International Journal of Business, Human and Social Sciences
ISSN: 2517-9411
Vol:9, No:6, 2015

Bounded Rational Heterogeneous Agents in Artificial Stock Markets: Literature Review and Research Direction

Talal Alsulaiman, Khaldoun Khashanah

Abstract—In this paper, we provided a literature survey on the artificial stock problem (ASM). The paper began by exploring the complexity of the stock market and the needs for ASM. ASM aims to investigate the link between individual behaviors (micro level) and financial market dynamics (macro level). The variety of patterns at the macro level is a function of the AFM complexity. The financial market system is a complex system where the relationship between the micro and macro level cannot be captured analytically. Computational approaches, such as simulation, are expected to comprehend this connection. Agent-based simulation is a simulation technique commonly used to build AFMs. The paper proceeds by discussing the components of the ASM. We consider the roles of behavioral finance (BF) alongside the traditionally risk-averse assumption in the construction of agent's attributes. Also, the influence of social networks in the developing of agents interactions is addressed. Network topologies such as a small world, distance-based, and scale-free networks may be utilized to outline economic collaborations. In addition, the primary methods for developing agents learning and adaptive abilities have been summarized. These incorporated approach such as Genetic Algorithm, Genetic Programming, Artificial neural network and Reinforcement Learning. In addition, the most common statistical properties (the stylized facts) of stock that are used for calibration and validation of ASM are discussed. Besides, we have reviewed the major related previous studies and categorize the utilized approaches as a part of these studies. Finally, research directions and potential research questions are argued. The research directions of ASM may focus on the macro level by analyzing the market dynamic or on the micro level by investigating the wealth distributions of the agents.

Keywords—Artificial stock markets, agent based simulation, bounded rationality, behavioral finance, artificial neural network, interaction, scale-free networks.

I. Introduction

THE stock price movements were and still are an unsolved puzzle. The contradiction between the fundamental value of a stock and the actual market price creates this problem. The traditional assumptions of asset values can be traced back to Adam Smith [1] and his concept of the invisible hand. In financial mathematics, it has been assumed that stock prices follow random walks that go in line with the Efficient Market Hypothesis [2]. In the same context and as a development of the Random Walk Hypothesis[1], the martingale property states that the expected future price given all the past information is the current price.

These theories in addition to other theories and models such as portfolio theory, capital asset pricing model (CAPM), arbitrage pricing theorem (APT) and other rational expectation models were able to predict the market in a certain epoch. Still, the financial markets have shown anomalies and behaviors that could not be explained by these theories, which lead to investigations of behavioral finance (BF). As early as Allais' paradox [5] that demonstrated the inconsistency of the independence axiom in expected utility theory and, in 1953, criticized the "rational man" in the American school of economics. In 1955, H. Simon [7][2] introduced the concept of bounded rationality to better approximate human capacity to deal with available information and not with full information. After all, the ideal "rational man" that produced optimal decisions with complete information did not exist. Under bounded rationality, agents search for *satisficing* (a portmanteau of satisfy and suffice) not optimal solutions.

The development of prospect theory in 1979 introduced behavioral economics and behavioral finance. In prospect theory, Tversky and Kahneman [12] experimentally showed that people tend to emphasize losses more than profit and thus they are more loss-averse than risk-averse.

It was in the 1980s when scholars began to compare between the volatility of stock markets and the anticipated volatility of the efficient market hypothesis (EMH). The comparisons show that stocks reveal some excess volatility that is due to non-fundamental reasons. For example, Dehnad [13] examined the EMH against the market data using some statistical tools. He reasoned that certain assertions of EMH and mathematical finance could be eliminated with a high degree of confidence. In the 1990s, academia tended to move toward developing models of human psychology to understand the price movement[3].

The BF models have shown that implementing arbitrage strategies might be both risky and costly, and thus noise traders may have a long impact on prices. These models provided the first building block of BF that is concerned with arbitrage. The second building block of BF is psychology. The different psyches of people such as overconfidence, optimism, representativeness (over-weighting of new information) and conservatism (under-weighting of new information) tend to form biased beliefs [14]. Pompian [15] have included twenty biases in his book; this includes overconfidence bias,

T. Alsulaiman is a PhD candidate in Financial Engineering at Stevens Institute of Technology, Hoboken NJ, USA (email:Talsulai@stevens.edu).
K. Khashanah is the Division Director of Financial Engineering at Stevens Institute of Technology, Hoboken NJ, USA (email: kkhashan@stevens.edu).

[1]See [3] and [4]

[2]See also [8]-[11]

[3]see [18]

representativeness bias, anchoring bias, cognitive dissonance bias, availability bias, self-attribution bias, illusion of control bias, conservatism bias, ambiguity bias, endowment bias, self-control bias, optimism bias, mental accounting bias, confirmation bias, hindsight bias, loss aversion bias, decency bias, regret aversion bias, farming bias, and status quo bias.

In addition to BF, Farmer and Lo [16] pointed out the role of biology in the modeling of stock market where they stated:

> *"One of the most promising directions is to view financial markets from a biological perspective and, specifically, within an evolutionary framework in which markets, instruments, institutions, and investors interact and evolve dynamically according to the "law" of economic selection. Under this view, financial agents compete and adapt, but they do not necessarily do so in an optimal fashion. Evolutionary and ecological models of financial markets are truly a new frontier whose exploration has just begun."*

In fact Lo [17] declared that the evolutionary perspective provides the missing ingredient to the concept of bounded rationality. More specifically, individuals will be able to determine the satisfactory point by learning from their mistakes in a given environment conditions. However, as a reconciliation between the EMH and BF, Lo [17] and [19] has proposed the Adaptive Market Hypothesis (AMH). AMH claimed that the anomalous that may be observed in the financial are only anomalous in the eyes of an economist (rational) while they are completely natural in biological perspective. The summary of the implications of the AMH as provided by Lo are:

1) The relationship between the risk and return are not stable over time but rather is affected by the population size and the preference of the majority.
2) The AMH states that up-normal returns occasionally take places.
3) Investment strategies are also influenced by the environment of the market.
4) While the EMH states that utility maximization is the objective of the investment, the AMH claims that survival is the ultimate goal for the investors.

Agents in stock markets are equipped with variable degrees of rationality, behaviors, risk preferences, information access and asymmetries, and variable computational capabilities. All of these attributes create the complexity of the markets. A complex system is defined by Mitchell [20] as:

" A system in which large networks of components with no central control and simple rules of operation give rise to complex collective behavior, sophisticated information processing, and adaptation via learning or evolution."

Agents interact by passing signals and information to each other. Based on these interactions they learn and evolve through adaptive processes. These attributes make the stock market a complex adaptive system (CAS)[4]. It is problematic to measure the state of CAS by using analytical methods without oversimplification; thus it may be appropriate to use computational methods to do so. The advantage of computational methods stands over analytical approaches in its flexibility and ability of modeling. It can capture much more elements of the system and simulate it more realistically. The computational family includes an approach that is called agent-based simulation models (ABSM). ABSM is a bottom up approach in the sense that it is not necessary to know the complex structure of the system but rather the system is described starting from agents in the CAS. We describe their attributes and how they may interact within a particular environment and how they are governed by given rules of interaction. The resulting patterns are what usually referred as emergence of outcomes. Simple rules imposed on a population may create complex patterns- a well-documented phenomenon in biological systems.

Since the financial and economic systems are classified as CAS, ABSM may be a good fit for building artificial stock markets (ASM) [21] and [20].

II. Literature Review

A. Components of Artificial Stock Markets

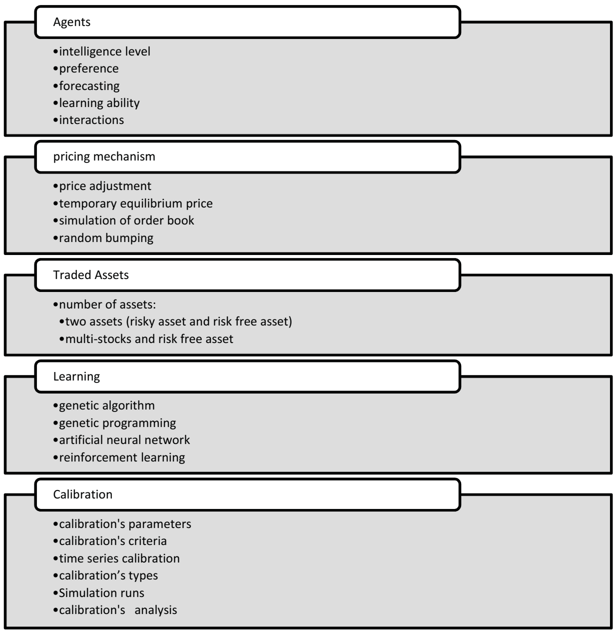

Fig. 1 Components of artificial stock markets

1) Agents: Researchers have discussed issues of agent design [22]-[26]. The main issues encountered in agent design are intelligence levels, decision-making processes, preferences and/or objective functions that individually and collectively influence agents' decisions, learning and interactions between agents.

The types of agents in the artificial financial markets vary from zero intelligence to sophisticated genetic programming [22]. However, the level of agents intelligence is highly related to the process of learning that is explained in the learning section in this paper. In the literature, agents

[4]Khashanah has provided a comprehensive taxonomy of existing systems [6]

in ASM may process their decisions through rule-based system. Rule based systems can be defined as an ”IF-Then” structure that relates the given information or facts. An example of a rule is that if stock price drop below a certain price then a buy signal is issued. Another method that can be used for the decision-making process is by using econometrics models, which include time series models and regression models. Moreover, agents may perform their forecasts by utilizing cognitive systems. Cognitive systems science is an interdisciplinary area that constructs artificial systems combining perception, action, reasoning, learning and communication. They might be built by implementing algorithms from various fields such as artificial intelligence, machine learning and data mining. These include algorithms such as artificial neural networks, genetic algorithms, genetic programming, adaptive boosting, support vector machine, Bayesian networks, reinforcement learning and classifier systems. In its sophisticated form, the rule-based system is classified as a cognitive system. It is important to notice that many of the techniques that are used for forecasting processes can also be used for learning processes; some of the most common cognitive methods are explained in more details in the learning section learning section.

The majority of agent-based models of financial markets assume that agents are risk averse and establish the objective function based on this assumption. Risk aversion implies that the agent would value certain outcomes over uncertain ones. Two ways are used to measure the risk aversion; they are relative risk aversion (RRA) and absolute risk aversion (ARA). We focus here on ARA that measures risk aversion in absolute terms to a loss relative to agents wealth. However, agents allocate their funds based on risk aversion characteristics of their utility function. The characteristics of risk aversion utility function are as follows:

- Increasing Absolute Risk Aversion (IARA): the agent tends to invest less in the risky asset as their wealth increase.
- Constant Absolute Risk Aversion (CARA): the agent tends to invest in the risky asset despite the level of change of their wealth increase.
- Decreasing Absolute Risk Aversion (DARA): the agent tends to invest more in the risky asset as its wealth increases.

In addition, mean/variance preferences have been used in many agent-based models e.g. [27]. In addition, it has been suggested by [23], to incorporate behavioral features into utility functions. Lovric [28] has combined some of these features into the microscopic simulation market that developed by Levy et al. [29]. He mainly investigates the market dynamic under the following behaviors:

- **Overconfidence:** overconfidence means overestimating self-ability and over trusting self-judgment. The overconfidence investors believe that they are superior in terms of picking the right assets and the right time to execute their transactions.
- **Sentiment (optimism, pessimism):** the optimism investor overestimates the future returns while the pessimism underestimates the future returns. The level of sentiment can be controlled using a specific parameter.
- **Self-attribution bias:** is a cognitive dissonance that refers the success to the individuals skills and failure to the bad luck. Overtime, self-attribution bias may lead to overconfident behavior.
- **Loss aversion:** is another term of Prospect Theory. It is a behavior where individuals value the losses more than gains. In other words, equal dollar amounts of loss or gain have unequal psychological impact on the same agent. More impact is assigned to losses than gains.
- **Recency and primacy effects:** are terms used in psychology to describe the effect of the order of presentation on the memory. The effect of primacy offers higher weight to the earlier information than the later one. On the other hand, the recency effect bases higher weight on the new information. Combining the two effects leads to decision’s bias that depend only on the initial and recent information with complete neglect or limit attentions to the information in-between.

Such behaviors may be common, and they can lead to irrational choices and, therefore, it is important to be considered when simulating the stock market.

For more realistic ASM, the literature identified two types of interactions in ABS referred to as phenotype and genotype interactions. Phenotype interactions mean that agents communicate through the aggregated level of the commodity price. On the other hand, genotype interactions imply that agents communicate with their neighbors, friends, advisory firms and share their investments opinions and/or strategies. In general, all ASM models must have phenotype interactions while genotype interactions provide a better approximation to reality and capture more of its complexity. Darley [25] stated that one should contemplate who interacts with whom, what the flow of information is and what the results of interactions are.

Two types of interactions have been defined by Vriend [30] that are endogenous and exogenous. These two types are metaphors to genotype and phenotype interactions. He looked for several of common styles to deal with interaction’s modeling.

- **Residential pattern [31]:** the agent in [31] model communicates with his Moore neighbors[5] in a lattice world[6]. If the ratio of similar agents in the neighbors does not meet the agent’s threshold he moves, otherwise he stays. The departures and arrivals of agents change the ratio in the neighborhoods. The agent decision is myopic response to the number of agents that are like him.
- **Resource gradient [32]:** the agents in [32] communicate with their von Neumann neighbors[7]. The agents trade for the resource that they need with however happiness to be their neighbors regardless the principles of gain and loss.
- **Predictor [33]:** the problem is that the agent will decide to go to the bar if there are only less than 60 individuals

[5]Moore neighbors are the eight agents surrounding the central agent.
[6]See next section
[7]Von Neumann neighbors are the four agents that are around the central agent

out of 100 total population. Each agent has a set of predictor in his mind. These predictors are based on the historical number of attendances. These predictors are evaluated based on the past performance. The agent will go to the bar if the predictor shows that the expected attendance is less than 60. The interaction is endogenous in term of that the decision is mad based on the past attendance which is so-called by Vriend [34] a past pattern of interaction.

- **Advertising signals [34]:** the firms communicate with the consumer by sending advertisement signals randomly to the population. These signals come along with additional cost that makes the determination of the number of signals to be sent a decision that evolve with time. The consumer can shop randomly, stay loyal to the current firm or follow the advertisement signals.
- **Expected payoff/familiarity [35] :** in this model, the buyer will observe the quality of the service and the prices in term of a payoff function and he will choose the seller that maximizes the expected payoff function. On the other hand, the seller may give some priority to some buyers if he familiar with them. The familiarity is represented as a weighted average of the past visits.

2) Networks: Network is one of the essential principles that characterize the complex systems. The dynamics of the networks are affected by the population and the structure of the network. These effects may be reflected in the aggregate level [36] and [37]. The network may be represented in term of nodes (agents) and links that connect them.

Wilhite [38] demonstrated some of the common networks structures that are used in economic literature. These structures are summarized below:

- **Complete network:** all the agents are connected with each other.
- **Star network** the agents are connected to a central agent but not to each other.
- **Ring network:** a network that has a circle structure where the agent is connected to the nearest two of the neighbors.
- **Grid (lattice) network:** it is a two-dimensional network on a discrete surface. The agent may be connected to von Neumann neighbors or Moore neighbors or has a connection that it is not related to his neighbors.
- **Tree network:** it combines two or more star networks into one network.
- **Small world network:** in small world network, the agents may communicate directly even though they are not neighbors.
- **Scale free (power) network:** the communications between the agents decay according to power-low distribution as the agent move away from the center.

3) Pricing Mechanism: Determine the asset prices and how the agents will trade is one of the most important issues that should be considered in the designing of artificial market. Four methods have been surveyed in the literature for pricing mechanism[8]:

[8]See [22] and [23]

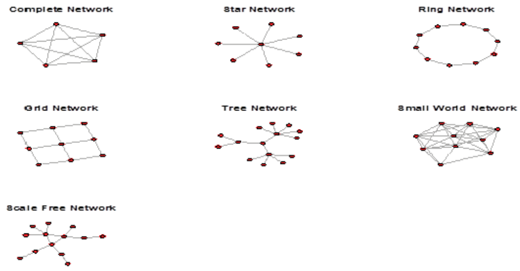

Fig. 2 Types of networks

- **Price Adjustment Method:** in the price adjustment method, the new price is determined based on the previous price and the difference between the aggregate demands and supplies multiplied by a sensitivity parameter.

$$p_{(t+1)} = p_t + \alpha(D_t - S_t)$$

where $p_{(t+1)}$ is the asset price at time t, p_t is the price at time t, D_t is the net of demands and S_t is the net of supplies and α is the sensitivity parameter.
- **Temporary Equilibrium Price:** in this method, demand functions would be developed by the ABM. These functions include all bids and ask (demands and supplies) from all agents. Then the clearing price would be determined numerically by solving

$$\sum_i x_t(p_t) = N$$

where i is the agents index, $x_t(p_t)$ is the demand function and N is the total number of assets shares in the market.
- **Simulation of Order Book:** in this method, more realistic approach is used where the actual trading mechanism is mimicked by which the limit orders and order crossing rules are simulated. This method is more challenging due to the level of attention that has to be given to many of scheming details. However, it is more suitable for simulating the market microstructure behavior.
- **Random Bumping:** in this method, the orders are matched randomly and the trade is executed if it is in favor of both parties.

4) Traded Assets: In general, in agent based financial market; the designer should determine what the types of assets that should be included. Would the market include different types of financial assets such as currencies, stocks, commodities and even more derivatives? Or is it limited to one asset. Also, what are the numbers of each included asset?

The agents in the majority of ASMs endeavored to allocate their wealth between the only two types of assets that are risky and risk free assets. Although, having only two assets may trigger some objections as unrealistic assumption but by interpreting the risky asset as market index may be reasonable [28] and [29]. However, some ASMs are implemented with multi-assets e.g. [39].

5) Learning: The development of learning in ASM varies from using zero intelligence agents with a budget constraint such as the work done by [40] to applying advanced artificial intelligence and machine learning techniques such as reinforcement learning [41], artificial neural network [42], genetic algorithm [43], and genetic programming [44]. Here we review some of the most popular learning algorithms that are used to build ASMs. Brenner [45] provided a comprehensive review of learning process in economic models. He divided the learning process into reinforcement learning and cognitive learning. The most important cognitive learning models that are presented by Brenner [45] are:

1) Bayesian Learning
2) Least Square Learning
3) Genetic Algorithm
4) Genetic Programming
5) Classifier System
6) Artificial Neural Network
7) Rule Learning
8) Stochastic Learning

In addition, Brenner [45] claimed that none of the above models continuously overcome the others and all of them are ad-hoc processes without scientific justification. However, these methods can be used in both learning and forecasting processes of the agents in ASM. We explore here the Genetic Algorithm, Genetic Programming, Artificial Neural Network and Reinforcement Learning.

- **Genetic Algorithm:**

Genetic Algorithms (GA)[9] was developed by Holland in 1975. GA is a stochastic search technique that emulate the biological evolution where the stronger tends to survive and the weak dies. The formal definition and mechanism description of the GA, which was provided by Goldberg, is as follows:

> *" Genetic Algorithms are search algorithms based on the mechanics of natural selection and natural genetics. They combine survival of the fittest among string structures with a structure yet randomized information exchange to form a search algorithm with some of the innovative flair of human search. In every generation, a new set of artificial creatures (string) is created using bits and pieces of the fittest of the old; an occasional new part is tried for good measure. While randomized, genetic algorithms are no simple random walk. They efficiently exploit historical information to speculate on new search points with an expected improved performance."*

The GA contains a population of chromosomes (also called string or individual). Each chromosome contains a number of genes (also called features, charters, alleles or decoders). The population evolves over time where the poorly performed rules according to the fitness function are replaced by new ones. The general procedure of genetic algorithms can be summarized as follows:

1) Initialize a population of chromosomes.

[9] For more details about GA, refer to [46], [47], [48], and [102]

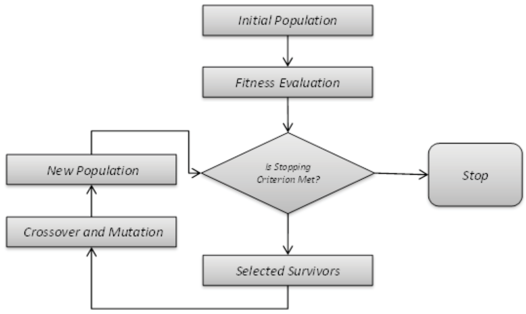

Fig. 3 Typical process of GA

2) Evaluate each chromosome according to a given fitness function.
3) Select the best-fitted chromosomes using stochastic selection procedures such as the roulette wheel selection, ranking selection or tournament selection. This is known as selection or reproduction process in GA literature.
4) Recombined the selected chromosomes by using crossover and mutation operators.
5) If the termination criteria are met then end the search, otherwise go to step 2.

Fifth, the crossover operation is implemented according to a given crossover rate. Crossover rate is critical to the success of genetic algorithms [49]. The crossover rate controls the exploitation capability of GA for local optima. Therefore, when the crossover very large, the GA will converge quickly to local optima and when it is too small the process of converging process will be very slow. Crossover operations might be implemented by using single-position crossover, two-position crossover, multi-position crossover, shuffle crossover [10], or uniform crossover[11].

Sixth, the second operator of GA, which is the mutation, alters one or more of the chromosome's genes randomly to ensure search diversification. It is performed upon mutation rate that controls the speed of GA in exploring new areas [49]. The small mutation rate is common among GA literature; however, some exception does exist e.g. [50].

Finally, several termination criteria might be adopted when design GA. One of them is the algorithm will stop if the maximum number of generations has been reached. Another criterion is that an acceptable best-fit individual has evolved. An additional criterion is that the average and/or maximum fitness value does not change significantly over the past specific number of generations. Fig. 3 Typical process of GA demonstrates the typical GA process.

- **Genetic Programming:**

Genetic Programming (GP)[12] was developed in its current form by Koza in 1992. Similar to GA, GP is a stochastic search technique based on biological evolution where it is programed

[10] Shuffle crossover first shuffles the crossover positions in two selected chromosomes, then it exchanges the segments between the crossover positions and finally un-shuffles the chromosomes

[11] Uniform crossover produced two new chromosomes by exchanging genes in two selected chromosomes according to crossover rate and a uniform random number given to the same gene in both chromosomes.

[12] For more details about GP, refer to [51], [52] and [103]

for an automated solution of a defined task. Poli et al. [103] have defined the GP as follows:

> *"Genetic programming (GP) is an evolutionary computation technique that automatically solves problems without requiring the user to know or specify the form or structure of the solution in advance. At the most abstract level GP is a systematic, domain-independent method for getting computers to solve problems automatically starting from a high-level statement of what needs to be done. "*

Just as in GA, GP has begun with generating a set of the initial population. However, in the GP the population is represented in term of the programs that are expressed as syntax trees. The syntax tree consists of terminal and function sets. The terminal sets are located in the leaves of the tree, and they usually take the form of variables or constants. The function sets are the nodes in the tree that may be represented by arithmetic, mathematical, logical or conditional function. For example consider the following statement, if variable x equals a constant number and y greater than another constant number then take action. Here the variables x and y and the two constants form the terminal sets while the conditions equal and greater than form the function sets.

After generating the initial programs, each program is subject to evaluation according to the fitness function. The candidate programs are then selected using the selection methods that are similar to the selection methods in GA. Crossover operation is implemented in the selected programs. However, in GP the crossover is done by exchange two randomly chosen sub-trees among the parents. The crossover point here is a function node and the crossover is occurring according to a given rate. In addition to crossover, mutation operation is performed on the selected programs according to a given mutation probability where we replace randomly chosen sub-tree by randomly generated tree. This continues until a complete set of new programs is generated and then we evaluate them a cording to the fitness function. The process is repeated till termination criteria are met.

the crossover is done by exchange two randomly chosen sub-trees among the parents. The crossover point here is a function node and the crossover is occurring according to a given rate. In addition to crossover, mutation operation is performed on the selected programs according to a given mutation probability where we replace randomly chosen sub-tree by randomly generated tree. This continues until a complete set of new programs is generated and then we evaluate them a cording to the fitness function. The process is repeated till we meet the termination criteria.

- **Artificial Neural Network:**

The artificial neural network (ANNT)[13] is an approximation method that has been inspired by the biological neural network. It is one of the most popular methods in financial forecasting. However, it may be used as a learning process in the agent-based model. The ANNT consists of input, output and hidden layers. Each of the layers contains one or more nodes. These nodes are connected by which each node in the input layer is connected to each node in the first hidden layer and each node in the first hidden layer is connected to each node in the second hidden layer (if a second hidden layer is needed). However, each node of the last hidden layer is connected with each node in the output layer. Each connection is assigned a random weight between 0 and 1 at the initial stage of the ANNT. In addition, each node in the hidden and output layers has an extra weight that is not associated with any node.

For the hidden layer, we apply a combination function by summing the multiplication of node inputs and the connection weights to find the net associated with each node in the hidden layer. The net is then used as input to an activation function. The obtained values of the activation function are then processed as inputs of the hidden layer nodes and the combination function is employed to determine the net value of each node in the output layer. The activation function is used to find out the final values of the output nodes.

The error rate is usually calculated by taking the squared difference between the actual values and the output values. In order to minimize the sum of square errors, the weights are optimized using techniques such as back propagation and GA.

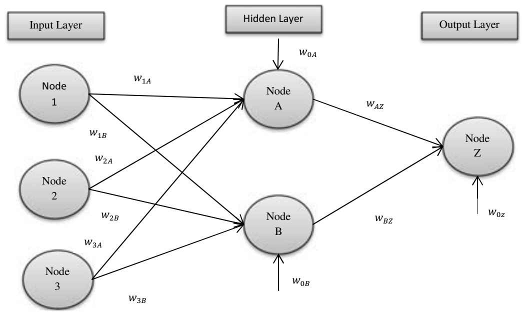

Fig. 4 Simple ANNT

- **Reinforcement Learning:**

Unlike evolutionary methods (GA & GP), Reinforcement Learning (RL) is not an algorithm by itself but rather a problem that required development of an algorithm to solve. RL is classified as a third learning next to supervised and unsupervised learning [59]. In the RL, agents learn via experience. Sutton et al. [60] defined RL as following:

> *"Reinforcement learning is learning what to do–how to map situations to actions–so as to maximize a numerical reward signal. The learner is not told which actions to take, as in most forms of machine learning, but instead must discover which actions yield the most reward by trying them "*

Agents buying and selling decisions are a typical RL problem where the selected actions by the agent affect and affected by the surrounding environment, the environment here is the asset prices over the time. The agent is then rewarded or punished for his action by receiving a positive or negative return. As time goes on, the agent develops a form of policy by implementing a mapping from states to probabilities of

[13] [53]-[59]

selecting an action in order to maximize his rewards over a long period. Any method that can effectively develop such a policy is RL method. Based on this classification the pre-described evolutionary methods may be considered as RL methods.

However, in their book Sutton et al. [60], focused on the methods that are structured around estimating a long run expectation of the reward. These include methods such as dynamic programming, Monte Carlo simulation and temporal difference learning. [45] claimed that models such as Bush-Mosteller Model, the principle of melioration and Roth-Erev model are widely used in economic and finance.

6) Validation and Calibration:: Two of the most important steps in the simulation, in general, and in agent-based simulation in particular are verification and validation. Verification is the process of ensuring that the model operates as intended. It is always required to debug a computer program to determine the location and source of errors. In addition, it is advisable to perform the model by more than one person that has simulation background, and it is recommended to start the model under simplified assumption and then extend it.

Validation is the process of ensuring that the model represents the reality. Usually, ABMs contain several parameters that are used to represent the system. These parameters may have uncertain values or a range of values, some of which may have a significant impact on the model behavior. The process of selecting these values is called parameterization and the selection should be based on empirical information. However, calibration is a special kind of parameterization in which we optimize only the most imperative parameters to create patterns that are close to the real system [61]. In the validation process, we fit the value of the essential parameters to match the real data. This is usually done by executing the model several times using various values of its parameters and examines which parameter values lead to similar patterns of the real system.

However, the process of validation can be implemented through three stages [62]. First, we need to determine what are the appropriate parameters for calibrations. The best method to find what the appropriate parameters are is by conducting sensitivity analysis [61]. Second, calibrate these parameters. We may refer to calibration process as inverse problem [62] meaning that instated of forward computing given input and parameter values to provide output values, we would have given input and output values and search for parameters that produce output values that are close to the given data. There are several considerations when calibrating the model. First, define the calibration criteria. At this stage, we need to specify the quantitative patterns for the model calibration e.g. stock returns. Second, determine how to use time series data for calibration. For example, in ASM, the change of prices over time may be measured; one may calibrate the model parameters by calculating the statistical properties of S&P 500. Third, determine if the parameter needs to have rang-of-fit or best-fit calibrations. The rang-of-fit calibration takes several values within a specific range. However, the best-fit calibration optimizes the parameter of a unique optimum value. In the third stage, we validate the model by running the simulation several times with different parameter values, compare it with calibration criteria and calculated statistical properties of actual data. Then, analyze the results statistically by which known as statistical validation [62].

B. Stylized Facts

Most of the developed agent based models in the financial markets aimed to explain the features of the stylized facts. Stylized facts are simplified presentation of the empirical findings. Cont [64] explained the stylized facts:

> *"The seemingly random variations of asset prices do share some quite nontrivial statistical properties. Such properties, common across a wide range of instruments, markets and time periods are called stylized empirical facts"*

Chen et al. [63] provided a table of thirty of the most analyzed stylized facts in finical markets. They divided the stylized facts into six groups. The first two groups are pertaining to return and trading volume using low frequency data. The rest of the groups deal with high frequency data, and they are concerned with the return, trading, duration, transaction size and bid-ask spread. Below is a brief summary of some of these stylized facts.

- **Absence of Autocorrelation:**the autocorrelation in financial time series is the correlation between the return r_t and its lagged values r_{t-l}[14].

$$\rho_l = \frac{Cov(r_t, r_{(t-l)})}{\sqrt{var(r_t)var(r_{t-l})}}$$

  The absence of correlation can be tested using Portmanteau test using the following hypothesis:

$$H_0 : \rho_1 = ... = \rho_m$$
$$H_1 : \rho_i \neq 0 \ for \ some \ i \in \{1, ..., m\}$$

- **Fat Tails:** the existing of fat tails means that the returns are normally distributed and thus excess losses and/or returns are not so rare.
- **Aggregational of Gaussianity:** as time scale increases, the returns distribution would be closer to normal distribution.
- **Calendar Effect:** is any market up-normal movement due to the effect of any type of seasonality or related days issues.
- **Gain/Loss Asymmetry:** the market tends to reach a negative return $-r$ faster than a positive return $+r$.
- **Leverage Effect:** the empirical evidences show that volatility seems to be higher in case of a large decrease than a large increasing in asset prices.
- **Volatility Clustering:** it is most likely a large variation follows when large variations occurred at prices. The volatility clustering can be measured by calculating the autocorrelation function of the return squared.

$$C_2(l) = corr(r_t^2, r_{(t-l)}^2)$$

[14]See [65]

- **Long Memory:** if the autocorrelation function of time series decays slowly to zero at a polynomial rate as the increase, the series has a long memory. Shiller [66] provided some statically evidences that support the existing volatility in the stock market.
- **Volatility Volume Correlations:** measure the relationship between volatility and volume and if the volume would cause volatility to cluster.
- **Bubbles and Crashes:** the prices are considered to be in a bubble / crash state when they rise/ drop dramatically with obvious reasons.
- **U shape:** the existing of U shape is an indication of heavier trading at the day's terminals and thinner trading in the middle of the day.
- **Price Duration Over-dispersion:** the price duration is the required time for the price to change by a given amount. The over-dispersion occurs when the standard deviation of price duration is greater than the mean.
- **Trade Duration Over-dispersion:** trade duration measures the time intervals between trades. The over-dispersion of trade duration indicates that trading intensity varies through the day.

TABLE I
Stylized Facts From [63]

Low Frequency	Return	1	Absence of Autocorrelations
		2	Aggregational of Gaussianity
		3	Bubbles and Crashes
		4	Calendar Effect
		5	Conditional Heavy Tails
		6	Equity Premium Puzzle
		7	Excess Volatility
		8	Fat Tails
		9	Gain/Loss Asymmetry
		10	Leverage Effect
		11	Long Memory
		12	Power Law Behavior of Return
		13	Power Law Behavior of Volatility
		14	Volatility Clustering
		15	Volatility Volume Correlations
Low Frequency	Volume	16	Power Low Behavior of Volume
		17	Long Memory of Volume
High Frequency	Return	18	Absence of Autocorrelations
		19	Fat Tails of Return
		20	Long Memory
		21	Periodic Effect
		22	Bursts
High Frequency	Trading Duration	23	Clustering of Trade Duration
		24	Long Memory
		25	Duration Over-dispersion
High Frequency	Volume	26	Power Law Behavior of Trades
		27	U Shape
		28	Price Change/Spread correlation
		29	Thinness and large Spread
		30	Turn-of-the-year Declining

C. Related Previous Studies

Although the models of segregation [67] and [31][15] are not directly related to the stock market but rather to minority game modeling [23] but it is hard to start any literature survey on agent-based simulation without mentioning them. These works considered pioneer works in the field of agent based modeling [38]. These works investigated the phenomena of segregation. Schelling developed agent-based models where his agents (people) behave according to given rules and he observed the outcomes of following these rules on the aggregate level. Each agent will count the number of the similar agents in the neighborhood. If the number the similar agents exceed the threshold-based rule, the agent will stay. Otherwise, the agent will leave the neighborhood. The outcomes are that there is no segregation if the threshold is set to be low; a very segregated environment would occur if the threshold is set to be reasonable (around 50%) and the system would not reach any steady state level if the threshold is set too high. This implies that the macro-behavior may not be a reflection of micro-motive.

However, the agent based computational economics models (ACE) has been classified into three categories [23]. The first category is known as few type models where the analysts examine a small number of strategies that are used by the agents to trade risky assets. In his survey, he included the works of Frankel et al. [69] and [70].

The increase of dollar value in 1984 despite the fact that the interest rate differentials have not been expanded has induced Frankel & Froot (1986) to study this discrepancy. They developed a model composed of three types of agents. The first agent is the fundamental analysts who provide their recommendations based on rational expectations. The second agent is those who follow the technical trends and then provide trading suggestions. The third agent is the portfolio managers who take the market positions based on the weighted average of the recommendations given by first two agents. The weighted average is updated periodically based on the performance of the recommendations of the fundamental and technical agents. As a result of the interactions between heterogeneous agents, the exchange rates become erratic.

Kim and Markowitz [70] model was motivated by the financial crash in 1987. The objective of their model is to determine if the behavior of investors (agents) would effect on the trading volumes, fluctuation in the prices or not. They assumed that there are only two types of investors, the investors with re-balancing strategy, where 50% of their wealth is allocated in risky assets and 50% is allocated in risk free assets, and the investors that are motivated by Constant Proportion of Portfolio Insurance (CPPI). The investors in Kim & Markowitz model do not have a direct interaction but they interact through the asset prices. Based on the simulation results, they conclude that there is a positive correlation between the second type of investors and the trading volume and price volatility.

The second category of ACE is many types models. It is called many type models because that the numbers of trading rules are much larger than few type models. LeBaron [23] enclosed in his survey the works of Arifovic [71], Lettau [72] and Routledge [73].

Arifovic [71] tried to solve an economic maximization problem of exchange rates using GA. The agents in her

[15] See also [68]

experiment are keen to maximize their income subject to their budget constraints overs two-period. The objective of her study was to examine if the equilibrium of the exchange rates is achieved using GA. The simulation results show that a stationary is not attained when agents learn through GA.

Lettau [72] develop a model of two versions. In the first version, He assumed that the population of agents is fixed. In the second version, the population of gents is mutable where new agents may enter the market and existing ones may leave. The objective of his study is to compare between the investment decisions that are made by boundedly rational agents and the optimal decision based on rational expectations. The agents can invest in a risky asset that pays normally distributed random dividends or to not invest. In addition, he assumed that the risky asset prices are not influenced by the agents but rather they are determined externally. Because of the normal distribution assumption of the dividends, an optimal solution is obtainable. He tested the capability of the agents to find the optimal solution by implementing GA. He remarked that the agents would be able to find the optimal solution if the lifespan of the agent and the population of decision rules are adequately determined. In other words, short time periods or small sample size would prevent agents from discovering the optimal solution.

The third category of ACE is emergence and many types models (Chen called it autonomous agent models; see [63]. LeBaron [23] specified objectives of these models:

> *"In emergence and many type models, the artificial market models moves farther from testing specific models and more towards understanding which types of strategies will appear in a dynamic trading environment. All have at their core a philosophy of building a kind of dynamic ecology of trading strategies and of examining their coevolution over time. This methodology attempts to determine which strategies will survive, and which will fail. Also, one observes which strategies will emerge from a random soup of starting strategies, and which are capable of self-reinforcing themselves, so that survival is possible. They also attempt to perform a very direct exploration into the dynamics of market efficiency the market moves into a state where certain inefficiencies appear, then the hope is that the evolutionary process will find new strategies to capitalize on this. The objective is to explore a market that may not be efficient in the textbook sense, but is struggling toward informational efficiency"*

Gode & Sunder [40], Observed the behavior of the market under profit motivated traders and zero intelligence (ZI) traders. ZI traders generate random bids and offers for their transactions. They performed five experiments for five different schedules of bids and offers to discovery if the prices tend to converge to a particular level. The ZI trades performed their trades with and without budget constraint. The experiments showed that the stock prices for unconstrained ZI traders do not converge to equilibrium though profit-motivated traders drive the prices to steady state level.

However, in the case of constrained zero intelligence traders the series showed that there is no learning sign. Also, the volatility level is higher than profit motivated case but lower than ZI traders case and the prices eventually reach an equilibrium state but slower than a profit-motivated case. In terms of allocative efficiency, the average among all the experiments for profit-motivated traders was 97.9%. Similarly, the average allocative efficiency of constrained ZI traders was 98.7%. Such results indicated that the style of the actual market is close to the behavior of ZI traders with budget constrained. In the other hand, the allocative efficiency dropped to 78.3% when the budget constraint is relaxed for the zero intelligence traders. The results of the experiment show that the Adam Smith Invisible Hand may be more powerful than some may have thought.

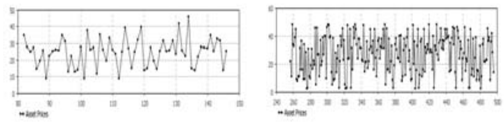

Fig. 5 Replication of Gode and Sunder [40] market prices with budget constraint (left) and without budget constraint (right)

The Santa Fe Artificial Stock Market Model (SFI-ASM) transpired as an illuminated model in this field. The debate of whether the adaptive agents would soon coverage in an equilibrium state and thus the market becomes efficient lead to the first version of SFI-ASM [74]. SFI-ASM has been modified and extended in later versions, see [43] and [50]. The objective of the SFI-ASM was to find an answer to settle the long outstanding issues between theorist and market participants. The theory states that the market is efficient and thus there is no possibility for making a continuous up-normal return. On the other hand, market participants claim that the market is driven by emotions essentially fear and greed and thus continuous profits may exist.

<u>Notations:</u>

The authors have constructed an artificial market consists of heterogeneous agents. Each agent has a choice to invest in a risky stock that pays stochastic dividend or/and in a risk free bond that pays a fixed rate of return. Each agent aims to maximize their wealth by allocating his portfolio between these two assets at each time period. The model is driven by notations and assumptions. Here we present these elements.

- $N : Number\ of\ Traderes\ (Agents)$
- $i : Trader\ index\ (i = 1, 2, ..., N)$
- $t : Time\ index (t = 1, 2, ...)$
- $p_t : Stock\ price\ at\ timet$
- $d_t : Divdend\ paid\ by\ stock\ at\ time\ t$
- $x_{it} : Amount\ of\ stock\ hold\ by\ agent\ i\ at\ time\ t$
- $W_{it} : Wealth\ of\ agent\ i\ at\ time\ t$
- $r_f : Riskfreerateofthebond$
- $\varepsilon \sim N(0, \sigma^2) : stochastic\ shock\ at\ time\ t$
- $\lambda : Degree\ of\ risk\ aversion$
- $\rho : Speed\ of\ mean\ reversion$

<u>Assumptions:</u>

- The time is discrete.

- The risk free rate of the bond is in infinite supply.
- The stock is limited with a finite supply.
- The dividend is following a stochastic process similar to random walk and it is represented by Autoregressive model with order of one.
- The agent does not inform other agents of his expectations and his trading intentions.
- The utility functions are indistinguishable for all traders.
- The stock return is normally distributed.

As stated, the objective for each agent is to maximize his utility function (constant absolute risk aversion) subject to his budgets; therefor the model may take mathematical program form:

$$Max\ U(W_{it+1}) = -e^{-\lambda W_{it+1}}$$

$$W_{it+1} = x_{it}(p_{t+1} + d_{t+1}) + (1 + r_f)(W_{it} - p_i x_{it})$$

where d_{t+1} and x_{it} can be estimated using:

$$d_{t+1} = \bar{d} + \rho(d_t - \bar{d}) + \varepsilon_{t+1}$$

$$\hat{x}_{it} = \frac{E_{it}[p_{t+1} + d_{t+1}] - p_t(1 + r_f)}{\lambda \rho^2_{t,p+d}}$$

where $\rho^2_{t,p+d}$ is the empirical observed variance of the stock combined price plus dividend time series. Given the basic structure of the artificial market, the agents start with a set of forecasting rules in order to predict buying and selling signals where they carry on the form of conditional rules that are based on Holland's classifier system. These rules would be evaluated at each time period. The survival rules are then combined with new forecasting rules while the poorly performed rules are eliminated. GA was implemented for rules evaluations and evolutions over time. The authors designed two experiments to test their hypothesis. The difference between the two experiments is centered on the learning rate parameter. Substantially, they placed fast and slow learning rate where in faster learning rate the agents learn on average every 250 time periods and in the slow learning rate the agents learn on average every 1000 time periods.

The simulation results show that when agents learn slowly the market coverages to the rational expectations equilibrium. Oppositely, in fast learning case the agents do not ended into any equilibrium form.

The SFI-ASM has persuaded many researchers to pursue their studies in the same directions. Joshi et al. [75] used SFI-ASM to study the effect of technical analysis on the market. The stated hypothesis is what strategy is best for the investor: using a combination of technical and fundamental analysis or focusing only on the fundamental analysis. The experiment to forward as follows, the agent believes that all other agents would or wouldn't include the technical analysis in their strategy. This assumption leads the experiment to take an economical game theory mode. The aim is, then, to find out if there is a dominating strategy.

They simulate the SFI-ASM using fast learning rate of 100. The simulation results find out the adding technical analysis would dominate the strategy of using fundamental analysis alone. Understandably, all the other agents will include the technical analysis in their investment analysis since it is superior to each of them individually, which creates Nash equilibrium. However, it has been noted that the expected wealth of the agent in the state where technical analysis is used by him and all other agents as well is less than the expected agent wealth in the case where both players do not use the technical analysis. This creates a prisoners dilemma situation. The authors discussed, the introducing of technical analysis increases the market volatility, noise and causes bubbles and crashes.

TABLE II
Strategies Table of An Agent in Joshi et al. [75]

		All other agents	
		Technical and Fundamental	Fundamental only
Agent	Technical and Fundamental	Expected Agents Wealth 1	Expected Agents Wealth 2
	Fundamental only	Expected Agents Wealth 3	Expected Agents Wealth 4

Tay and Linn [76], have altered Holland's classifier system by the genetic-fuzzy classifier. The objective of their study was to replicate the experiments using the new classifier and explore if the simulation results remain the same. The motivation was their hypothesis that agents are unable to handle a large number of trading rules, and therefore fuzzy approach is more appropriate. Similar to Lebaron et al. [43], they have conducted experiments with fast and slow learning rate. They conclude that an efficient market resulted when the agents learn slowly.

Bak et al. [77] constructed an ASM of two types of agents and a single stock that pays a random dividends that follow Bernoulli distribution. The types of agents are noise traders who may mimic others decisions and fundamental traders who aim to maximize their utility function. However, the agents are heterogeneous by which fundamentals traders will have different risk appetite and noise traders will either estimate the price randomly or randomly select another trader and imitate his estimation. They have investigated the market under different conditions. First, they simulated the market with fundamentals traders and found that the market will reach to equilibrium after some times and no more trade will occur. Second, they simulated the market with noise traders only. However, they conclude that the prices will vary according to reaction diffusion process if there is no direct interaction between the agents. On the other hand, if the traders imitate each other actions, the market would expose crowd behavior. In addition, they have investigated the market when 2% of the population is fundamental and 98% is noise traders. Under this environment they have detected a bubble phenomenon . However, if the portion of fundamental investors increases to 20%, the deviation from the rational price is insignificant.

Chan et al. [78] developed an agent based of financial market. The model consists of a single stock that pays dividends. The dividend payment appears in three states that are 0, 1 or 2 with probability of 1/3 for each state. The agents are classified as informed traders where the coming state of the dividend is known for them, partially informed traders who know that one of the states will not occur for sure and uniformed traders that have no clue about the future state of the dividend. The agents place their orders based on one of three strategies. They may use empirical Bayesian strategy (fundamental), momentum strategy (technical) or they may place their orders based on nearest-neighbors heuristic (sophisticated technical). The technical and sophisticated technical traders disregard the available information of the dividend while the fundamental traders do not. However, the authors have constructed six experiments to explore the dynamic of market. Table III summaries these experiments. The experiments tested if the market's information is reflected in the prices or not. They do that by taking the mean absolute difference between the transaction prices that obtained by simulation and the prices that are obtained by rational expectation equilibrium (REE). Also, they tested the hypothesis by measuring the spread between bid and ask and trading volume. The first two experiments showed that the prices would converge to the REE. However, the second experiments do not show evidences of convergence. This indicates that when agents are heterogeneous, the prices would be driven far from the REE. In the fifth experiment, the authors reported that the deviation from the REE decreases as the time goes on because those agents would learn over time. However, they stated that as the number of the momentum traders increases, the deviation would increase. In the sixth experiment, the prices converge to the REE. However, in the sixth experiment, they have examined the predictability of the market by stating a hypothesis about the wealth differences between the empirical Bayesian traders and the nearest-neighbors traders. The results displayed that the prices may be predicted where the mean wealth of nearest-neighbors traders was significantly higher than the mean wealth of the empirical Bayesian traders.

TABLE III
The Experiments of Chan et al. (EB.=Empirical Bayesian, NN.= Nearest-neighbors

Ex.	Population	Preference	Information
Ex1.	20	homogenous	partially informed
Ex2.	20	homogenous	10 informed & 10 uniformed
Ex3.	20	heterogeneous	partially informed
Ex4.	20	heterogeneous	10 informed & 10 uniformed
Ex5.	20-to-170	homogenous	20 EB. & 0-150 Momentum
Ex6.	20	homogenous	15 EB. & 10 NN.

LeBaron [79] built an ASM that is akin to the SFI-ASM, however, the model comes with paramount differences. For example, the CARA utility function was superseded by CRRA that will increment the impact of the wealthier agents. Additionally, the classifier system was superseded by ANNT. The classifier systems were replaced by ANNT. The third major change is in the trading rules where in this model the trading rules are supplied through public sources such as mutual fund advisor or investment newsletter. The agents are heterogeneous in term of how they evaluate the performance of trading rules based on the historical performance. Some of the agents are utilizing long-horizon while others are using short-horizon. He considered the agent who use long-horizon a rational trader while the agent that use short-horizon as a noise trader. The agent and rules evolve over time. The evolution of agents is done by replacing one of the most five poorest agents randomly with a new agent that has a new feature. The rules evolve by substituting the rules that are not used for the last 10 periods by rules that are generated from GA. Two experiments have been performed. In the first experiments, the agent horizons were uniformly distributed between 5 and 100. The horizons distribution in the second experiments was uniform between 75 to 100. The objective was to examine if using longer horizon will lead to an equilibrium state of the market. The results demonstrated that the utilization of long-horizon will drive the market to an equilibrium state. Furthermore, the results showed that the wealth variations between agents are varied when the horizon range is expanded.

Chen et al. [44], introduce the concept of school into the artificial market. The role of the school is to generate rules that are different from agent's rules. A significant difference between the school and agents rules is in their fitness functions. The agent will evaluate his performance at evaluation date and consult the school if the performance is substandard. He would follow the school rules if it proves to be better than his. The objective was to examine some series' properties of prices and returns such as the normality assumption, existence of uniting root for the prices and if the returns are identically and independently distributed (iid). The simulation results showed the existence of the fat tails and thus neither prices nor returns are normal. Also, the unit root hypothesis is rejected, and returns are iid. Further, they explored if the traders will eventually follow the efficient market hypothesis and if they will evolve to be more experienced as time forwards. The outcomes showed the traders are not much into efficient market hypothesis; they keep looking profit opportunities; therefore, they consult the school at a considerable rate. In addition, no evidence of sophisticated behavior is shown.

Raberto et al. [80], developed the Genoa artificial stock market which consists of a set of naive agents. The agents trade randomly by creating random limit orders. The model is subjected to a budget constraint. The objective was to study some stylized facts of the financial time series. The conclusion confirmed previous study results that the independent returns have fat tail distributions and the existing of price volatility. However, they claimed that the volatility tends to vanish by increasing the number of agents.

Similar to Arthur et al. [50], Hommes [27] constructed an artificial market that he called it Adaptive Belief Systems (ABS). The objectives were to observe the level of volatility in the heterogeneous financial market, test if the market with heterogeneous agents would converge to the efficient state and find out if the technical traders have the opportunity to make profits. He concluded that the volatility is low when the market tends to be efficient. The volatility increases if the number of technical traders increase or news on fundamentals appear. In addition, he claimed that the market seems to be inefficient, and trend followers have a chance for profits making.

Farmer [81], explored an agent based model to examine stylized facts of financial time series. They introduced market maker as a third agent alongside with fundamentals and technical traders. The market maker balances the market, whenever buy and sell orders not matched; market maker restrains the excess. The study emphasized on market orders and assumed that the market maker does not consume any risk and the traders can invest in a single risky asset or save the cash with zero interest. They summarized that returns have fat tails and the excess kurtosis is relatively high compared to the normal distribution. Furthermore, they show that volume and volatility are positively correlated, and the increase of technical traders activities may increase the frequency of oscillations in mispricing.

Iori [82], modified the Ising model to simulate the financial market in order to observe the stylized facts. The decision process of the trader combined the trader signal along with the signals that he received from the four nearest neighborhoods, which would outcome a consolidated signal that is compared with a threshold. Based on the consolidated signal and the threshold, each period the agent may buy, sell, or hold. At the beginning of the trading, every trader starts with an equal amount of capital and number of shares that are subjected to capital constrain. The model includes market maker for price adjusting. The findings confirm the existing of volatility clustering. However, the volatility and volume have a negative correlation when the agents do not accept recommendations from neighbors and positive correlation whenever the neighbors are involved in the decision making process. Further, they observed that the threshold is a crucial parameter for volatility clustering. Moreover, the return showed a high independence level from its lagged.

Neuberg and Bertels [83] created an ASM market that connects between the news/information and the evolution of the stock prices. The market is exposed to information arrival where it varies from negative news (-3) to positive news (+3). The reference time series of the prices is calculated as a function of information and information sensitivity. The agents are divided into two groups; the first group is normal agents who learn the reference time series of the prices, the second group is the perturbating that are digressed the reference time series. The perturbating agents can be inverse by which they opposite the normal agents, filter by which they attach less criticalness to the extreme values of information arriving at the market and crazy agents who behave in a zero-intelligence way. The agent predicted the future prices using classifier system like that is utilized within the SFI-ASM. They simulated the markets under three different regimes that are normal, crashes and bubbles. The normal regime implies that the information varies between (-3) to (3) in a normal distribution fashion while, in other two regimes, the information are biased toward the negative and positive sides accordingly.

The simulation results showed that the correlation coefficient between the market prices and time series reference decrease sharply in the normal regime as the number of perturbating agents increase. However, the declined in the correlation coefficient was less significant in the crash regime and minutely diminutive in the bubble regime. In addition, the highest volatility appears in the bubble regime, and the least volatility appears in the normal regime. Likewise, the volatility decreases as the number of perturbating increase. The excess kurtosis in the return is higher in the bubble regime, and it grows as the number of perturbating increases. Finally, the

TABLE IV
The Experiments of Neuberg and Bertels [83](N=Normal Regime, B= Bubble Regime, C= Crash Regime

	Normal	Inverse	Filter	Crazy	Regime
1	10	0	0	0	N
2	7	1	1	1	N
3	5	2	2	1	N
4	4	2	3	1	N
5	3	3	3	1	N
6	2	3	3	2	N
7	10	0	0	0	B
8	7	1	1	1	B
9	5	2	2	1	B
10	4	2	3	1	B
11	3	3	3	1	B
12	2	3	3	2	B
13	10	0	0	0	C
14	7	1	1	1	C
15	5	2	2	1	C
16	4	2	3	1	C
17	3	3	3	1	C
18	2	3	3	2	C

skewness shows up if the market has more heterogenous agents and becomes higher in the bubble regime.

Takahashi et al.[84], assessed the asset prices by investigating the relationship between micro-rules and macro-behavior using agent based approach. The model was implemented using fundamentals (rational) and technical (behavioral) agents. The objective is to verify if it is true that the rational dominate irrational investors and, therefore, they survive according to the principle of natural selection. There are two types of assets, the stock that pays dividends and the risk free rate of returns, which the investor has to allocate his budget over them. The technical agents are classified as trend followers and overconfident traders. Also, investors based on prospect theory are introduced as an agent in the market. The prospect theory investors do not ponder in term of net asset but rather in term of gain and loss where the fear of loss overwhelmed the avarice for winning. The traded prices are determined according to Arthur et al. [50] method. They have conducted several simulation experiments, the results summary are:

1) All the investors in the market are the fundamentals: the efficient market hypothesis is confirmed.
2) 50% are fundamental traders and 50% are trend followers: the efficient market hypothesis is confirmed, the fundamentals eliminate the trend followers.
3) 10% are fundamental traders and 90% are trend followers: the efficient market hypothesis is invalidated, the prices are driven by its authentic values and the fundamental traders are dominated.
4) 30% are fundamental traders and 70% are trend followers, and there is a limit on holding ratio of stocks: the efficient market hypothesis is invalidated.
5) 30% are fundamental traders and 70% are trend followers, and there is a limit on holding ratio of stocks, but 1% of the fundamental traders: the efficient market hypothesis is confirmed.
6) 50% are fundamental traders and 50% are overconfident speculators: As the parameter of confidence level decreases (The smaller the value of the parameter the

higher degree of confidence), the market tends to be inefficient.

7) 30% are fundamental traders and 70% fundamental traders with prospect theory: the prices are driven from its fundamental value.
8) 80% are fundamental traders and 20% trend followers with prospect theory: the prices are driven from its fundamental value.

Chen et al. [39], extended the artificial market to comprise multiple assets in order to simulate the evolution of portfolio behavior. Agents, in this model, at each time point allocate their investments between portfolios including a number of stocks and saving account that pays a fixed risk free rate. Various agents compete to maximize eight different utility functions. One type of these agents believes in capital asset pricing model (CAPM) while the other seven have diverse beliefs. The agents' beliefs evolved using GA, and they deemed as machine learning agents where they preserved the recent days for validation purpose. The agents' believe have been tested against the validation period, and it has been noted that the accuracy of the beliefs is improved by increasing the period of validation and the simulation process become closer to the real world. Also, the authors compare between the roles of utility functions and agents believes and found that the utility functions play a more important role in agents' survivability. Further, they claimed that the existing of risk free rate asset lower the probability of survives for CAPM believers.

Kaizoji [85] defined three types of agents in his ASM. These agents are fundamental, chartists and noise traders. The fundamental traders are classified as rational traders while chartists and noise traders are classified as irrational traders. The agents in the developed ASM may switch their strategy if the performance is less than the opposed strategy. The addressed questions are under what conditions speculative bubbles may be observed, who has the highest probability to survive under such environment and does the distribution of the returns has a fat tail if the market is exposed to speculative bubbles. They have concluded that the nonlinearity of the excess demand function could cause the speculative bubbles phenomenon. Also, the irrational investors most probably will derive the rational investors out of the market under speculative bubbles environment. Finally, the returns distribution is proven to be fat-tailed.

The agents in Li and Rosser model [110] can be either fundamentalists or noise traders that use technical analysis,e.g. moving average, for trading. The traders may switch their trading strategy based on historical performance according to a given transition probability and switching sensitivity parameters. The simulation of the asset price confirmed that the return series have shown a volatility cluster, absence of autocorrelation in the short run and the existence of autocorrelation in the long run. Furthermore, the fat tail phenomenon is manifested.

Chen and Liao [86], studied the relationship between stock returns and trading volume. They have adapted the same ASM that developed by Chen et al. [44]. In the study, three experiments have been designed. The dividends in the first experiments are following normal distribution, and the parameter for risk aversion is 0.1 (investors are willing to accept higher risk). The second experiment differed in the parameter for risk aversion. It has been increased to 0.5, which indicates the risk appetite is low. The third experiment is similar to the first experiment except that the dividends are uniformly distributed. Three simulation runs were performed for each experiment, which brought the total number of experiments to nine. Based on the obtained results, they examined the casual relationship between stocks returns and trading volumes using linear Granger causality test and Baek and Brock test. They observed the relation at the macro (market) and micro (investors) levels and then compared the outcomes to see the consistency of the relation between macro and micro levels.

At the macro level, the existence of causality is indecisive. Similar results are found in the micro level. The relation between the macro and micro levels is consistent whenever the macro phenomena validate the micro behavior and inconsistent otherwise. In other words, if the macro and micro levels mutually rejected or accepted the null hypothesis of non-existence of causality, then the relation is consistent. Four experiments showed that the relation is consistent though six experiments showed the relation is inconsistent. These results induced the authors to conclude that the relations between stock returns and trading volume couldn't be fully understood unless the feedback relation between macro and micro levels is comprehensible.

TABLE V
Experimental Results of Chen et al. [86]

	Fail to Reject H_0	Reject H_0
Fail to Reject H_0	Ex2, Ex3, Ex4	Ex7, Ex8
Reject H_0	Ex1, Ex5, Ex9	Ex6

Cincotti et al. [87] have elongated the Genoa artificial stock market to explore the market of multi-assets. The agents are zero-intelligence in the developed market. However, the simulations are executed when the stocks are paying dividends and when they are not. The returns have exposed volatility clustering and returns fat-tailed distributions. Besides that the statistical tests have rejected the hypothesis of the random walks. However, the existence of unit root was not rejected if there are dividend payments. They have concluded that the ability to replicate stylized facts that are similar to the real markets with only zero-intelligence agents is worth more consideration.

> *"In our view, zero-intelligence behavior still deserves much interest for its simplicity and the possibility to focus the attention more on the structural aspects than on the behavioral features."*

Derveeuw [88] has modified the SFI-ASM by partitioning the agents into fundamental and technical traders in order to offer the agents with minimal economic rationale. He led the first experiment with fundamental traders only. The outcomes substantiated the random walks of the stock prices. The second experiment is executed with a mixture of Fundamental (25%) and technical (75%) traders. The results demonstrated higher volatility and the null hypothesis of random walks is rejected.

In addition, it has been asserted that the returns are not normally distributed.

Shimokawa et al. [89] built up a model of a risky asset and risk free asset. The risky asset does not pay any dividend and the expected return of the riskless asset is zero. The marketplace consists of informed traders, loss averse traders, and noise traders who invest randomly. Unlike the loss averse traders and noise traders, the informed traders are subject to a private signal of the future fundamental value of the asset; however, this signal is combined with random white noise. The optimal holding of the risky asset of the loss averse investors is set dependently upon a given parameter of a reference point that is computed by taking the moving average of the precedent prices throughout a designated period. The simulation results indicated that the existence of loss averse traders would run to increase the volatility and excess kurtosis. However, the return autocorrelation was feeble in both cases, but the autocorrelation function of return volatility was signed up to the 12th lags, when loss averse traders are presented, which attest the volatility clustering phenomenon.

Chen and Huang [90] investigated the survivability of variant agents by utilizing agent based simulation. The agents aim to allocate their capital between savings account and a portfolio of multi-assets. The agents evolve through GA where they update their investment decisions according to the update of their beliefs at a designated time horizon. The update of the beliefs is accounted to learning level of the GA while the update of the decision process is accounted to the optimization level of the GA. The agents are divided into CAPM believers (that do not evolve) and autonomous traders. The autonomous traders are differed only in their risk preferences in the first developed experiment while they differed in their risk preferences and belief formation process in the second experiment. The population size in the experiments was 40 with 5 agents of CAPM and 35 traders divided over seven different utility functions. Table V shows the preferences of investors in the model. The wealth of traders of type 1 is overriding the other traders wealth in the first experiment. The reason cannot be referred to forecasting precision since all traders are statistically having an equal forecasting accuracy. In addition, the saving rate was examined, and it has been found that type 1 agents have the lowest mean and variation on saving rate. Furthermore, the performance of the portfolio of traders of type 1 was not superior to other investors. Thus, the authors have returned the advantage of type 1 traders to their preference. The second experiment allowed variations in forecasting parameters which will produce different forecasting accuracy among the agents. The comparison between agents of type 1 and all the other agents showed that the wealth amount is still in favor of the agents of type 1, although their forecasting accuracy is less accurate. However, they have concluded that forecasting accuracy may matter when agents have a homogeneous risk preference.

Martinez-Jaramillo and Tsang [91] name their artificial market ”The Co-evolutionary Heterogeneous Artificial Stock Market” (CHASM). The investors in the CHASM can invest in a risky asset or keep their cash. The investors are fundamental, technical and noise traders. The traders actions are buy, sell or do nothing where they will convert the actions to bids and offers as a fraction of their current holding. The noise traders settle on their choices as stated by probability parameters that are corresponding to each potential action. The fundamental traders will adjust their holding if the deviation between the market price and the fundamental value of the asset is greater than a given threshold. The technical traders will forecast the future prices by using diverse sets of technical and momentum indicators that establish the decision trees. The technical indicators will, then, evolve by utilizing GP. By carrying on various experiments, they have reasoned out that the learning would ameliorate the wealth of the traders. However, the learning mechanism does not affect the statistical properties of returns. An exemption has occurred when the Red Queen[16] constraint is introduced where the statistical properties of prices become more authentic.

TABLE VI
Preferences of Traders in Chen and Huang Model [90]

Type	Preference
CAPM	No Preference
Type1	$u(c) = log(c)$
Type2	$u(c) = \sqrt{c}$
Type3	$u(c) = \alpha_1 + \beta_1 c$
Type4	$u(c) = (\alpha_1/\beta_1)e^{(\beta_2 c)}$
Type5	$u(c) = \left(\frac{1}{(\gamma_3+1)\beta_3}\right)(\alpha_3 + \beta_3 c)^{\gamma_3+1}$
Type6	$u(c) = c - (\frac{\alpha_4}{2})c^2$
Type7	$u(c) = a_0 + \sum_{i=1}^{6} a_i c^i$

LeBaron [92] has developed a market of two assets; one of them is a risky asset that pays stochastic dividend while the other is a riskless asset that pays a constant interest. The agents share similar preference; that is constant relative risk aversion but the differed in the formation of expectations. The first type of agents uses the adaptive linear forecast, the second type of agents utilizes log price dividend ratio regression, the third type uses linear regression, and fourth type buys and holds using long run mean of returns. The analysis of wealth showed that the adaptive agents frequently control about 45% of wealth fraction. Next to the adaptive agents, the buy and hold agents control about 40% of the wealth fraction, and they are followed by the fundamental agents with 10% and last the regression agents who control only 5% of wealth fraction.

Kurmar et al. [93] market divided into three segments that are the market maker, traders and news manager. The market maker role is to organize and control the market. The traders are either informed traders, who trade based on a given signal about the fundamental value of the asset, or uninformed (noise) traders who will trade randomly. There are two kinds of informed traders in this market that are perfectly informed traders, who monitor the true value of the assets, and nosily informed traders who capture a deformed fundamental value. The fundamental value of the asset remains constant most of the time; however, it is exposed to a jump according to a given probability. The jump process is a random normal process with mean zero and a given variance. Once the jump

[16]Red Queen is an analogy to co-valuation, where the trader has an access to the performance of other traders which permit him to compare his performance to and amend the investment rules accordingly

happens, the news manager will advise the informed traders of the new fundamental value and the market maker of the jump occurrence.

Liu et al. [94] constructed an ASM to study the impact of switching strategies on market volatility. In other words, what will happen in the market if the investors decided to pay for the information and move from the uniformed state to the informed state. In general, the market consists of informed traders, uniformed traders, switcher traders and noise traders. The informed traders know the current fundamental value of the stock. On the other hand, the uniformed traders know the fundamental value of the stock at lag time, and they predict the stock price using GA. The switcher traders are uninformed traders who evaluate the cost of information and if they found that it is worth, they switch to be informed traders. The noise traders are zero- intelligence traders. They are similar to uninformed traders except that they do not use GA to optimize their prediction. the authors have designed five simulation experiments where the differences between the experiments are in the percentage of uninformed and switcher traders. Table VI shows the distribution of population size amid traders types. The results showed that there is no significant change in the average returns of the informed traders. However, the average returns of the uniformed and switcher traders declined sharply as the percentage of switcher traders increases. Furthermore, the market has unveiled higher volatility along with the augmentation in the extent of switcher traders.

TABLE VII
The Experiments of Liu et al. [94]

	Informed	Uninformed	Zero-Intellgence	Switchers
Exp.1	12%	30%	58%	0%
Exp.2	12%	23%	58%	7%
Exp.3	12%	15%	58%	15%
Exp.4	12%	8%	58%	22%
Exp.5	12%	0%	58%	30%

Manahov and Hudson [95] constructed four experiments by which they isolated the agents into ”best agents” and ”all agents”. The population size of best agents was 1%, 5%, 10% and 20% of the aggregate population size in the four experiments respectively. The agents can exchanges in only one risky asset. Each agent will attempt to predict the future price in a precise manner. The forecasting accuracy for the trading rules is measured in term of strength by figuring the conditional variance. The selection of the trading rules and the evolving over time is implemented using GP.

Ke and Chen [96] constructed an artificial stock market that comprises of heterogenous agents who use distinctive sets of trading strategies that are classified as rational and irrational trading strategies. The rational trading strategies involve fundamental analysis and technical analysis while the irrational trading strategies include disposition effect, herding behavior, chase sell and absolute execution strategy. The trader learns by evaluating his forecasting accuracy. He proceeds with the current strategy if it fulfills the threshold; otherwise, he supplants the used strategy by another from the pool or by selecting an alternate set of indicators with likelihood 0.5 each. The investors are structured into institutional and individual where the extent of individual is larger, yet the buying/selling power of the institutional investors is higher. The simulation experiments examine the effect of the transaction cost, investors formation, tick size and price limit system. The observations are:

1) The trading volume will diminish alongside the increment in the transaction cost. Likewise, the volatility will lessen inferring that the investors become more cautious.
2) The increase of the proportion of institutional investors will decrease the stock volatility until the proportion reaches 90%. After that, the effect will be diminished.
3) The increase in tick size will increment the market volatility.
4) Decreasing the price limit will decrement market volatility.

Moving forward on ACE development, the stylized facts are then used to build the ACE. More specifically, they have been utilized for empirical estimation of the model parameters. Chen et al. [63], list the major contribution of calibrations and/or estimations. They stated that three approaches are most used in the literature. The approaches are the method of moments, maximum likelihood and least squares. Yet, these approaches work once the aggregation function can be driven analytically; otherwise simulation is exercised for estimation. Furthermore, they estimated parameters along with the artificial models that have been used for forecasting purposes by some researchers. De Jong et al. [97], estimated the three types ABS model and used it for forecasting the exchange rate. The model has been estimated with and without the switching mechanism. The types of agents are fundamentals, chartists and moving average traders. They calibrated the behavioral coefficients such as mean reverting, extrapolation and intensity choice for all types of agents. The mean reverting and extrapolation coefficients were significantly different among all agents while no significant difference is shown in intensity choice. Although the forecasting returns exceed the returns of a random walks, the results are required to be validated by using testing data.

Boswijk et al. [98], estimated the model parameters of ABS using S&P 500 yearly data from 1871 to 2003; namely, the estimated reverting coefficient, extrapolating coefficient and intensity coefficient. These parameters were estimated for chartists and fundamentals agents where the results showed significant results of the first two parameters and insignificant results for the third parameter. This indicates cohabitation of fundamentals and chartists in the market and underrates the learning behavior. Amilon [99], extended the ABS model by adding the agents perceived risk of investment, risk preference, fitness of measure and the noise structure. Also, he estimated the parameters of the modified ABS. This includes the estimation of mean reverting coefficients, extrapolation coefficient, and intensity of choice and memory parameters. All the parameters are found to be statistically significant.

In less spread pattern, ACE has been utilized in the advantage of econometrics. In this case, the ACE models are used as data-generation mechanism to test several econometric hypotheses. The objective is to compare between the standard behaviors of econometrics at aggregation level with its behavior when the micro level has more realistic configuration e.g. heterogeneous agents. For example, Chen [100] used the agent approach to estimate the elasticity in the individual and aggregate consumptions equations.

TABLE VIII
Research Considerations

Investment Strategies	
Fundamental	[27][29][39][41][42][72][73][69][75][77][78][81][84][85][110][88][90][91][92][96][99][104]
Technical	[27][42][69][75][77][78][81][84][88][91][94][96][99][105]
Mixed between fundamental and technical	[43][50][74][75][69][83]
Zero-intelligence	[40][78][80][83][87][89]
Minority/Majority game	[106][107]
Balance strategy and constant proportion of portfolio insurance	[70]
Informed and uniformed	[89][94]
Interactions	
No direction interaction	[43][50][70][74][77][83][88][99]
Direct interaction	[44][82][86][104]
Number of assets	
One risky asset and one risk free asset	[29][39][42][43][50][69][70][72][73][74][75][76][79][83][85][88][89][92][99][108]
Portfolio of risky assets	[80][87][90][109]
One risky asset	[41][72][77][78][81][91][94][96][105][107][109]
Pricing Mechanism	
Temporary equilibrium	[29][43][50][79][83][84][92][109]
Price adjustment	[74][85][110][88][91]
Order book	[40][78][80][105]
Dividend process	
Random walks	[29][88][92][99]
Auto-regressive process	[42][43][50][73][74][76][79]
Probability distribution	[72][77][86]
Behaviors	
Overconfidence	[28][84]
Prospect theory (loss aversion)	[28][84][89]
Optimists and Pessimist	[28]
Herding Behavior	[82][96][104][108]
Learning and Evolution	
Genetic algorithm	[39][43][50][71][72][74][79][83][88][90][105]
Genetic programming	[44][86][91][108][83]
Classifier System	[43][50][74]
Artificial neural network	[42][79]
Fuzzy rules	[76]
Bayesion	[78]
Reinforcement learning	[41][106]
Output	
Bubbles and crashes	[28][43][50][74][83][85][92][104]
Returns (fat-tail and correlation)	[28][29][40][43][44][50][74][76][77][79][80][81][82][83][84][110][87][88][89][92][94][104][107][108]
Returns (volatility clustering)	[27][28][29][42][43][50][76][77][79][80][81][82][83][84][110][87][88][89][92][94][104][107][108][109]
Volume	[28][29][42][43][50][77][82][84][85][86][89][96]

III. Research Direction

A. Motivation and Purpose

Despite the enormous number of works done on the markets dynamic, many questions remain unsolved. Is the market efficient or not? What are the real reasons that cause bubbles and crashes? Who has the higher probability to survive in a competitive market (risk takers, conservative investors, fundamental investors, technical traders etc.)? Is technical analysis useful? These questions and many yet need to be resolved. LeBaron [101] has stated:

> *"The fact that asset prices can move from simple benchmark rational pricing levels and then stay far from these levels for some time is a major puzzle"*

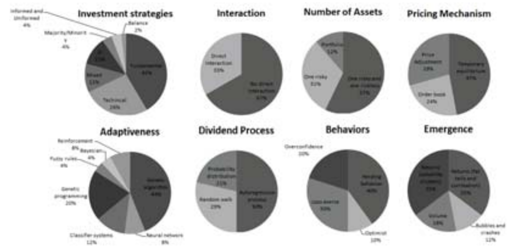

Fig. 6 Pie charts of research concentrations

The anomalies of stock movements and their contradictions with the traditional assumptions have inspired us to perform this study. The limitations of analytical methods due to the environments complexity are the main motive for analyzing the markets dynamic using agent based simulation in general and in this work in particular. Although agent based simulations might be less precise than the traditional approaches but they are much more flexible which provides representation that is more tangible to real life situations. We borrow here the quotation of Harry Markowitz as a motivation for using computational approaches:

> *"If we restrict ourselves to models which can be solved analytically, we will be modeling for our mutual entertainment, not maximize explanatory or predictive power"*

The purpose of the study is not to survey all the works that have been done to model and solve economic problems, but rather to study the related researches that have been done to model and solve artificial stock market problem. Another major focus of this research is the control of dynamic stochastic environment of the stock market. The control of this environment is accomplished by an integrated model that utilizes agent-based simulation, heuristic forecasting, learning techniques and pricing mechanism.

B. Research Questions

The aim of this study is to investigate different types of agents and trading environments in order to examine their impact on the aggregate market dynamic and conversely the impact of market dynamic on the agents' decisions. The primary objective of this research is to answer the following questions:

1) What are the models most important parameters that reproduce patterns that observed in the real market and what are the optimal values of theses parameters?
2) What is the effect of population size of a certain type of agent?

3) Is there a significant effect of wealth amount at the beginning of the investment period on wealth amount at the end of investment period?
4) What is the effect of communications and interactions between the agents on the statistical properties of the asset and the wealth of agents?
5) Are stylized facts fixed among the experiments?

C. Stated Hypotheses

The first question will be answered by performing sensitivity analysis (see Fig. 7). However, to answer the rest of the questions we stated the following hypotheses:

- **Question 2:**

$$H_0 : W_T \ when \ the \ P \ is \ large = W_T \ when \ the \ P \ is \ small$$

$$H_1 : W_T \ when \ the \ P \ is \ large \neq W_T \ when \ the \ P \ is \ small$$

where W_T is the wealth at the end of trading and P is population size.

- **Question 3:**

$$H_0 : W_T \ when \ W_0 \ large = W_T \ when \ W_0 \ is \ small$$

$$H_1 : W_T \ when \ W_0 \ large \neq W_T \ when \ W_0 \ is \ small$$

where W_0 is the wealth at the beginning of trading.

- **Question 4:**

$$H_0 : W_T \ (if \ I = 1) = W_T \ (if \ I = 0)$$

$$H_1 : W_T \ (if \ I = 1) \neq W_T \ (if \ I = 0)$$

where I is the interaction between the agents. It is a binary integer wehre it can either 0 (if there is no direct interaction) or 1(if there is a direct) interaction.

- **Question 5:**

$$H_0 : r_{\mu 1} = r_{\mu 2} = r_{\mu 3} = r_{\mu 4} = r_{\mu 5} = r_{\mu 6}$$

$$H_1 : r_{\mu 1} \neq r_{\mu 2} \neq r_{\mu 3} \neq r_{\mu 4} \neq r_{\mu 5} \neq r_{\mu 6}$$

where $r_{\mu 1}$ is the mean daily return of the stock price when the population is large, $r_{\mu 2}$ is the mean daily return of the stock price when the population is small, $r_{\mu 3}$ is the mean daily return of the stock price when W_0 is large, $r_{\mu 4}$ is the mean daily return of the stock price when W_0 is small, $r_{\mu 5}$ is the mean daily return of the stock price when there is communication between agents and$r_{\mu 6}$ is the mean daily return of the stock price when there is no communication between agents.

To test these hypotheses, the research has to go through several stages. First, we need to define and formulate the market components. This includes how many types of agents are there, what are their preferences, how they are going to base their investment decision and who they interact. In addition, we need to determine the heterogeneity and homogeneity of their behaviors. Also, we determine the pricing mechanism, traded assets, process of dividend and we defined the parameters of the models. Second, we need to produce the agent based simulation model and verify it using computer software.

Third, we perform a sensitivity analysis to ascertain the most important parameters that affect the model reality. Sensitivity analysis can be done using fractional factorial design. After that, we calibrate the models parameters to reproduce patterns that are close to real market. The calibration will be done against real data of S&P500, some liquid stocks such as Apple Inc. (AAPL) and Microsoft Corporation (MSFT), and some illiquid stocks such as Isramco, Inc (ISRL) and Optibase, Ltd. (OBAS). Fifth, we designed a set of experiments to examine our hypotheses. The experiments will be analyzed statistically using Analysis of Variance (ANOVA). Fig. 7, demonstrate the basic flow chart of the research.

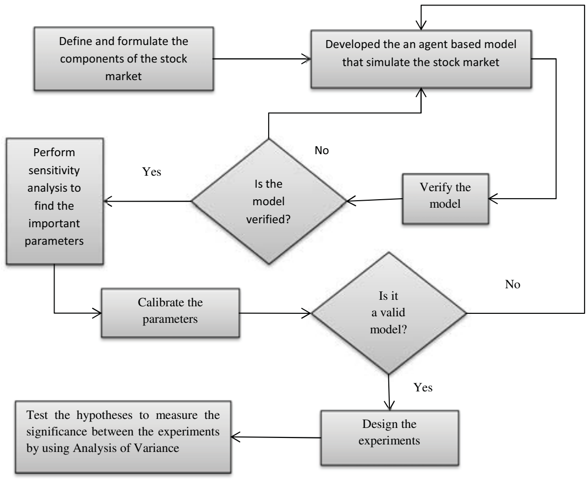

Fig. 7 Flow chart of the research

IV. Conclusion

In this paper, a review of agents based models and artificial stock markets problem has been provided. The review includes a description of the components of the artificial stock and the used methodologies to design these components. The designing of agents involves taking care their preferences, behavior, intelligence level, interaction and decision process. The assets in the artificial stock markets are divided into risky asset (stock) and risk free asset (bond). However, few models utilized a portfolio of risky assets. The asset prices occur as an aggregation of bids and offers of the agents. Four methods for determining asset prices are presented that are price adjustment method, temporary equilibrium price, simulation of order book and random bumping. The agents in the ASM literature learn through reinforcement or cognitive learning methods. The cognitive learning method implicates genetic algorithm, genetic programming and artificial neural network. The calibration of the models involves determining the most important parameters and then solve for the optimal values of these parameters by minimizing the error function of the model to the real market assets.

The field of ASM is growing rapidly. Many questions are waiting for answers. However, Agent based simulating seems to adequate to approach the problem. In this paper, we proposed our profound questions and our initial hypotheses where we would like to see the effect of the model parameters on the model output. In addition, we would like to examine the effect of the population size, amount of initial wealth and the interaction between agents on the agent's wealth at the end of the simulation. Also, we would like to observe the how these changes would affect the stylized facts of asset prices.

The objective of our study would be achieved by defining and formulating the agent based model and simulate the model. Once the developed model is simulated we would measure the sensitivity of the parameters using fractional factorial design. After that, the selected parameters would be calibrated, and the experiments would be designed to test the stated hypotheses.

ACKNOWLEDGMENT

The authors would like to thank Dr.Ali Mostashari for his help and comments.

REFERENCES

[1] A. Smith, "The wealth of nations (1776)," *New York: Modern Library*, vol. 740, 1937.
[2] B. G. Malkiel and E. F. Fama, "Efficient capital markets: A review of theory and empirical work*," *The journal of Finance*, vol. 25, no. 2, pp. 383–417, 1970.
[3] P. A. Samuelson, "Proof that properly anticipated prices fluctuate randomly," *Industrial management review*, vol. 6, no. 2, pp. 41–49, 1965.
[4] S. F. LeRoy, "Efficient capital markets and martingales," *Journal of Economic Literature*, pp. 1583–1621, 1989.
[5] M. Allais, "Le comportement de l'homme rationnel devant le risque: critique des postulats et axiomes de l'école américaine," *Econometrica: Journal of the Econometric Society*, pp. 503–546, 1953.
[6] K. Khashanah, "Systems taxonomy," in *4th International Engineering Systems Symposium*, 2014.
[7] H. A. Simon, "A behavioral model of rational choice," *The quarterly journal of economics*, vol. 69, no. 1, pp. 99–118, 1955.
[8] H. A. Simon, "From substantive to procedural rationality," *25 Years of Economic Theory*, pp. 65–86, 1976.
[9] H. A. Simon, "Rational choice and the structure of the environment." *Psychological review*, vol. 63, no. 2, p. 129, 1956.
[10] H. A. Simon, "Theories of bounded rationality," *Decision and organization*, vol. 1, pp. 161–176, 1972.
[11] H. A. Simon, "Models of bounded rationality: Empirically grounded economic reason," 1982.
[12] D. Kahneman and A. Tversky, "Prospect theory: An analysis of decision under risk," *Econometrica: Journal of the Econometric Society*, pp. 263–291, 1979.
[13] K. Dehnad, "Behavioral finance and technical analysis," *One company has the precision and focus to help you redefine value in a competitive market.*, p. 107, 2011.
[14] N. Barberis and R. Thaler, "A survey of behavioral finance," *Handbook of the Economics of Finance*, vol. 1, pp. 1053–1128, 2003.
[15] M. M. Pompian, "Behavioral finance and wealth management," *How to build optimalportfoliosforprivate clients*, 2006.
[16] J. D. Farmer and A. W. Lo, "Frontiers of finance: Evolution and efficient markets," *Proceedings of the National Academy of Sciences*, vol. 96, no. 18, pp. 9991–9992, 1999.
[17] A. W. Lo, "The adaptive markets hypothesis," *The Journal of Portfolio Management*, vol. 30, no. 5, pp. 15–29, 2004.
[18] R. J. Shiller, "From efficient markets theory to behavioral finance," *Journal of economic perspectives*, pp. 83–104, 2003.
[19] A. W. Lo, "Reconciling efficient markets with behavioral finance: the adaptive markets hypothesis," *Journal of Investment Consulting*, vol. 7, no. 2, pp. 21–44, 2005.
[20] M. Mitchell, *Complexity: A guided tour*. Oxford University Press, 2009.
[21] J. H. Miller and S. E. Page, *Complex Adaptive Systems: An Introduction to Computational Models of Social Life: An Introduction to Computational Models of Social Life*. Princeton University Press, 2009.
[22] B. LeBaron, "A builders guide to agent-based financial markets," *Quantitative Finance*, vol. 1, no. 2, pp. 254–261, 2001.
[23] B. LeBaron, "Agent-based computational finance," *Handbook of computational economics*, vol. 2, pp. 1187–1233, 2006.
[24] R. Marks, "Market design using agent-based models," *Handbook of computational economics*, vol. 2, pp. 1339–1380, 2006.
[25] V. Darley and A. V. Outkin, *Nasdaq Market Simulation: Insights on a Major Market from the Science of Complex Adaptive Systems*. World Scientific Publishing Co., Inc., 2007.
[26] S. Martinez-Jaramillo, "Artificial financial markets: an agent based approach to reproduce stylized facts and to study the red queen effect," Ph.D. dissertation, University of Essex, 2007.
[27] C. H. Hommes, "Financial markets as nonlinear adaptive evolutionary systems," 2001.
[28] M. Lovric, *Behavioral Finance and Agent-Based Artificial Markets*. Erasmus Research Institute of Management (ERIM), 2011, no. EPS-2011-229-F&A.
[29] H. Levy, M. Levy, and S. Solomon, *Microscopic simulation of financial markets: from investor behavior to market phenomena*. Academic Press, 2000.
[30] N. J. Vriend, "Ace models of endogenous interactions," *Handbook of computational economics*, vol. 2, pp. 1047–1079, 2006.
[31] T. C. Schelling, "Dynamic models of segregation," *Journal of mathematical sociology*, vol. 1, no. 2, pp. 143–186, 1971.
[32] J. M. Epstein, *Growing artificial societies: social science from the bottom up*. Brookings Institution Press, 1996.
[33] W. B. Arthur, "Inductive reasoning and bounded rationality," *American Economic Review*, vol. 84, no. 2, pp. 406–411, 1994.
[34] N. J. Vriend, "Self-organization of markets: An example of a computational approach," *Computational Economics*, vol. 8, no. 3, pp. 205–231, 1995.
[35] A. P. Kirman and N. J. Vriend, "Evolving market structure: An ace model of price dispersion and loyalty," *Journal of Economic Dynamics and Control*, vol. 25, no. 3, pp. 459–502, 2001.
[36] M. O. Jackson, *Social and economic networks*. Princeton University Press, 2010.
[37] D. Easley and J. Kleinberg, "Networks, crowds, and markets," *Cambridge Univ Press*, vol. 6, no. 1, pp. 6–1, 2010.
[38] A. Wilhite, "Economic activity on fixed networks," *Handbook of computational economics*, vol. 2, pp. 1013–1045, 2006.
[39] S.-H. Chen and Y.-C. Huang, "Simulating the evolution of portfolio behavior in a multiple-asset agent-based artificial stock market," in *9th International Conference on Computing in Economics and Finance, University of Washington, Seattle, USA*, 2003.
[40] D. K. Gode and S. Sunder, "Allocative efficiency of markets with zero-intelligence traders: Market as a partial substitute for individual rationality," *Journal of political economy*, vol. 101, no. 1, p. 119, 1993.
[41] N. T. Chan and C. Shelton, "An electronic market-maker," 2001.
[42] J. Yang, "The efficiency of an artificial double auction stock market with neural learning agents," in *Evolutionary Computation in Economics and Finance*. Springer, 2002, pp. 85–105.
[43] B. LeBaron, W. B. Arthur, and R. Palmer, "Time series properties of an artificial stock market," *Journal of Economic Dynamics and control*, vol. 23, no. 9, pp. 1487–1516, 1999.
[44] S.-H. Chen and C.-H. Yeh, "Evolving traders and the business school with genetic programming: A new architecture of the agent-based artificial stock market," *Journal of Economic Dynamics and Control*, vol. 25, no. 3, pp. 363–393, 2001.
[45] T. Brenner, "Agent learning representation: advice on modelling economic learning," *Handbook of computational economics*, vol. 2, pp. 895–947, 2006.
[46] D. E. Goldberg and J. H. Holland, "Genetic algorithms and machine learning," *Machine learning*, vol. 3, no. 2, pp. 95–99, 1988.
[47] Z. Michalewicz, *Genetic algorithms+ data structures= evolution programs*. springer, 1996.
[48] M. Mitchell, *An introduction to genetic algorithms*. MIT press, 1998.
[49] W.-Y. Lin, W.-Y. Lee, and T.-P. Hong, "Adapting crossover and mutation rates in genetic algorithms," *J. Inf. Sci. Eng.*, vol. 19, no. 5, pp. 889–903, 2003.
[50] W. Arthur, J. Holland, B. LeBaron, and R. Palmer, "P. tayler, 1997,asset pricing under endogenous expectations in an artificial stock market," *The Economy as an Evolving Complex System II, Reading, Ma: Addison-Wesley*, pp. 15–44.
[51] J. R. Koza, "Genetic programming as a means for programming computers by natural selection," *Statistics and Computing*, vol. 4, no. 2, pp. 87–112, 1994.
[52] R. Poli, W. B. Langdon, N. F. McPhee, and J. R. Koza, "A field guide to genetic programming," 2008.
[53] M. Smith, *Neural networks for statistical modeling*. Thomson Learning, 1993.
[54] J. A. Anderson, *An introduction to neural networks*. MIT press, 1995.

[55] R. Rojas, *Neutral Networks: A Systematic Introduction*. Springer, 1996.
[56] K. Gurney, *An introduction to neural networks*. CRC press, 2003.
[57] P. D. McNelis, *Neural networks in finance: gaining predictive edge in the market*. Elsevier Acad. Press, 2005.
[58] D. T. Larose, *Discovering knowledge in data: an introduction to data mining*. John Wiley & Sons, 2005.
[59] R. O. Duda, P. E. Hart, and D. G. Stork, *Pattern classification*. John Wiley & Sons, 2012.
[60] A. G. Barto, "Reinforcement learning: An introduction," 1998.
[61] S. F. Railsback and V. Grimm, *Agent-based and individual-based modeling: a practical introduction*. Princeton University Press, 2011.
[62] F. Klügl, "A validation methodology for agent-based simulations," in *Proceedings of the 2008 ACM symposium on Applied computing*. ACM, 2008, pp. 39–43.
[63] S.-H. Chen, C.-L. Chang, and Y.-R. Du, "Agent-based economic models and econometrics," *The Knowledge Engineering Review*, vol. 27, no. 02, pp. 187–219, 2012.
[64] R. Cont, "Empirical properties of asset returns: stylized facts and statistical issues," 2001.
[65] R. S. Tsay, *Analysis of financial time series*. John Wiley & Sons, 2005, vol. 543.
[66] R. J. Shiller, "Market volatility," 1992.
[67] T. C. Schelling, "Models of segregation," *The American Economic Review*, pp. 488–493, 1969.
[68] T. C. Schelling, *Micromotives and macrobehavior*. WW Norton & Company, 2006.
[69] J. A. Frankel and K. A. Froot, "Explaining the demand for dollars: International rates of return and the expectations of chartists and fundamentalists," *Department of Economics, UCB*, 1986.
[70] G.-r. Kim and H. M. Markowitz, "Investment rules, margin, and market volatility," *The Journal of Portfolio Management*, vol. 16, no. 1, pp. 45–52, 1989.
[71] J. Arifovic, "The behavior of the exchange rate in the genetic algorithm and experimental economies," *Journal of Political Economy*, pp. 510–541, 1996.
[72] M. Lettau, "Explaining the facts with adaptive agents: The case of mutual fund flows," *Journal of Economic Dynamics and Control*, vol. 21, no. 7, pp. 1117–1147, 1997.
[73] B. R. Routledge, "Genetic algorithm learning to choose and use information," *Macroeconomic dynamics*, vol. 5, no. 02, pp. 303–325, 2001.
[74] R. Palmer, W. Brian Arthur, J. H. Holland, B. LeBaron, and P. Tayler, "Artificial economic life: a simple model of a stockmarket," *Physica D: Nonlinear Phenomena*, vol. 75, no. 1, pp. 264–274, 1994.
[75] S. Joshi, J. Parker, and M. A. Bedau, "Technical trading creates a prisoners dilemma: Results from an agent-based model," *Computational Finance*, vol. 99, pp. 465–479, 1999.
[76] N. S. Tay and S. C. Linn, "Fuzzy inductive reasoning, expectation formation and the behavior of security prices," *Journal of Economic Dynamics and Control*, vol. 25, no. 3, pp. 321–361, 2001.
[77] P. Bak, M. Paczuski, and M. Shubik, "Price variations in a stock market with many agents," *Physica A: Statistical Mechanics and its Applications*, vol. 246, no. 3, pp. 430–453, 1997.
[78] N. T. Chan, B. LeBaron, A. W. Lo, T. Poggio, A. W. L. Yy, and T. P. Zz, "Agent-based models of financial markets: A comparison with experimental markets," 1999.
[79] B. LeBaron, "Evolution and time horizons in an agent-based stock market," *Macroeconomic Dynamics*, vol. 5, no. 02, pp. 225–254, 2001.
[80] M. Raberto, S. Cincotti, S. M. Focardi, and M. Marchesi, "Agent-based simulation of a financial market," *Physica A: Statistical Mechanics and its Applications*, vol. 299, no. 1, pp. 319–327, 2001.
[81] J. D. Farmer and S. Joshi, "The price dynamics of common trading strategies," *Journal of Economic Behavior & Organization*, vol. 49, no. 2, pp. 149–171, 2002.
[82] G. Iori, "A microsimulation of traders activity in the stock market: the role of heterogeneity, agents interactions and trade frictions," *Journal of Economic Behavior & Organization*, vol. 49, no. 2, pp. 269–285, 2002.
[83] L. Neuberg and K. Bertels, "Heterogeneous trading agents," *Complexity*, vol. 8, no. 5, pp. 28–35, 2003.
[84] H. Takahashi and T. Terano, "Agent-based approach to investors' behavior and asset price fluctuation in financial markets," *Journal of artificial societies and social simulation*, vol. 6, no. 3, 2003.
[85] T. Kaizoji, "Speculative bubbles and fat-tail phenomena in a heterogeneous agent model," in *International Symposia in Economic Theory and Econometrics*, vol. 14. Emerald Group Publishing Limited, 2004, pp. 259–275.
[86] S.-H. Chen and C.-C. Liao, "Agent-based computational modeling of the stock price–volume relation," *Information Sciences*, vol. 170, no. 1, pp. 75–100, 2005.
[87] S. Cincotti, L. Ponta, and M. Raberto, "A multi-assets artificial stock market with zero-intelligence traders," *WEHIA 2005 (13-15 June 2005)*, 2005.
[88] J. Derveeuw, "Market dynamics and agents behaviors: a computational approach," in *Artificial Economics*. Springer, 2006, pp. 15–26.
[89] T. Shimokawa, K. Suzuki, and T. Misawa, "An agent-based approach to financial stylized facts," *Physica A: Statistical Mechanics and its Applications*, vol. 379, no. 1, pp. 207–225, 2007.
[90] S.-H. Chen and Y.-C. Huang, "Risk preference, forecasting accuracy and survival dynamics: Simulations based on a multi-asset agent-based artificial stock market," *Journal of Economic Behavior & Organization*, vol. 67, no. 3, pp. 702–717, 2008.
[91] S. Martinez-Jaramillo and E. P. Tsang, "An heterogeneous, endogenous and coevolutionary gp-based financial market," *IEEE Transactions on Evolutionary Computation*, vol. 13, no. 1, p. 33, 2009.
[92] B. LeBaron, "A real minsky moment in an artificial stock market," 2012.
[93] P. Kumar, R. Rupika, S. Vennilla, K. Abinaya, and V. Mohandas, "Implementing an agent based artificial stock market model in jade-an illustration." *International Journal of Engineering & Technology (0975-4024)*, vol. 5, no. 3, 2013.
[94] Y.-F. Liu, C. Xu, W. Zhang, and J. V. Andersen, "Impact of information cost and switching of trading strategies in an artificial stock market," *arXiv preprint arXiv:1311.4274*, 2013.
[95] D. Hirshleifer and S. Hong Teoh, "Herd behaviour and cascading in capital markets: A review and synthesis," *European Financial Management*, vol. 9, no. 1, pp. 25–66, 2003.
[96] J. Ke and Y. Chen, "Modeling and simulation of the artificial stock market trading system." *Applied Mathematics & Information Sciences*, vol. 7, no. 4, 2013.
[97] E. de Jong, W. F. Verschoor, and R. C. Zwinkels, "Heterogeneity of agents and exchange rate dynamics: Evidence from the ems," *Journal of International Money and Finance*, vol. 29, no. 8, pp. 1652–1669, 2010.
[98] H. P. Boswijk, C. H. Hommes, and S. Manzan, "Behavioral heterogeneity in stock prices," *Journal of Economic Dynamics and Control*, vol. 31, no. 6, pp. 1938–1970, 2007.
[99] H. Amilon, "Estimation of an adaptive stock market model with heterogeneous agents," *Journal of Empirical Finance*, vol. 15, no. 2, pp. 342–362, 2008.
[100] S. Chen, Y. Huang, and J. Wang, "Bounded rationality and the elasticity puzzle: An analysis of agent-based computational consumption capital asset pricing models," *Routledge, New York*, 2009.
[101] B. LeBaron, "Heterogeneous gain learning and long swings in asset prices," 2010.
[102] S. Sivanandam and S. Deepa, "Introduction to genetic algorithms. 2008."
[103] J. R. Koza, *Genetic programming: on the programming of computers by means of natural selection*. MIT press, 1992, vol. 1.
[104] T. Lux, "The socio-economic dynamics of speculative markets: interacting agents, chaos, and the fat tails of return distributions," *Journal of Economic Behavior & Organization*, vol. 33, no. 2, pp. 143–165, 1998.
[105] B. LeBaron and R. Yamamoto, "Long-memory in an order-driven market," *Physica A: Statistical Mechanics and its Applications*, vol. 383, no. 1, pp. 85–89, 2007.
[106] D. Challet and T. Galla, "Price return autocorrelation and predictability in agent-based models of financial markets," *Quantitative Finance*, vol. 5, no. 6, pp. 569–576, 2005.
[107] F. F. Ferreira, V. M. de Oliveira, A. F. Crepaldi, and P. R. Campos, "Agent-based model with heterogeneous fundamental prices," *Physica A: Statistical Mechanics and its Applications*, vol. 357, no. 3, pp. 534–542, 2005.
[108] V. Manahov and R. Hudson, "Herd behaviour experimental testing in laboratory artificial stock market settings. behavioural foundations of stylised facts of financial returns," *Physica A: Statistical Mechanics and its Applications*, vol. 392, no. 19, pp. 4351–4372, 2013.
[109] F. Ghoulmie, R. Cont, and J.-P. Nadal, "Heterogeneity and feedback in an agent-based market model," *Journal of Physics: condensed matter*, vol. 17, no. 14, p. S1259, 2005.

[110] H. Li and J. Rosser, “Market dynamics and stock price volatility,” *The European Physical Journal B-Condensed Matter and Complex Systems*, vol. 39, no. 3, pp. 409–413, 2004.