International Journal of Electrical, Electronic and Communication Sciences
ISSN: 2517-9438
Vol:9, No:8, 2015

Cognitive Relaying in Interference Limited Spectrum Sharing Environment: Outage Probability and Outage Capacity

Md Fazlul Kader, Soo Young Shin

Abstract—In this paper, we consider a cognitive relay network (CRN) in which the primary receiver (PR) is protected by peak transmit power $\bar{P}_{ST}$ and/or peak interference power Q constraints. In addition, the interference effect from the primary transmitter (PT) is considered to show its impact on the performance of the CRN. We investigate the outage probability (OP) and outage capacity (OC) of the CRN by deriving closed-form expressions over Rayleigh fading channel. Results show that both the OP and OC improve by increasing the cooperative relay nodes as well as when the PT is far away from the SR.

Keywords—Cognitive relay, outage, interference limited, decode-and-forward (DF).

I. INTRODUCTION

DUE to the rapid growth of wireless applications, the demand of radio spectrum has been increased dramatically [1]. To fulfill this demand, cognitive radio (CR) [2] is an exciting and emerging technology to improve both spectrum efficiency and utilization. CR takes the advantage of unoccupied or partially-occupied frequency bands to improve the spectrum utilization. In CR network, secondary users (SUs) or CR users may coexist with the primary users (PUs) either on non-interference basis (in white spaces) which is called opportunistic spectrum access [3] or interference tolerant basis (in gray spaces) which is called underlay spectrum access [4].

In interference temperature limit systems, the SUs are allowed to use primary spectrum as long as the interference caused by the SU to the PR falls below some certain threshold. In [4], the authors analysed the fundamental limits of non-cooperative CR systems under interference limited environment. Closed-form expressions of the capacities as well as capacities along with the power allocation problems of such systems are investigated in [5] and [6] respectively.

In a telecommunication system, outage can be defined as a service condition in which a user is completely deprived of service by the system or a service condition below a threshold of acceptable performance. Outage probability (OP) and outage capacity (OC) are two important metrics for measuring performance in wireless communications over fading channels. The term OP can be defined as the probability that an outage will occur within a specified time period. On the contrary, OC can be defined as the data rate that can be achieved if outage is allowed to occur with probability ε.

In cooperative communication system, each wireless user may act as a cooperative relay for another user as well as able to transmit its own data. Recently, cooperative relaying technique has attracted enormous interest to the research community to increase performance of the interference temperature limited systems. In [7]–[10], the authors analysed the cooperative relaying for such systems. OP of the cognitive relay network (CRN) considering peak interference power constraint is analysed in [7]. They have proposed two protocols such as distributed space time code (DSTC) and relay selection methods for cooperative relaying. The evaluation of OP based on a suitable relay selection criterion in underlay CR network is analysed in [8]. In [9], the authors investigated the OP of the CR wireless networks where secondary receivers are not suffered any interference from the PT. Closed-form expression of the capacity of CRN is derived in [10], where PR is protected by interference power constraint. In [11] and [12], the authors investigated the OP and OC of the primary network with cooperative relaying as well as OC of the secondary network without cooperative relaying respectively under spectrum sharing environments.

However, in existing works, the authors did not analyse how the interference effect from the PT to the cognitive relay (R) and secondary receiver (SR) has an impact on the performance of the CRN. So, the main focus of this paper is to analyse the OP and OC of the CRN in multi-relay scenario by considering two different cases such as: (1) Peak transmit power constraint $\bar{P}_{ST}$ at ST and showing how the position of the PT with respect to R and SR has an impact on the performance of CRN as well as (2) showing the impact of the predefined interference threshold Q to the PR on the performance of the CRN considering fixed interference from the PT to R and SR.

The rest of this paper is organized as follows. In Section II, we introduce the cooperative relay based system and channel models of a CRN. In Section III, closed-form expressions of the OP and OC are derived. Theoretical results verified with simulation results are presented in Section IV, and finally we conclude this paper in Section V.

II. SYSTEM AND CHANNEL MODELS

We consider an underlay spectrum sharing environment, where a secondary transmitter (ST) - receiver (SR) pair co-exists with a primary transmitter (PT) - primary receiver (PR) pair. It is assumed that the direct link between the ST and SR is not strong enough due to fading or shadowing. So, a

Md Fazlul Kader and Soo Young Shin are with the Wireless and Emerging Network System Laboratory (WENS Lab.), Kumoh National Institute of Technology, Republic of Korea e-mail: (f.kader@kumoh.ac.kr, wdragon@kumoh.ac.kr).

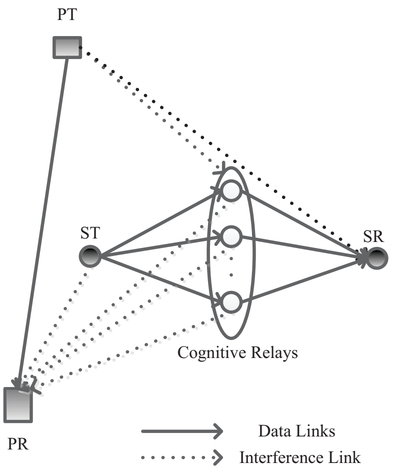

Fig. 1. System Model under consideration.

group of L relays assist the secondary transmission. The relays may be dedicated relays or idle ST. The proposed system model is shown in Fig. 1. Relays adopt the same transmit power as the ST to forward the secondary information. One of the relays satisfying secondary target rate I_{ST} will be selected as a best decode-and-forward (DF) relay. Both the primary and secondary networks are half duplex radio and single antenna systems. Assume that PRs are protected by peak transmit power $\bar{P}_{ST}$ and/or peak interference power Q constraints. Also assume that R$_i$, $i \in \{1,2,...,L\}$ and SRs are suffered by interference from the PT.

Consider that the channels over all the links are subject to Rayleigh flat fading plus additive white Gaussian noise. Let, link gain $G_{p,q}$ between nodes p and q is an exponentially distributed random variable with mean value $\lambda_{p,q}$. To show the effect of the position of the PT with respect to the SR, we consider $\lambda_{p,q} = (\frac{1}{d_{p,q}})^n$ where $d_{p,q}$ denotes the distance between any nodes p and q as well as n is the path loss exponent. Assume, all the signals follow independent fading path. The transmit power of ST, R$_i$ and PT is denoted as P_{ST}, P_{R_i}, and P_{PT} respectively. Moreover, we consider signal-to-interference ratio (SIR) over all the links and relays adopt the same transmit power as the ST.

In interference limited environment, noise power is negligible [13]. So the achievable rate of the links ST$\longrightarrow$R$_i$ and R$_i$ $\longrightarrow$SR can be formulated as

$$I_{ST-R_i} = \frac{1}{2}\log_2(1 + SIR_{ST-R_i}) \tag{1}$$

$$I_{R_i-SR} = \frac{1}{2}\log_2(1 + SIR_{R_i-SR}) \tag{2}$$

The scaling factor $\frac{1}{2}$ in (1) and (2) is due to the fact that the overall transmission is divided into two phases. The SIR of the links ST$\longrightarrow$R$_i$ and R$_i$ $\longrightarrow$SR can be approximated by considering two cases as follows.

A. Case 1 (C1)

Here, we investigate how the position of the PT with respect to the R and SR has an impact on the performance of the CRN by considering $\bar{P}_{ST}$ at ST. So, the SIR of phase-1 and phase-2 in C1 can be represented as follows

- Phase-1: $0 \leq P_{ST} \leq \bar{P}_{ST}$

$$SIR_{ST-R_i} \cong \frac{G_{ST-R_i}P_{ST}}{G_{PT-R_i}P_{PT}} \tag{3}$$

- Phase-2: $0 \leq P_{R_i} \leq \bar{P}_{ST}$

$$SIR_{R_i-SR} \cong \frac{G_{R_i-SR}P_{r_i}}{G_{PT-SR}P_{PT}} \tag{4}$$

B. Case 2 (C2)

Here, we investigate how the Q has an impact on the performance of the CRN considering fixed interference from the PT to R and SR. So, the SIR of phase-1 and phase-2 in C2 can be represented as follows

- Phase-1: $G_{ST-PR}P_{ST} \leq Q$

$$SIR_{ST-R_i} \cong \frac{G_{ST-R_i}Q}{G_{ST-PR}P_{INT}} \tag{5}$$

- Phase-2: $G_{R_i-PR}P_{r_i} \leq Q$

$$SIR_{R_i-SR} \cong \frac{G_{R_i-SR}Q}{G_{R_i-PR}P_{INT}} \tag{6}$$

where P_{INT} denotes interference caused by the PT at R and SR.

C. Relay Selection

We consider a similar relay selection procedure as in [14]. Let, the set of relays S that can be considered for relay selection as

$$S = \{i | i \in L, (min\{I_{ST-R_i}, I_{R_i-SR}\} > I_{ST})\} \tag{7}$$

Therefore, the best relay can be selected as follows

$$R_{best} = arg\max_{i \in S}(min\{I_{ST-R_i}, I_{R_i-SR}\} > I_{ST}) \tag{8}$$

After the selection of the best relay, the ST transmits the message to the best relay in the first time slot. If the best relay is able to decode the message successfully then it will forward this message to the SR in the second time slot. Otherwise, the best relay remains silent and the system declares an outage.

III. PERFORMANCE EVALUATION

In this Section, closed-form expressions of the OP and OC of CRN are derived. The secondary system is in outage if none of the relays satisfy I_{ST} i.e. $|S| = 0$. So, the OP can be expressed as

$$\begin{aligned} P^S_{OUT} &= P_r\{|S| = 0\} \\ &= \prod_{i=1}^{L} P_r\{min(I_{ST-R_i}, I_{R_i-SR}) < I_{ST})\} \\ &= \prod_{i=1}^{N} [1 - (1-Y)(1-Z)] \end{aligned} \tag{9}$$

where $Y = P_r(I_{ST-R_i} < I_{ST})$ and $Z = P_r(I_{R_i-SR} < I_{ST})$

Let, G_0 and G_1 are two exponential distributed random variables with means λ_0 and λ_1 respectively. Then the probability density function (PDF) and cumulative density function(CDF) of $X = \frac{G_0}{G_1}$ can be expressed as [15]

$$f_X(x) = \frac{\lambda_0\lambda_1}{(\lambda_0+\lambda_1 x)^2}, x > 0 \tag{10}$$

$$\begin{aligned} F_X(\rho_{i,j}) &= \int_0^{\rho_{i,j}} \frac{\lambda_0\lambda_1}{(\lambda_0+\lambda_1 x)^2}dx \\ &= \frac{\rho_{i,j}\lambda_1}{\lambda_0+\rho_{i,j}\lambda_1} \end{aligned} \tag{11}$$

Similarly, all the link gains assumed in this paper are exponentially distributed random variables with their corresponding mean values which are defined in Sect. II. So, the OP and the OC of the corresponding links in case 1 and case 2 are given as follows.

A. Case 1 (C1)

According to (11), the OP of the links ST⟶R$_i$ and R$_i$ ⟶SR in C1 can be written as

$$\begin{aligned} P_r(I_{ST-R_i} < I_{ST}) &= P_r(\frac{G_{ST-R_i}}{G_{PT-R_i}} < \rho_{ST-R_i}) \\ &= \frac{\rho_{ST-R_i}\lambda_{PT-R_i}}{\lambda_{ST-R_i}+\rho_{ST-R_i}\lambda_{PT-R_i}} \end{aligned} \tag{12}$$

$$\begin{aligned} P_r(I_{R_i-SR} < I_{ST}) &= P_r(\frac{G_{R_i-SR}}{G_{PT-SR}} < \rho_{R_i-SR}) \\ &= \frac{\rho_{R_i-SR}\lambda_{PT-SR}}{\lambda_{R_i-SR}+\rho_{R_i-SR}\lambda_{PT-SR}} \end{aligned} \tag{13}$$

where $\rho_{ST-R_i} = (2^{2I_{ST}}-1)\times(1/\frac{P_{ST}}{P_{PT}})$ and $\rho_{R_i-SR} = (2^{2I_{ST}}-1)\times(1/\frac{P_{R_i}}{P_{PT}})$. Now, substituting (12) and (13) into (9), we can get the OP of the CRN of C1.

Outage capacity can be defined as the data rate that can be achieved if outage is allowed to occur with probability ε. Therefore, the OC of C1 associated with given ε is written as [11]

$$C_{OUT}^{C1} = \log_2(1+\rho_{ST-R_i-SR}^{C1}\frac{P_{ST}}{P_{PT}})(1-\varepsilon) \tag{14}$$

where the value of $\rho_{ST-R_i-SR}^{C1}$ is the solution of (9). In [11], the authors derived the closed-form expression of the OC for the primary cooperative network. Therefore, the value of $\rho_{ST-R_i-SR}^{C1}$ can be written as [[11], (22)] to calculate the OC in C1 of the CRN considered in this work.

$$\rho_{ST-R_i-SR}^{C1} = \frac{-B+\sqrt{B^2-4AC}}{2A} \tag{15}$$

where
$A = \lambda_{PT-R_i}\lambda_{PT-SR}$
$B = \lambda_{ST-R_i}\lambda_{PT-SR} + \lambda_{PT-R_i}\lambda_{R_i-SR}$
$C = -\frac{\lambda_{ST-R_i}\lambda_{R_i-SR}\sqrt[L]{\varepsilon}}{1-\sqrt[L]{\varepsilon}}$

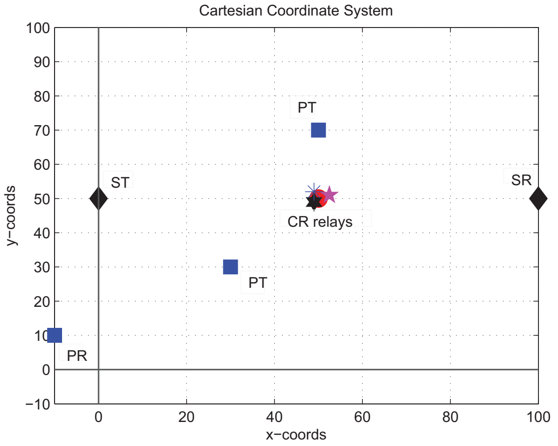

Fig. 2. An instance of our simulation scenario. Two locations (30, 30) and (50, 70) of the PT are considered. Relays are located at (50, 50).

B. Case 2 (C2)

Similarly of C1, the OP of the links ST⟶R$_i$ and R$_i$ ⟶SR in C2 can be written as

$$\begin{aligned} P_r(I_{ST-R_i} < I_{ST}) &= P_r(\frac{G_{ST-R_i}}{G_{ST-PR}} < \rho_{ST-R_i}) \\ &= \frac{\rho_{ST-R_i}\lambda_{ST-PR}}{\lambda_{ST-R_i}+\rho_{ST-R_i}\lambda_{ST-PR}} \end{aligned} \tag{16}$$

$$\begin{aligned} P_r(I_{R_i-SR} < I_{ST}) &= P_r(\frac{G_{R_i-SR}}{G_{R_i-PR}} < \rho_{R_i-SR}) \\ &= \frac{\rho_{R_i-SR}\lambda_{R_i-PR}}{\lambda_{R_i-SR}+\rho_{R_i-SR}\lambda_{R_i-PR}} \end{aligned} \tag{17}$$

where $\rho_{ST-R_i} = (2^{2I_{ST}}-1)\times(1/\frac{Q}{P_{INT}})$ and $\rho_{R_i-SR} = (2^{2I_{ST}}-1)\times(1/\frac{Q}{P_{INT}})$. Now, substituting (16) and (17) into (9), we can get the OP of the CRN of C2.

Therefore, the OC of C2 associated with given ε is written as [11]

$$C_{OUT}^{C2} = \log_2(1+\rho_{ST-R_i-SR}^{C2}\frac{Q}{P_{INT}})(1-\varepsilon) \tag{18}$$

where the value of $\rho_{ST-R_i-SR}^{C2}$ is the solution of (9). Therefore, the value of $\rho_{ST-R_i-SR}^{C2}$ can be written as (15) to calculate the OC in C2.

$$\rho_{ST-R_i-SR}^{C2} = \frac{-B+\sqrt{B^2-4AC}}{2A} \tag{19}$$

where
$A = \lambda_{ST-PR}\lambda_{R_i-PR}$
$B = \lambda_{ST-R_i}\lambda_{R_i-PR} + \lambda_{ST-PR}\lambda_{R_i-SR}$
$C = -\frac{\lambda_{ST-R_i}\lambda_{R_i-SR}\sqrt[L]{\varepsilon}}{1-\sqrt[L]{\varepsilon}}$

IV. RESULTS AND DISCUSSIONS

In this Section, we have presented theoretical and Monte Carlo simulation results to evaluate the performance of the CRN over Rayleigh fading channel. For visualizing our result, we have considered a two dimensional space in Cartesian

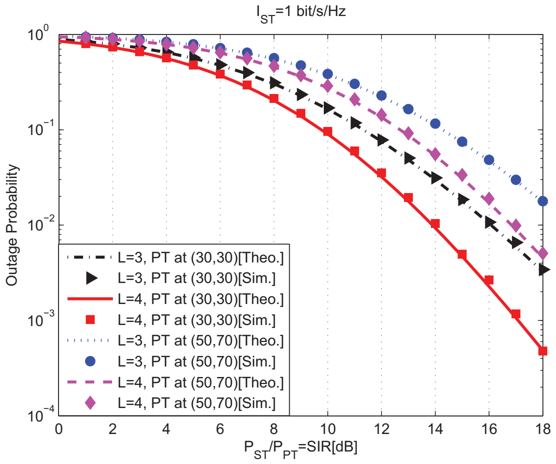

Fig. 3. Outage probability of C1.

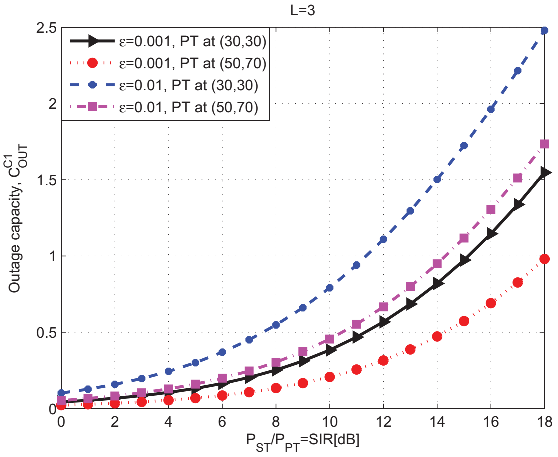

Fig. 4. Outage capacity of C1.

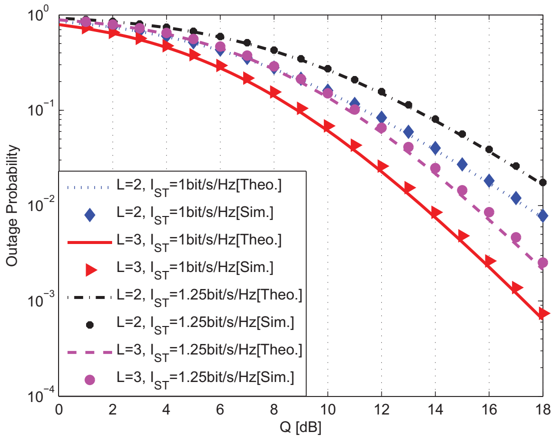

Fig. 5. Outage probability of C2.

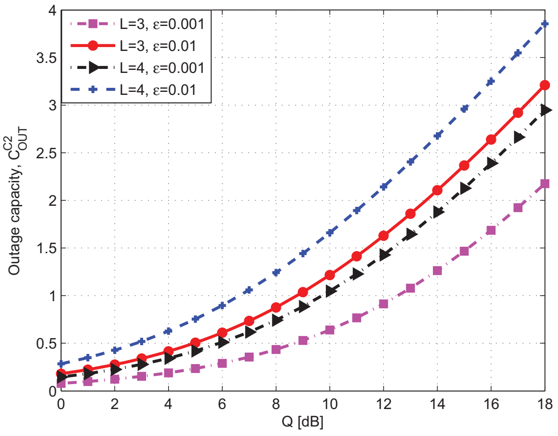

Fig. 6. Outage capacity of C2.

coordinate system as a simulation scenario as shown in Fig. 2. Assume that the position of the ST, SR and PR are located at (0, 50), (100, 50) and (-10,10) respectively. Relays are located at (50, 50). Although, we have assumed some fixed locations of PT, PR, ST, R and SR but our scheme works for any location of PT, PR, ST, R and SR as well as any distance among them. As in [16], to reduce the numbers of model parameters, the position of all the relays are considered at approximately the same distance between ST and SR. Moreover, we consider variable location ((30, 30), (50, 70)) of PT to show that the OP and OC of the CRN are affected by the position of the PT with respect to the SR. Furthermore, slow fading is considered in this paper. So, to get the effect of all relays, fading co-efficient of each relay is independent to each other. Assume, path loss exponent n=2 and meter is the unit of distance here.

In Fig. 3 and 4, we depict the OP and OC of the CRN in C1 respectively as a function of SIR. Assume $SIR = P_{ST}/P_{PT} = P_{R_i}/P_{PT}$ and SIR varies from 0 to 18 dB. Two different location of the PT at (30, 30) and (50, 70) respectively are considered to show the impact of the interference from the PT on the performance of the CRN. Fig. 3 depicts the OP when I_{ST}=1 bit/s/Hz as well as L=3 and L=4 whereas Fig. 4 depicts the OC when L=3 as well as ε=0.01 and ε=0.001. It is clear from Fig. 3 that OP decreases with increasing SIR as well as L as expected. Increasing L means more number of nodes participate in the cooperation process. As a result, the number of independent paths between the ST and SR increases showing greater diversity. It is also shown that both the OP and OC degrade as the PT becomes closer to the SR. When the PT becomes nearby to the SR, it causes more interference to the SR. Thus, resulting performance degradation of the CRN. Moreover, it is clear from Fig. 4 that OC improves with increasing ε.

In Fig. 5 and 6, we depict the OP and OC of the CRN in C2 respectively as a function of Q, where Q varies from 0 to 18 dB. We let, L=2, 3 and I_{ST} =1, 1.25 bits/s/Hz in Fig. 5 whereas L=3, 4 and ε =0.01, 0.001 in Fig. 6 respectively. It is observed that both the OP and OC improve as the Q as well as L increase. Increasing Q represents that the PR can tolerate more interference from the ST and R. Thus, resulting performance improvement of the CRN. It is also shown in Fig. 5 that OP increases with increasing I_{ST}. This means that, for the same set of parameters if the target rate increases, there is a high probability that the CRN will be in a deep fade or outage. Moreover, it is clear from Fig. 6 that OC improves with increasing ε.

V. CONCLUSION

In this paper, we have investigated the OP and OC of the CRN in interference limited environments over Rayleigh fading channel. Closed-form expressions of the OP and OC are derived. We found that performance of the CRN improves as the number of cooperative relay increases as well as when the PT is far away from the SR. It is also found that increasing ε and Q improve OP and OC. Moreover, with the same set of parameters, increasing I_{ST} degrades performance of the CRN. Some other issues such as end-to-end delay and bit error rate for the CRN will be investigated in future.

ACKNOWLEDGMENT

This research was supported by the MSIP (Ministry of Science, ICT and Future Planning), Korea, under the CITRC (Convergence Information Technology Research Center) support program (IITP-2015-H8601-15-1011) supervised by the IITP (Institute for Information and communications Technology Promotion).

REFERENCES

[1] G. Staple and K. Werbach, “The end of spectrum scarcity [spectrum allocation and utilization],” *IEEE Spectrum*, vol. 41, no. 3, pp. 48–52, Mar. 2004.

[2] J. Mitola and G. Maguire, “Cognitive radio: making software radios more personal,” *IEEE Personal Communications*, vol. 6, no. 4, pp. 13–18, Aug. 1999.

[3] Q. Zhao and A. Swami, “A decision-theoretic framework for opportunistic spectrum access,” *IEEE Wireless Communications*, vol. 14, no. 4, pp. 14–20, Aug. 2007.

[4] A. Ghasemi and E. Sousa, “Fundamental limits of spectrum-sharing in fading environments,” *IEEE Transactions on Wireless Communications*, vol. 6, no. 2, pp. 649–658, Feb. 2007.

[5] L. Musavian and S. Aissa, “Ergodic and outage capacities of spectrum-sharing systems in fading channels,” in *IEEE Global Telecommunications Conference, 2007. GLOBECOM ’07.*, Nov. 2007, pp. 3327–3331.

[6] X. Kang, Y.-C. Liang, A. Nallanathan, H. Garg, and R. Zhang, “Optimal power allocation for fading channels in cognitive radio networks: Ergodic capacity and outage capacity,” *IEEE Transactions on Wireless Communications*, vol. 8, no. 2, pp. 940–950, Feb. 2009.

[7] Asaduzzaman, H. Y. Kong, and K. Lyum, “Cooperative relaying in interference limited cognitive radio networks,” in *IEEE 6th International Conference on Wireless and Mobile Computing, Networking and Communications (WiMob)*, Oct. 2010, pp. 280–285.

[8] J. Lee, H. Wang, J. Andrews, and D. Hong, “Outage probability of cognitive relay networks with interference constraints,” *IEEE Transactions on Wireless Communications,*, vol. 10, no. 2, pp. 390–395, Feb. 2011.

[9] F. Khan, T. Ratnarajah, and Z. Ding, “Outage performance of cognitive radio wireless network with secondary relaying,” in *International Conference on Computer Systems and Industrial Informatics (ICCSII)*, Dec. 2012, pp. 1–5.

[10] A. Gopalakrishna and D. B. Ha, “Capacity analysis of cognitive radio relay networks with interference power constraints in fading channels,” in *International Conference on Computing, Management and Telecommunications (ComManTel)*, Jan. 2013, pp. 111–116.

[11] M. F. Kader, Asaduzzaman, and M. M. Hoque, “Outage capacity analysis of a cooperative relaying scheme in interference limited cognitive radio networks,” *Wireless Personal Communications*, vol. 79, no. 3, pp. 2127–2140, Dec. 2014.

[12] M. F. Kader, Asaduzzaman, and S. Y. Shin, “Outage capacity analysis of a secondary network in interference limited cognitive radio spectrum sharing system,” in *17th International Conference on Computer and Information Technology (ICCIT)*, Dec. 2014, pp. 47–52.

[13] H. Kim, S. Lim, H. Wang, and D. Hong, “Optimal power allocation and outage analysis for cognitive full duplex relay systems,” *IEEE Transactions on Wireless Communications*, vol. 11, no. 10, pp. 3754–3765, Oct. 2012.

[14] M. F. Kader, Asaduzzaman, and M. M. Hoque, “Hybrid spectrum sharing with cooperative secondary user selection in cognitive radio networks,” *KSII Transactions on Internet and Information Systems (TIIS)*, vol. 7, no. 9, pp. 2081–2100, Sept. 2013.

[15] H. Wang, J. Lee, S. Kim, and D. Hong, “Capacity enhancement of secondary links through spatial diversity in spectrum sharing,” *IEEE Transactions on Wireless Communications*, vol. 9, no. 2, pp. 494–499, Feb. 2010.

[16] J. Zhang and Q. Zhang, “Stackelberg game for utility-based cooperative cognitiveradio networks,” in *tenth ACM international symposium on Mobile ad hoc networking and computing*. ACM, May 2009, pp. 23–32.

Md Fazlul Kader was born in 1982. He received his B. Sc. and M. Sc. in computer science and engineering (CSE) from Chittagong University of Engineering and Technology (CUET), Bangladesh in 2005 and 2014 respectively. Now, he is working towards the PhD at WENS Lab., Kumoh National Institute of Technology, South Korea. From 2007 onwards he is a faculty member of the Dept. of Applied Physics, Electronics and Communication Engineering, University of Chittagong, Bangladesh. His major research interests include cognitive radio networks, cooperative communications, MIMO, computer network, NOMA, spatial modulation etc.

Soo Young shin (S’01-M’07) was born in 1975. He received the B.S., M.S., and Ph.D. degrees in electrical engineering and computer science from Seoul National University, Seoul, Korea, in 1999, 2001, and 2006, respectively. From July 2006 to June 2007, he was a Visiting Scholar at FUNLab, University of Washington, Seattle, WA, USA. For 3 years, he was working at WiMAX Design Lab, Samsung Electronics, Suwon City, Korea, and is currently an Assistant Professor in the School of Electronics, Kumoh National Institute of Technology, Gumi City, Korea, since September 2010. His research interests include next generation mobile wireless broadband networks, cognitive radio networks, wireless LAN and PAN, wireless sensor networks, molecular network, and industrial and military network.