Cognitive Relaying in Interference Limited Spectrum Sharing Environment: Outage Probability and Outage Capacity

Md Fazlul Kader, Soo Young Shin

Abstract—In this paper, we consider a cognitive relay network (CRN) in which the primary receiver (PR) is protected by peak transmit power $\bar{P}_{ST}$ and/or peak interference power Q constraints. In addition, the interference effect from the primary transmitter (PT) is considered to show its impact on the performance of the CRN. We investigate the outage probability (OP) and outage capacity (OC) of the CRN by deriving closed-form expressions over Rayleigh fading channel. Results show that both the OP and OC improve by increasing the cooperative relay nodes as well as when the PT is far away from the SR.

Keywords—Cognitive relay, outage, interference limited, decode-and-forward (DF).

I. Introduction

DUE to the rapid growth of wireless applications, the demand of radio spectrum has been increased dramatically [1]. To fulfill this demand, cognitive radio (CR) [2] is an exciting and emerging technology to improve both spectrum efficiency and utilization. CR takes the advantage of unoccupied or partially-occupied frequency bands to improve the spectrum utilization. In CR network, secondary users (SUs) or CR users may coexist with the primary users (PUs) either on non-interference basis (in white spaces) which is called opportunistic spectrum access [3] or interference tolerant basis (in gray spaces) which is called underlay spectrum access [4].

In interference temperature limit systems, the SUs are allowed to use primary spectrum as long as the interference caused by the SU to the PR falls below some certain threshold. In [4], the authors analysed the fundamental limits of non-cooperative CR systems under interference limited environment. Closed-form expressions of the capacities as well as capacities along with the power allocation problems of such systems are investigated in [5] and [6] respectively.

In a telecommunication system, outage can be defined as a service condition in which a user is completely deprived of service by the system or a service condition below a threshold of acceptable performance. Outage probability (OP) and outage capacity (OC) are two important metrics for measuring performance in wireless communications over fading channels. The term OP can be defined as the probability that an outage will occur within a specified time period. On the contrary, OC can be defined as the data rate that can be achieved if outage is allowed to occur with probability ε.

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In cooperative communication system, each wireless user may act as a cooperative relay for another user as well as able to transmit its own data. Recently, cooperative relaying technique has attracted enormous interest to the research community to increase performance of the interference temperature limited systems. In [7]–[10], the authors analysed the cooperative relaying for such systems. OP of the cognitive relay network (CRN) considering peak interference power constraint is analysed in [7]. They have proposed two protocols such as distributed space time code (DSTC) and relay selection methods for cooperative relaying. The evaluation of OP based on a suitable relay selection criterion in underlay CR network is analysed in [8]. In [9], the authors investigated the OP of the CR wireless networks where secondary receivers are not suffered any interference from the PT. Closed-form expression of the capacity of CRN is derived in [10], where PR is protected by interference power constraint. In [11] and [12], the authors investigated the OP and OC of the primary network with cooperative relaying as well as OC of the secondary network without cooperative relaying respectively under spectrum sharing environments.

However, in existing works, the authors did not analyse how the interference effect from the PT to the cognitive relay (R) and secondary receiver (SR) has an impact on the performance of the CRN. So, the main focus of this paper is to analyse the OP and OC of the CRN in multi-relay scenario by considering two different cases such as: (1) Peak transmit power constraint $\bar{P}_{ST}$ at ST and showing how the position of the PT with respect to R and SR has an impact on the performance of CRN as well as (2) showing the impact of the predefined interference threshold Q to the PR on the performance of the CRN considering fixed interference from the PT to R and SR.

The rest of this paper is organized as follows. In Section II, we introduce the cooperative relay based system and channel models of a CRN. In Section III, closed-form expressions of the OP and OC are derived. Theoretical results verified with simulation results are presented in Section IV, and finally we conclude this paper in Section V.

II. System and Channel Models

We consider an underlay spectrum sharing environment, where a secondary transmitter (ST) - receiver (SR) pair co-exists with a primary transmitter (PT) - primary receiver (PR) pair. It is assumed that the direct link between the ST and SR is not strong enough due to fading or shadowing. So, a
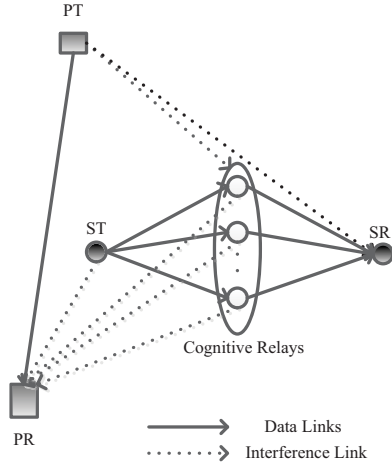
Fig. 1. System Model under consideration.

group of L relays assist the secondary transmission. The relays may be dedicated relays or idle ST. The proposed system model is shown in Fig. 1. Relays adopt the same transmit power as the ST to forward the secondary information. One of the relays satisfying secondary target rate I_{ST} will be selected as a best decode-and-forward (DF) relay. Both the primary and secondary networks are half duplex radio and single antenna systems. Assume that PRs are protected by peak transmit power $\bar{P}_{ST}$ and/or peak interference power Q constraints. Also assume that R_i, $i \in \{1, 2, ..., L\}$ and SRs are suffered by interference from the PT.

Consider that the channels over all the links are subject to Rayleigh flat fading plus additive white Gaussian noise. Let, link gain $G_{p,q}$ between nodes p and q is an exponentially distributed random variable with mean value $\lambda_{p,q}$. To show the effect of the position of the PT with respect to the SR, we consider $\lambda_{p,q} = (\frac{1}{d_{p,q}})^n$ where $d_{p,q}$ denotes the distance between any nodes p and q as well as n is the path loss exponent. Assume, all the signals follow independent fading path. The transmit power of ST, R_i and PT is denoted as P_{ST}, P_{R_i}, and P_{PT} respectively. Moreover, we consider signal-to-interference ratio (SIR) over all the links and relays adopt the same transmit power as the ST.

In interference limited environment, noise power is negligible [13]. So the achievable rate of the links ST⟶R_i and R_i⟶SR can be formulated as

$$I_{ST-R_i} = \frac{1}{2}\log_2(1 + SIR_{ST-R_i}) \quad (1)$$

$$I_{R_i-SR} = \frac{1}{2}\log_2(1 + SIR_{R_i-SR}) \quad (2)$$

The scaling factor $\frac{1}{2}$ in (1) and (2) is due to the fact that the overall transmission is divided into two phases. The SIR of the links ST⟶R_i and R_i⟶SR can be approximated by considering two cases as follows.

A. Case 1 (C1)

Here, we investigate how the position of the PT with respect to the R and SR has an impact on the performance of the CRN by considering $\bar{P}_{ST}$ at ST. So, the SIR of phase-1 and phase-2 in C1 can be represented as follows

• Phase-1: $0 \leq P_{ST} \leq \bar{P}_{ST}$

$$SIR_{ST-R_i} \cong \frac{G_{ST-R_i}P_{ST}}{G_{PT-R_i}P_{PT}} \quad (3)$$

• Phase-2: $0 \leq P_{R_i} \leq \bar{P}_{ST}$

$$SIR_{R_i-SR} \cong \frac{G_{R_i-SR}P_{r_i}}{G_{PT-SR}P_{PT}} \quad (4)$$

B. Case 2 (C2)

Here, we investigate how the Q has an impact on the performance of the CRN considering fixed interference from the PT to R and SR. So, the SIR of phase-1 and phase-2 in C2 can be represented as follows

• Phase-1: $G_{ST-PR}P_{ST} \leq Q$

$$SIR_{ST-R_i} \cong \frac{G_{ST-R_i}Q}{G_{ST-PR}P_{INT}} \quad (5)$$

• Phase-2: $G_{R_i-PR}P_{r_i} \leq Q$

$$SIR_{R_i-SR} \cong \frac{G_{R_i-SR}Q}{G_{R_i-PR}P_{INT}} \quad (6)$$

where P_{INT} denotes interference caused by the PT at R and SR.

C. Relay Selection

We consider a similar relay selection procedure as in [14]. Let, the set of relays S that can be considered for relay selection as

$$S = \{i|i \in L, (min\{I_{ST-R_i}, I_{R_i-SR}\} > I_{ST})\} \quad (7)$$

Therefore, the best relay can be selected as follows

$$R_{best} = \arg\max_{i \in S}(min\{I_{ST-R_i}, I_{R_i-SR}\} > I_{ST}) \quad (8)$$

After the selection of the best relay, the ST transmits the message to the best relay in the first time slot. If the best relay is able to decode the message successfully then it will forward this message to the SR in the second time slot. Otherwise, the best relay remains silent and the system declares an outage.

III. PERFORMANCE EVALUATION

In this Section, closed-form expressions of the OP and OC of CRN are derived. The secondary system is in outage if none of the relays satisfy I_{ST} i.e. $|S| = 0$. So, the OP can be expressed as

$$\begin{aligned} P_{OUT}^S &= P_r\{|S| = 0\} \\ &= \prod_{i=1}^{L} P_r\{min(I_{ST-R_i}, I_{R_i-SR}) < I_{ST}\} \\ &= \prod_{i=1}^{N}[1 - (1 - Y)(1 - Z)] \end{aligned} \quad (9)$$

where $Y = P_r(I_{ST-R_i} < I_{ST})$ and $Z = P_r(I_{R_i-SR} < I_{ST})$

Let, G_0 and G_1 are two exponential distributed random variables with means λ_0 and λ_1 respectively. Then the probability density function (PDF) and cumulative density function(CDF) of $X = \frac{G_0}{G_1}$ can be expressed as [15]

$$f_X(x) = \frac{\lambda_0\lambda_1}{(\lambda_0 + \lambda_1 x)^2}, x > 0 \qquad (10)$$

$$\begin{aligned} F_X(\rho_{i,j}) &= \int_0^{\rho_{i,j}} \frac{\lambda_0\lambda_1}{(\lambda_0 + \lambda_1 x)^2} dx \\ &= \frac{\rho_{i,j}\lambda_1}{\lambda_0 + \rho_{i,j}\lambda_1} \end{aligned} \qquad (11)$$

Similarly, all the link gains assumed in this paper are exponentially distributed random variables with their corresponding mean values which are defined in Sect. II. So, the OP and the OC of the corresponding links in case 1 and case 2 are given as follows.

A. Case 1 (C1)

According to (11), the OP of the links ST⟶R_i and R_i⟶SR in C1 can be written as

$$\begin{aligned} P_r(I_{ST-R_i} < I_{ST}) &= P_r(\frac{G_{ST-R_i}}{G_{PT-R_i}} < \rho_{ST-R_i}) \\ &= \frac{\rho_{ST-R_i}\lambda_{PT-R_i}}{\lambda_{ST-R_i} + \rho_{ST-R_i}\lambda_{PT-R_i}} \end{aligned} \qquad (12)$$

$$\begin{aligned} P_r(I_{R_i-SR} < I_{ST}) &= P_r(\frac{G_{R_i-SR}}{G_{PT-SR}} < \rho_{R_i-SR}) \\ &= \frac{\rho_{R_i-SR}\lambda_{PT-SR}}{\lambda_{R_i-SR} + \rho_{R_i-SR}\lambda_{PT-SR}} \end{aligned} \qquad (13)$$

where $\rho_{ST-R_i} = (2^{2I_{ST}} - 1) \times (1/\frac{P_{ST}}{P_{PT}})$ and $\rho_{R_i-SR} = (2^{2I_{ST}} - 1) \times (1/\frac{P_{R_i}}{P_{PT}})$. Now, substituting (12) and (13) into (9), we can get the OP of the CRN of C1.

Outage capacity can be defined as the data rate that can be achieved if outage is allowed to occur with probability ε. Therefore, the OC of C1 associated with given ε is written as [11]

$$C_{OUT}^{C1} = \log_2(1 + \rho_{ST-R_i-SR}^{C1}\frac{P_{ST}}{P_{PT}})(1-\varepsilon) \qquad (14)$$

where the value of $\rho_{ST-R_i-SR}^{C1}$ is the solution of (9). In [11], the authors derived the closed-form expression of the OC for the primary cooperative network. Therefore, the value of $\rho_{ST-R_i-SR}^{C1}$ can be written as [[11], (22)] to calculate the OC in C1 of the CRN considered in this work.

$$\rho_{ST-R_i-SR}^{C1} = \frac{-B + \sqrt{B^2 - 4AC}}{2A} \qquad (15)$$

where
$A = \lambda_{PT-R_i}\lambda_{PT-SR}$
$B = \lambda_{ST-R_i}\lambda_{PT-SR} + \lambda_{PT-R_i}\lambda_{R_i-SR}$
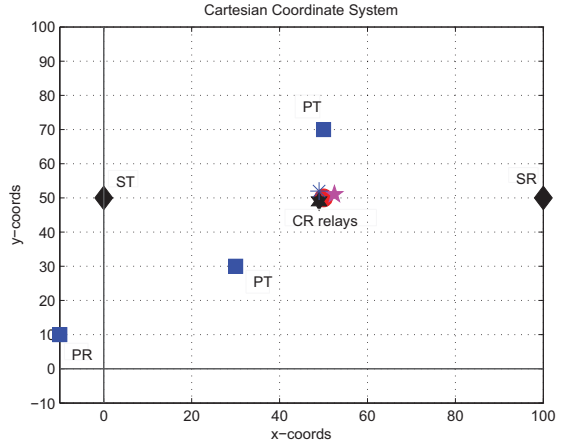$C = -\frac{\lambda_{ST-R_i}\lambda_{R_i-SR}\sqrt[L]{\varepsilon}}{1 - \sqrt[L]{\varepsilon}}$

Fig. 2. An instance of our simulation scenario. Two locations (30, 30) and (50, 70) of the PT are considered. Relays are located at (50, 50).

B. Case 2 (C2)

Similarly of C1, the OP of the links ST⟶R_i and R_i⟶SR in C2 can be written as

$$\begin{aligned} P_r(I_{ST-R_i} < I_{ST}) &= P_r(\frac{G_{ST-R_i}}{G_{ST-PR}} < \rho_{ST-R_i}) \\ &= \frac{\rho_{ST-R_i}\lambda_{ST-PR}}{\lambda_{ST-R_i} + \rho_{ST-R_i}\lambda_{ST-PR}} \end{aligned} \qquad (16)$$

$$\begin{aligned} P_r(I_{R_i-SR} < I_{ST}) &= P_r(\frac{G_{R_i-SR}}{G_{R_i-PR}} < \rho_{R_i-SR}) \\ &= \frac{\rho_{R_i-SR}\lambda_{R_i-PR}}{\lambda_{R_i-SR} + \rho_{R_i-SR}\lambda_{R_i-PR}} \end{aligned} \qquad (17)$$

where $\rho_{ST-R_i} = (2^{2I_{ST}} - 1) \times (1/\frac{Q}{P_{INT}})$ and $\rho_{R_i-SR} = (2^{2I_{ST}} - 1) \times (1/\frac{Q}{P_{INT}})$. Now, substituting (16) and (17) into (9), we can get the OP of the CRN of C2.

Therefore, the OC of C2 associated with given ε is written as [11]

$$C_{OUT}^{C2} = \log_2(1 + \rho_{ST-R_i-SR}^{C2}\frac{Q}{P_{INT}})(1-\varepsilon) \qquad (18)$$

where the value of $\rho_{ST-R_i-SR}^{C2}$ is the solution of (9). Therefore, the value of $\rho_{ST-R_i-SR}^{C2}$ can be written as (15) to calculate the OC in C2.

$$\rho_{ST-R_i-SR}^{C2} = \frac{-B + \sqrt{B^2 - 4AC}}{2A} \qquad (19)$$

where
$A = \lambda_{ST-PR}\lambda_{R_i-PR}$
$B = \lambda_{ST-R_i}\lambda_{R_i-PR} + \lambda_{ST-PR}\lambda_{R_i-SR}$
$C = -\frac{\lambda_{ST-R_i}\lambda_{R_i-SR}\sqrt[L]{\varepsilon}}{1 - \sqrt[L]{\varepsilon}}$

IV. RESULTS AND DISCUSSIONS

In this Section, we have presented theoretical and Monte Carlo simulation results to evaluate the performance of the CRN over Rayleigh fading channel. For visualizing our result, we have considered a two dimensional space in Cartesian
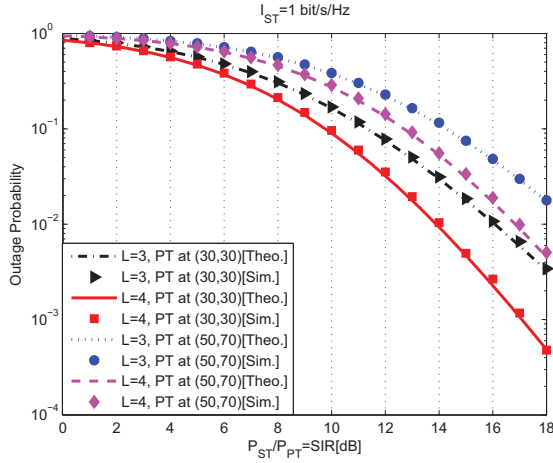
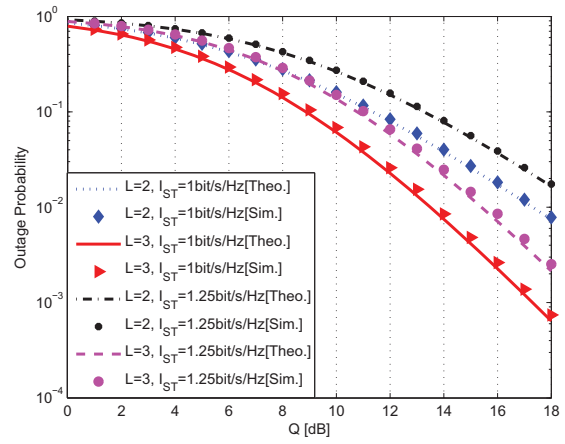Fig. 3.   Outage probability of C1.

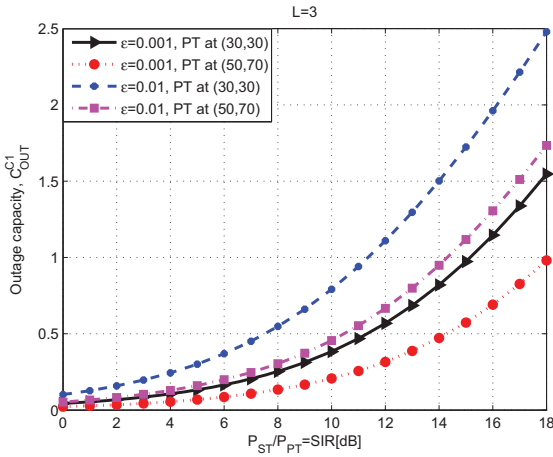Fig. 5.   Outage probability of C2.

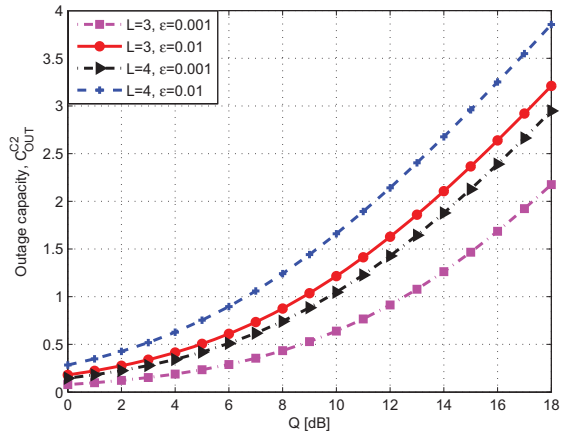Fig. 4.   Outage capacity of C1.

Fig. 6.   Outage capacity of C2.

coordinate system as a simulation scenario as shown in Fig. 2. Assume that the position of the ST, SR and PR are located at (0, 50), (100, 50) and (-10,10) respectively. Relays are located at (50, 50). Although, we have assumed some fixed locations of PT, PR, ST, R and SR but our scheme works for any location of PT, PR, ST, R and SR as well as any distance among them. As in [16], to reduce the numbers of model parameters, the position of all the relays are considered at approximately the same distance between ST and SR. Moreover, we consider variable location ((30, 30), (50, 70)) of PT to show that the OP and OC of the CRN are affected by the position of the PT with respect to the SR. Furthermore, slow fading is considered in this paper. So, to get the effect of all relays, fading co-efficient of each relay is independent to each other. Assume, path loss exponent $n=2$ and meter is the unit of distance here.

In Fig. 3 and 4, we depict the OP and OC of the CRN in C1 respectively as a function of SIR. Assume $SIR = P_{ST}/P_{PT} = P_{R_i}/P_{PT}$ and SIR varies from 0 to 18 dB. Two different location of the PT at (30, 30) and (50, 70) respectively are considered to show the impact of the interference from the

PT on the performance of the CRN. Fig. 3 depicts the OP when $I_{ST}=1$ bit/s/Hz as well as $L=3$ and $L=4$ whereas Fig. 4 depicts the OC when $L=3$ as well as $\varepsilon=0.01$ and $\varepsilon=0.001$. It is clear from Fig. 3 that OP decreases with increasing SIR as well as L as expected. Increasing L means more number of nodes participate in the cooperation process. As a result, the number of independent paths between the ST and SR increases showing greater diversity. It is also shown that both the OP and OC degrade as the PT becomes closer to the SR. When the PT becomes nearby to the SR, it causes more interference to the SR. Thus, resulting performance degradation of the CRN. Moreover, it is clear from Fig. 4 that OC improves with increasing ε.

In Fig. 5 and 6, we depict the OP and OC of the CRN in C2 respectively as a function of Q, where Q varies from 0 to 18 dB. We let, $L=2$, 3 and $I_{ST}=1$, 1.25 bits/s/Hz in Fig. 5 whereas $L=3$, 4 and $\varepsilon=0.01$, 0.001 in Fig. 6 respectively. It is observed that both the OP and OC improve as the Q as well as L increase. Increasing Q represents that the PR can tolerate more interference from the ST and R. Thus, resulting

performance improvement of the CRN. It is also shown in Fig. 5 that OP increases with increasing I_{ST}. This means that, for the same set of parameters if the target rate increases, there is a high probability that the CRN will be in a deep fade or outage. Moreover, it is clear from Fig. 6 that OC improves with increasing ε.

V. Conclusion

In this paper, we have investigated the OP and OC of the CRN in interference limited environments over Rayleigh fading channel. Closed-form expressions of the OP and OC are derived. We found that performance of the CRN improves as the number of cooperative relay increases as well as when the PT is far away from the SR. It is also found that increasing ε and Q improve OP and OC. Moreover, with the same set of parameters, increasing I_{ST} degrades performance of the CRN. Some other issues such as end-to-end delay and bit error rate for the CRN will be investigated in future.

Acknowledgment

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