r, \bar{\lambda}_m, \lambda_f)$, $(\bar{\lambda}_s, \bar{\lambda}_r, \lambda_{dm})$, $(\bar{\lambda}_s, \lambda_{dr}, \bar{\lambda}_m)$, $(\lambda_{ds}, \bar{\lambda}_r, \bar{\lambda}_m)$

Mixed models:

$(\bar{i}_s, \bar{i}_r, \lambda_{ds})$, $(\bar{i}_s, \bar{i}_r, \lambda_{dr})$, $(\bar{i}_s, \bar{i}_r, \lambda_{dm})$, $(\bar{i}_s, \bar{i}_r, \lambda_f)$, $(\bar{i}_s, \bar{i}_m, \lambda_{dr})$, $(\bar{i}_s, \bar{i}_m, \lambda_f)$, $(\bar{i}_r, \bar{i}_m, \lambda_{ds})$, $(\bar{i}_r, \bar{i}_m, \lambda_f)$.

$(\bar{\lambda}_s, \bar{\lambda}_r, i_{ds})$, $(\bar{\lambda}_s, \bar{\lambda}_r, i_{dr})$, $(\bar{\lambda}_s, \bar{\lambda}_r, i_{dm})$, $(\bar{\lambda}_s, \bar{\lambda}_r, i_f)$, $(\bar{\lambda}_s, \bar{\lambda}_m, i_{dr})$, $(\bar{\lambda}_s, \bar{\lambda}_m, i_f)$, $(\bar{\lambda}_r, \bar{\lambda}_m, i_{ds})$, $(\bar{\lambda}_r, \bar{\lambda}_m, i_f)$.

$(\bar{i}_s, \bar{\lambda}_s, i_{dr})$, $(\bar{i}_s, \bar{\lambda}_s, i_f)$, $(\bar{i}_s, \bar{\lambda}_m, i_{dr})$, $(\bar{i}_s, \bar{\lambda}_m, i_f)$, $(\bar{i}_r, \bar{\lambda}_r, i_{ds})$, $(\bar{i}_r, \bar{\lambda}_r, i_f)$, $(\bar{i}_m, \bar{\lambda}_s, i_{dr})$, $(\bar{i}_m, \bar{\lambda}_s, i_f)$, $(\bar{i}_m, \bar{\lambda}_r, i_{ds})$, $(\bar{i}_m, \bar{\lambda}_r, i_f)$, $(\bar{i}_s, \bar{\lambda}_r, i_{dr})$, $(\bar{i}_s, \bar{\lambda}_r, i_{dm})$, $(\bar{i}_s, \bar{\lambda}_r, i_f)$, $(\bar{i}_r, \bar{\lambda}_s, i_{ds})$, $(\bar{i}_r, \bar{\lambda}_s, i_{dm})$, $(\bar{i}_r, \bar{\lambda}_s, i_f)$, $(\bar{i}_r, \bar{\lambda}_m, i_{ds})$, $(\bar{i}_r, \bar{\lambda}_m, i_f)$.

$(\bar{i}_s, \bar{\lambda}_s, \lambda_{dr})$, $(\bar{i}_s, \bar{\lambda}_s, \lambda_f)$, $(\bar{i}_s, \bar{\lambda}_m, \lambda_{dr})$, $(\bar{i}_s, \bar{\lambda}_m, \lambda_f)$, $(\bar{i}_r, \bar{\lambda}_r, \lambda_{ds})$, $(\bar{i}_r, \bar{\lambda}_r, \lambda_f)$, $(\bar{i}_m, \bar{\lambda}_s, \lambda_{dr})$, $(\bar{i}_m, \bar{\lambda}_s, \lambda_f)$, $(\bar{i}_m, \bar{\lambda}_r, \lambda_{ds})$, $(\bar{i}_m, \bar{\lambda}_r, \lambda_f)$, $(\bar{i}_s, \bar{\lambda}_r, \lambda_{dr})$, $(\bar{i}_s, \bar{\lambda}_r, \lambda_{dm})$,

$(\bar{i}_s, \bar{\lambda}_r, \lambda_f)$, $(\bar{i}_r, \bar{\lambda}_s, \lambda_{ds})$, $(\bar{i}_r, \bar{\lambda}_s, \lambda_{dm})$, $(\bar{i}_r, \bar{\lambda}_s, \lambda_f)$, $(\bar{i}_r, \bar{\lambda}_m, \lambda_{ds})$, $(\bar{i}_r, \bar{\lambda}_m, \lambda_f)$.

III. The Approach to Main Flux Saturation Modeling

The proposed common approach to introduce magnetic saturation in any d-q existing model, for the synchronous machines, relies only on the knowledge of the winding currents saturated model. For that reason, it will be first shown how to obtain it.

The leakage inductances in (4)-(6) are assumed to be constant, only the main flux $\bar{\lambda}_m$ is subject to saturation. Deriving stator and rotor linkage fluxes, in (4)-(6), leads to the time derivative of the magnetizing flux $\bar{\lambda}_m$.

$$\frac{d\bar{\lambda}_{s,r,f}}{dt} = l_{s,r,f}\frac{d\bar{i}_{s,r,f}}{dt} + \frac{d\bar{\lambda}_m}{dt} \tag{23}$$

Therefore, $\frac{d\bar{\lambda}_m}{dt}$ has to be described by means of the winding currents.

$$\frac{d\bar{\lambda}_m}{dt} = \frac{d(\lambda_m e^{i\alpha})}{dt} = e^{i\alpha}\left(\frac{d\lambda_m}{dt} + \lambda_m\frac{d\alpha}{dt}\right) \tag{24}$$

α is the angular position of $\bar{\lambda}_m$ with respect to the d-axis. It also characterizes the position of the magnetizing current $\bar{i}_m$ in the air gap, since the hysteresis angle is neglected. Writing $\bar{i}_m$, α and $\frac{d\lambda_m}{dt}$ as:

$$i_m = \sqrt{i_{dm}^2 + i_{qm}^2} \tag{25}$$

$$\alpha = \tan^{-1}\frac{i_{qm}}{i_{dm}} \tag{26}$$

$$\frac{d\lambda_m}{dt} = \frac{d\lambda_m}{di_m}\frac{di_m}{dt} = L_{mdy}\frac{di_m}{dt} \tag{27}$$

We get:

$$\begin{aligned}\frac{d\lambda_{dm}}{dt} &= L_{d1}\frac{di_{dm}}{dt} + L_{dq1}\frac{di_{qm}}{dt}\\ \frac{d\lambda_{qm}}{dt} &= L_{dq1}\frac{di_{dm}}{dt} + L_{q1}\frac{di_{qm}}{dt}\end{aligned} \tag{28}$$

$L_{mdy} = \frac{d\lambda_m}{di_m}$ is called dynamic inductance by analogy with $L_m = \frac{\lambda_m}{i_m}$ said static inductance. Both coefficients are given by the conventional nonlinear magnetization curve, determined experimentally.

$$L_{dq1} = (L_{mdy} - L_m)\cos\alpha\sin\alpha \tag{29}$$

$$L_{d1} = L_m + L_{dq1}\cot\alpha \tag{30}$$

$$L_{q1} = L_m + L_{dq1}\tan\alpha \tag{31}$$

After necessary manipulations, it is not difficult to have:

$$\frac{d\lambda_{ds}}{dt} = (l_s + L_{d1})\frac{di_{ds}}{dt} + L_{dq1}\frac{di_{qs}}{dt} + L_{d1}\frac{di_{dr}}{dt} + L_{dq1}\frac{di_{qr}}{dt} + L_{d1}\frac{di_f}{dt} \tag{32}$$

$$\frac{d\lambda_{qs}}{dt} = L_{dq1}\frac{di_{ds}}{dt} + (l_s + L_{q1})\frac{di_{qs}}{dt} + L_{dq1}\frac{di_{dr}}{dt} + L_{q1}\frac{di_{qr}}{dt} + L_{dq1}\frac{di_f}{dt} \tag{33}$$

$$\frac{d\lambda_{dr}}{dt} = L_{d1}\frac{di_{ds}}{dt} + L_{dq1}\frac{di_{qs}}{dt} + (l_r + L_{d1})\frac{di_{dr}}{dt} + L_{dq1}\frac{di_{qr}}{dt} + L_{d1}\frac{di_f}{dt} \tag{34}$$

$$\frac{d\lambda_{qr}}{dt} = L_{dq1}\frac{di_{ds}}{dt} + L_{q1}\frac{di_{qs}}{dt} + L_{dq1}\frac{di_{dr}}{dt} + (l_r + L_{q1})\frac{di_{qr}}{dt} + L_{dq1}\frac{di_f}{dt} \tag{35}$$

$$\frac{d\lambda_f}{dt} = L_{d1}\frac{di_{ds}}{dt} + L_{dq1}\frac{di_{qs}}{dt} + L_{d1}\frac{di_{dr}}{dt} + L_{dq1}\frac{di_{qr}}{dt} + (l_f + L_{d1})\frac{di_f}{dt} \tag{36}$$

The main differential system can be represented by

$$[V] = [A][\dot{X}] + [B][X]$$

with:

$$[V] = [v_{ds}\ v_{qs}\ 0\ 0\ v_f\ ]^t \tag{37}$$

$$[X] = [i_{ds}\ i_{qs}\ i_{dr}\ i_{qr}\ i_f\ ]^t \tag{38}$$

$$[A] = \begin{bmatrix} l_s + L_{d1} & L_{dq1} & L_{d1} & L_{dq1} & L_{d1}\\ L_{dq1} & l_s + L_{q1} & L_{dq1} & L_{q1} & L_{dq1}\\ L_{d1} & L_{dq1} & l_r + L_{d1} & L_{dq1} & L_{d1}\\ L_{dq1} & L_{q1} & L_{dq1} & l_r + L_{q1} & L_{dq1}\\ L_{d1} & L_{dq1} & L_{d1} & L_{dq1} & l_f + L_{d1} \end{bmatrix} \tag{39}$$

$$[B] = \begin{bmatrix} R_s & -\omega L_s & 0 & -\omega L_m & 0\\ \omega L_s & R_s & \omega L_m & 0 & \omega L_m\\ 0 & 0 & R_r & 0 & 0\\ 0 & 0 & 0 & R_r & 0\\ 0 & 0 & 0 & 0 & R_f \end{bmatrix} \tag{40}$$

It is clear that the winding currents model, which is the most known and used, is the heaviest one to compute. All 25 elements of its matrix $[A]$ are present and saturation dependent. Also, it contains all kinds of magnetic couplings along the d-axis L_{d1}, the q-axis L_{q1}, and the d–q axis L_{dq1}. On the contrary, the winding currents model, not therefore advised, is exploited here to derive any other saturation model whatever the state-space variables.

The proposed approach consists of three stages. First, a combination of state-space vectors is chosen among the sixty three remaining possibilities. Second, the d–q components of linkage fluxes and winding currents are described in terms of these selected variables using (12)–(15) and (19)–(21). Third, by ordinary manipulations of (32)–(36) time derivatives of the d–q components of the linkage fluxes are written as functions of the chosen variables.

IV. Derivation of the Saturated Models

Obviously, it is not possible to report in one paper all results related to the remaining sixty four models. In return, special interest will be done to novel models or to models cited in the literature but cannot be developed by previous works [16].

A. Model with $(\bar{i}_s, \bar{i}_m, i_f)$, as State-Space Variables

The choice of a pair of currents, for example $(\bar{i}_s, \bar{i}_m, i_f)$, among the six possible ones, imposes the description of fluxes $\bar{\lambda}_s, \bar{\lambda}_r$ and λ_f as well as their derivatives in terms of the selected vectors.

The leakage inductances in (4)-(6) are assumed to be constant, only the main flux $\bar{\lambda}_m$ is subject to saturation. Deriving stator and rotor linkage fluxes, in (4)-(6), leads to the time derivative of the magnetizing flux $\bar{\lambda}_m$.

$$\begin{aligned} \frac{d\bar{\lambda}_s}{dt} &= l_s \frac{d\bar{i}_s}{dt} + \frac{d\bar{\lambda}_m}{dt} \\ \frac{d\bar{\lambda}_r}{dt} &= l_r \frac{d\bar{i}_r}{dt} + \frac{d\bar{\lambda}_m}{dt} \\ \frac{d\lambda_f}{dt} &= l_f \frac{di_f}{dt} + \frac{d\lambda_{dm}}{dt} \end{aligned} \tag{41}$$

with

$$i_{dr} = i_{dm} - i_{ds} - i_f \tag{42}$$

$$i_{qr} = i_{qm} - i_{qs} \tag{43}$$

Therefore, $\frac{d\bar{\lambda}_m}{dt}$ has been defined in (25)-(28), so we have:

$$\begin{aligned} \frac{d\lambda_{ds}}{dt} &= l_s \frac{di_{ds}}{dt} + L_{d1}\frac{di_{dm}}{dt} + L_{dq1}\frac{di_{qm}}{dt} \\ \frac{d\lambda_{qs}}{dt} &= l_s \frac{di_{qs}}{dt} + L_{q1}\frac{di_{qm}}{dt} + L_{dq1}\frac{di_{dm}}{dt} \end{aligned} \tag{44}$$

$$\begin{aligned} \frac{d\lambda_{dr}}{dt} &= -l_r \frac{di_{ds}}{dt} - l_r \frac{di_{ds}}{dt} + \left(l_r + L_{d1}\right)\frac{di_{dm}}{dt} + L_{dq1}\frac{di_{qm}}{dt} \\ \frac{d\lambda_{qr}}{dt} &= -l_r \frac{di_{qs}}{dt} + \left(l_r + L_{q1}\right)\frac{di_{qm}}{dt} + L_{dq1}\frac{di_{dm}}{dt} \end{aligned} \tag{45}$$

$$\frac{d\lambda_f}{dt} = l_f \frac{di_f}{dt} + L_{d1}\frac{di_{dm}}{dt} + L_{dq1}\frac{di_{qm}}{dt} \tag{46}$$

After having carried out handling which is essential, the main differential system can be represented by

$$[V] = [A]\left[\dot{X}\right] + [B][X]$$

with:

$$[V] = \left[v_{ds}\ v_{qs}\ 0\ 0\ v_f\ \right]^t \tag{47}$$

$$[X] = \left[i_{ds}\ i_{qs}\ i_{dm}\ i_{qm}\ i_f\ \right]^t \tag{48}$$

$$[A] = \begin{bmatrix} l_s & 0 & L_{d1} & L_{dq1} & 0 \\ 0 & l_s & L_{dq1} & L_{q1} & 0 \\ -l_r & 0 & l_r + L_{d1} & L_{dq1} & -l_r \\ 0 & -l_r & L_{dq1} & l_r + L_{q1} & 0 \\ 0 & 0 & L_{d1} & L_{dq1} & l_f \end{bmatrix} \tag{49}$$

$$[B] = \begin{bmatrix} R_s & -\omega l_s & 0 & -\omega L_m & 0 \\ \omega l_s & R_s & \omega L_m & 0 & 0 \\ -R_r & 0 & R_r & 0 & -R_r \\ 0 & -R_r & 0 & R_r & 0 \\ 0 & 0 & 0 & 0 & R_f \end{bmatrix} \tag{50}$$

B. Model with $(\bar{i}_s, \bar{\lambda}_r, \lambda_f)$ as State-Space Variables

It is well emphasized in previous that models, having $(\bar{i}_s, \bar{\lambda}_r, \lambda_f)$, $(\bar{i}_r, \bar{\lambda}_s, \lambda_f)$ or $(\bar{i}_s, \bar{i}_r, \lambda_f)$ as state variables, cannot be derived by differentiation. The major advantage of the proposed approach is its ability to undergo such limit and to develop any possible model. This is going to be verified with the $(\bar{i}_s, \bar{\lambda}_r, \lambda_f)$ model, for instance.

For this case, matrices $[B]$ and $[A]$ involves the description of $\bar{i}_r$, i_f and $\bar{\lambda}_s$ with the selected variables and $\frac{d\bar{\lambda}_s}{dt}$ with their time derivatives, respectively. Note that due to the absence of the q-axis excitation winding, the number of equations in the d and q axes is not the same. Thus, it is more convenient for synchronous machines to separate the space-vector equations in d-q ones.

Indeed, the procedure of deriving the present model is closely similar to that used for the previous one. Briefly, (5) and (6), if associated to (7) and (8), yield:

$$i_{dr} = -\frac{l_f}{L_1} i_{ds} + \frac{1}{L_2}\lambda_{dr} - \frac{1}{L_1}\lambda_f \tag{51}$$

$$i_{qr} = -\frac{L_m}{L_r} i_{qs} + \frac{1}{L_r}\lambda_{qr} \tag{52}$$

$$i_f = -\frac{l_r}{L_1} i_{ds} - \frac{1}{L_1}\lambda_{dr} + \frac{1}{L_3}\lambda_f \tag{53}$$

where in:

$$L_1 = l_r + l_f \frac{L_r}{L_m} \tag{54}$$

$$L_2 = l_r + l_f \frac{L_m}{L_f} \tag{55}$$

$$L_3 = l_f + l_r \frac{L_m}{L_r} \tag{56}$$

This is enough to express the stator flux components and then construct matrix $\left[B\right]$. Next, Time derivatives of stator flux components are described after eliminating, in (32) and (33), damper and excitation currents using (34) to (36). The consequent matrices are:

$$\left[X\right]=\left[i_{ds}\ i_{qs}\ \lambda_{dr}\ \lambda_{qr}\ \lambda_{f}\right]^{t} \tag{57}$$

$$\left[A\right]=\begin{bmatrix} l_s+L_{d3} & L_{dq3} & \frac{L_{d3}}{l_r} & \frac{L_{dq3}}{l_r} & \frac{L_{d3}}{l_f} \\ L_{dq3} & l_s+L_{q3} & \frac{L_{dq3}}{l_r} & \frac{L_{q3}}{l_r} & \frac{L_{dq3}}{l_f} \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{58}$$

$$\left[B\right]=\begin{bmatrix} R_s & -\omega L_4 & 0 & -\omega\frac{L_m}{L_r} & 0 \\ \omega L_5 & R_s & \omega\frac{l_f}{L_1} & 0 & \omega\frac{l_r}{L_1} \\ -R_r\frac{L_m}{L_r} & 0 & \frac{R_r}{L_2} & 0 & -\frac{R_r}{L_1} \\ 0 & -R_r\frac{L_m}{L_r} & 0 & \frac{R_r}{L_r} & 0 \\ -R_f\frac{l_r}{L_1} & 0 & \frac{R_f}{L_1} & 0 & \frac{R_f}{L_3} \end{bmatrix} \tag{59}$$

with:

$$L_{dq3}=l_f\,\frac{l_r^2(L_r^{-1}-L_{rdy}^{-1})\cos\alpha\sin\alpha}{L_3+l_r^2(L_r^{-1}-L_{rdy}^{-1})\cos\alpha^2} \tag{60}$$

$$L_{d3}=l_r\frac{l_f}{L_1}+\frac{l_f}{L_3}L_{dq3}\cot\alpha \tag{61}$$

$$L_{q3}=l_r\frac{L_m}{L_r}+\frac{L_3}{l_f}L_{dq3}\tan\alpha \tag{62}$$

and:

$$L_4=(l_s+l_r\frac{L_m}{L_r}) \tag{63}$$

$$L_5=(l_s+l_r\frac{l_f}{L_4}) \tag{64}$$

V. Experimental Determination of Static and Dynamic Saturation Coefficients

The magnetizing curve of the synchronous machine is the well-known nonlinear function $\lambda_m(i_m)$, Fig. 1. It is of prime importance in the present analysis. Its approximation by mathematical expressions was already performed in multiple forms. The most used fitting are polynomial, exponential, smoothing spline…

In transient studies of saturated induction machines, it is common to first approximate the experimental characteristic $\lambda_m(i_m)$ and second determine the expressions of the desired coefficients.

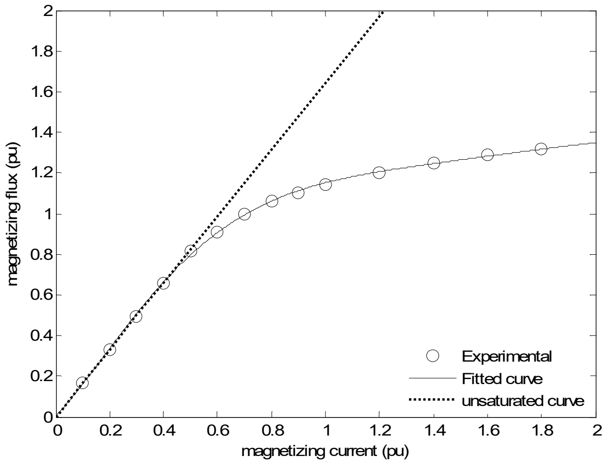

Fig. 1 Variation of λ_m vs. i_m

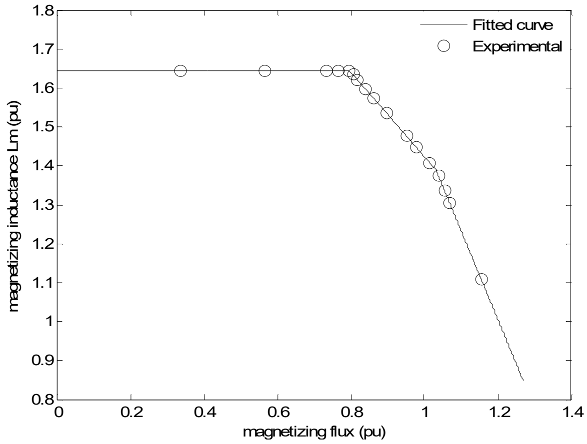

Fig. 2 Variation of L_m vs. λ_m

On the contrary, the alternative we propose in this contribution begins with the fitting of the characteristic relating the magnetizing inductance L_m, as obtained experimentally to the air gap flux. Fig. 2 shows such characteristic which, in general, possesses two portions. One corresponds to the unsaturated zone where L_m is constant and all the dynamic terms are null. The other corresponds to the saturated region and may be represented by a limited number of segments. In general, two segments give a sufficient accuracy. For each of them L_m is expressed as:

$$L_m=k_1-k_2\lambda_m \tag{65}$$

Constants k_1 and k_2 depend upon the machine. Relation (65) when associated to (7) gives:

$$i_m=\frac{\lambda_m}{k_1-k_2\lambda_m} \tag{66}$$

Hence, the dynamic inductance L_{mdy} becomes:

$$L_{mdy} = \frac{d\lambda_m}{di_m} = \frac{L^2{}_m}{k_1} \tag{67}$$

As seen, dynamic term L_{mdy} is also written in term of L_m is simply the magnitudes ratio of the magnetizing flux and current.

VI. Simulation Results and Discussion

Though, the theory of main flux saturation is well recognized, a short application on synchronous machine is added to verify the validity of proposed saturated models and the equivalence between them. Also, the objective of this application is to initially show comparison between models with cross magnetizing and without cross saturation moreover the equivalence between the sixty for models and then the influence of magnetic saturation on the machine operating.

For that purpose a critical application were selected. The abrupt three-phase short circuit of an isolated and damped alternator actuated at his speed of synchronism and initially excited. The generator per unit parameters are: $R_s = 0.003$, $R_r = 0.01334$, $R_f = 0.000927$, $l_s = 0.19$, $l_r = 0.08129$, $l_f = 0.1415$, $\omega = 120\pi$.

The following figures compare the novel saturated model with the model without cross saturation, for a short circuit. More specifically, Figs. 3, 5 and 7, show respectively the currents i_{ds}, i_f and i_{qs} for each scenario. Figs. 4, 6 and 8, prove respectively comparisons between the stator currents i_{ds}, i_f and i_{qs} with and without cross saturation, in same conditions of excitation. While the model without cross magnetizing is grossly in error, the novel saturated model shows good agreement with the so-called simulation data on the currents i_{ds}, i_f and i_{qs} .This suggests that, even when approximated, a simple ad hoc adjustment scheme should always be applied to the d-axis linear network in order to avoid transient results, which are largely unrealistic. Despite this effectiveness in predicting d-axis variables, the novel model unexpectedly displays differences up to some % of with the i_{ds} without cross saturation in Fig. 3.

VI. Conclusion

A complete and detailed synthesis of possible d-q models is established by changing the state space variables, which proves the existence of 64 possible models, in the electrical equations. The choice of state-space variables in modeling of saturated synchronous machine has been discussed and a number of models have been derived. The saturated analytical model with cross magnetizing and the saturated model without cross magnetizing are combined for the simulation of synchronous machines, where numerical integration can be used to solve the nonlinear state equations.

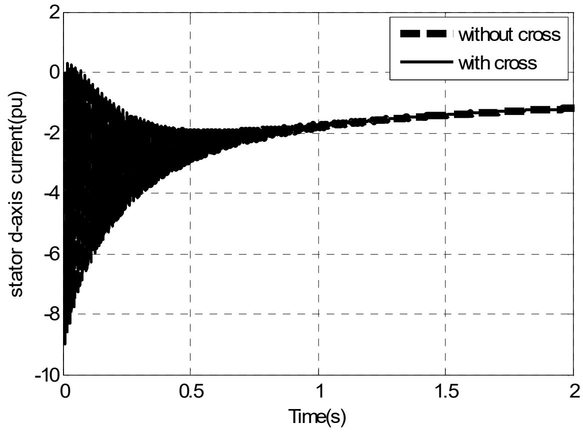

Fig. 3 Transient behavior of stator d-axis current following the short circuit of a damped alternator

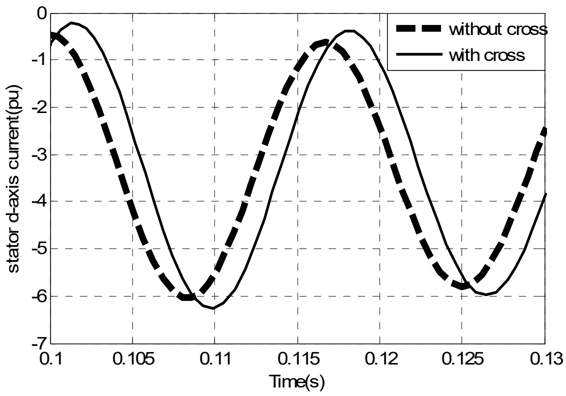

Fig. 4 Comparison of stator d-axis current obtained with two different full saturated synchronous machines with and without cross saturation for same conditions as in Fig. 3

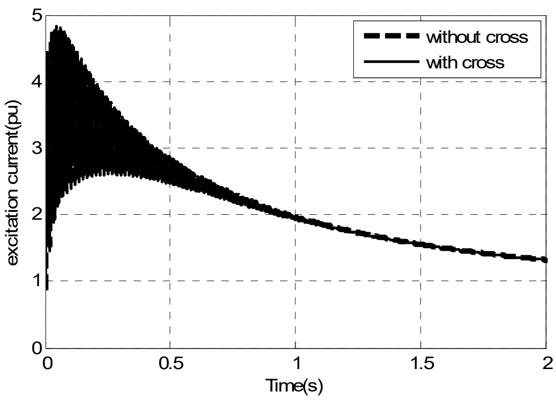

Fig. 5 Response of damped alternator of excitation current

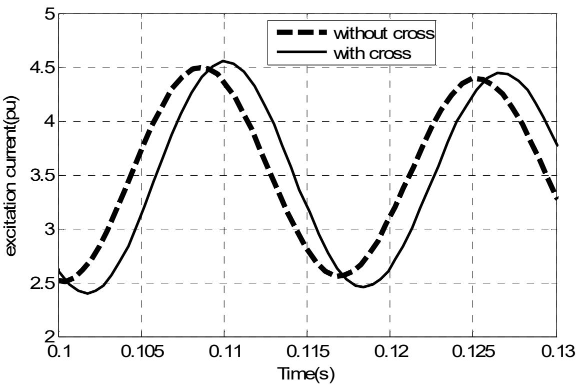

Fig. 6 Comparison of excitation current obtained with two different full saturated synchronous machines with and without cross saturation for same conditions as in Fig. 5

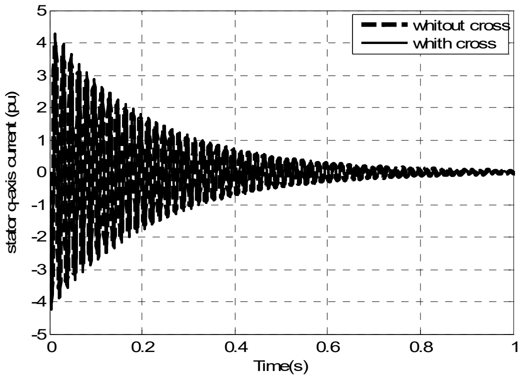

Fig. 7 Transient behavior of stator q-axis current following the short circuit of a damped alternator

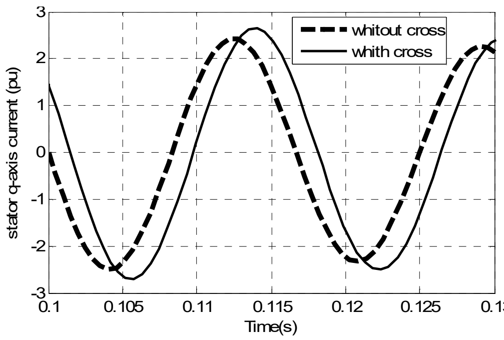

Fig. 8 Comparison of stator d-axis current obtained with two different full saturated synchronous machines with and without cross saturation for same conditions as in Fig. 7

ACKNOWLEDGMENT

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