International Journal of Business, Human and Social Sciences
ISSN: 2517-9411
Vol:9, No:8, 2015

Chaotic Behavior in Monetary Systems: Comparison among Different Types of Taylor Rule

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***Abstract*—**The aim of the present study is to detect the chaotic behavior in monetary economic relevant dynamical system. The study employs three different forms of Taylor rules: current, forward, and backward looking. The result suggests the existence of the chaotic behavior in all three systems. In addition, the results strongly represent that using expectations in policy rule especially rational expectation hypothesis can increase complexity of the system and leads to more chaotic behavior.

***Keywords*—**Chaos theory, GMM estimator, Lyapunov Exponent, Monetary System, Taylor Rule.

I. Introduction

CHAOTIC behavior that can put some limitation on the predictability of the future of a series has captured attention of many mathematical economists in these years. May in his semantic paper argued that a very simple nonlinear model can possess extraordinary rich and complex dynamical behavior [14].

Historically, the chaos theory dates back over 120 years ago to the work of Henry Poincare [11], but it started in 1960 when Edward Lorenz created his numerical atmospheric turbulence model at the MIT. The basic theoretical explanations of chaotic dynamics were provided by [20], [21] and in the early 1970s by [19].

Broadly Speaking, chaos can be defined as "stochastic behavior occurring in a deterministic system" (Royal Society, London, 1986). In 1989 Devaney presented a definition of chaos which is probably one of the most popular in the mathematical text books [12]. He proposed three properties for chaotic behavior [1]:

Definition: Let v be an interval. We say that $f: v \to v$ is chaotic on v if:

i. f has sensitive dependent on initial condition.
ii. f is transitive.
iii. Periodic points are dense in v.

In the most of the economic models we accepted that the external noise is the source of the randomness and the volatility in the behavior of the dynamical system, but the chaos revolution shows that we can have another source for this behavior. This behavior can provide difficulties for both policy designer and economic analyst. Detecting chaos in economic series can help the policy designer to have a better understanding about the impact of monetary policy on real economy.

In recent years there has been a growing interest in the search for evidence of nonlinear dynamics, and in particular chaos, in economic data. Most of these study employed deferent tests such as correlation dimension, the BDS[1] test, largest Lyapunov exponent (LLE), and Kolmogorov entropy, such as [9], [10], [3], [2], [25], [24], [18], and many others.

The main objective of this study is to detect and compare the chaotic behavior among different types of Taylor policy rules. Specifically, this study discovers the possible occurrence of chaotic behavior in the outcomes of economic model after using any of these types of monetary policy rules. The rest of this study organized as follow. Section two describes the structure of dynamical model with three different types of Taylor rules: Backward, Current, and Forward looking model. Estimation and evaluation of the model, simulation of the outcomes under different types of Taylor rules and detecting chaos with the LLE are presented in section three. Finally, the last section expresses summary and the conclusion remarks.

II. The Structure of Monetary Model

This part discusses the structure of the models. The base model that we employ is based on the modified Moosavi and Kilicman's model used for monetary policy analysis in an open economy [15], [16].

A. The Base Model

The base model employs in this study is composed of:

$$y_t = E_t y_{t+1} + \alpha_1 (i_t - E_t \pi_{t+1}) + \alpha_2 e_t + \varepsilon_t^d; \alpha_1 \& \alpha_2 < 0 \quad (1)$$

$$\pi_t = \beta_1 E_t \pi_{t+1} + \beta_2 y_t + \varepsilon_t^s; \quad 0 \le \beta_1 \le 1, \beta_2 > 0 \quad (2)$$

$$e_t = \gamma_0 + \gamma_1 (E_t \pi_{t+1}^* - E_t \pi_{t+1}) + \varepsilon_t^f; \gamma_0 > 0, \gamma_1 < 0 \quad (3)$$

where y shows the gap between actual output from potential output (steady state), π is the domestic inflation rate, e is the real exchange rate, π^* shows the world inflation rate. In the above system of equations E_t represents the mathematical expectation conditioned on period t information. Expectations are assumed to be rational [16][2]. Finally, $\varepsilon_t^j \sim iid(0, \sigma^2)$; $\forall j = d, s, f$ shows the white noise demand, supply, and foreign

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[1]Brock, Dechert, and Scheinkman
[2]The rational expectation hypothesis means that the prediction should be consistent with the data generation model, condition on all information that is available at that time [23], [17].

shocks, respectively.

The first equation shows the demand side of the economy. It is the forward looking rational expectation of the IS curve. The second one is the supply side of the economy. This is the augmented Phillips curve. The last equation is the purchasing power parity. This equation helps to analyze the impacts of foreign side of the economy.

B. Taylor Rule

Taylor argued that a good policy rule calling for changes in the money supply, monetary base, or short term interest rate in response to change in the price level and/or change in real income [22]. Nowadays, most of the central banks use the nominal interest rate (i) as their monetary policy instrument. The main assumption of this model is that the central banks have target for this variable based on the structure of the economy (rule with feedback).

The remaining necessary equations of our model are the monetary policy rule. In this study we employ and analyze three different types of Taylor rule. The first one is the current looking Taylor rule:

$$i_t = \varphi_1^c \pi_t + \varphi_2^c y_t \qquad \varphi_1^c > 0, \varphi_2^c > 0 \tag{4}$$

In this equation the reaction of the central bank is depend on the current value of the variables.

The second type of the Taylor rule which is analyzed here has the backward looking:

$$i_t = \varphi_1^b \pi_{t-1} + \varphi_2^b y_{t-1} \qquad \varphi_1^b > 0, \varphi_2^b > 0 \tag{5}$$

The above equation shows that in this rule the reaction of the policymakers is depend on the past value of the variables. We can say that in this rule the policymakers behave as they accepted the adaptive expectation hypothesis.

The last type is the forward looking Taylor rule:

$$i_t = \varphi_1^f E_t \pi_{t+1} + \varphi_2^f E_t y_{t+1} \qquad \varphi_1^f > 0, \varphi_2^f > 0 \tag{6}$$

Conducting monetary policy by use of this equation needs the rational expectation hypothesis to be accepted by the central bankers.

In all above equations φ_1^j and φ_2^j for $j = c, b, f$ are the coefficients of the central bank's reaction of inflation and output gap, respectively.

Equations (1)-(3) in combination with one of the monetary policy rules that are described above shows a small open macroeconomic model.

III. Empirical Results

Various numbers of tests proposed in chaos literature to detect the chaotic behavior in a time series data. The LLE is the most popular among of that. In this study and after estimating the above models, we simulate the real output series under different types of Taylor rules. After removing conditional heteroscedastisity and linear dependency from the simulated output, the LLE test can lead us to detect whether a time series data came from a chaotic data generation process.

The data set are obtained from the IFS software. These data are quarterly time series from 1980:1-2014:2, and related to the US. We employed the Zivot-Andrews's stationary test to check the unit root [26], and the GMM[3] estimator to simultaneously estimate the coefficient of the economic systems. The instrument set includes lags of output gap, exchange rate, domestic and world inflation, and as well as the same amount of lags of expected domestic and world inflation. Four lags of instrument are used. We employed the HP[4] filter to find the potential output and expected inflation rate. For removing linear relations the ARMA[5] model is employed. Finally, the LLE test ran on the each filtered simulated outcomes to detect the chaotic behavior.

A. The Zivot-Andrews Test

Results of the Zivot-Andrews test confirm that at the 5% significance level all of the variables in the model are non-stationary and we cannot reject the null hypothesis of unit root in the different series. In other words, all variables are stationary at first difference, i.e. statistically they are all $I(1)$.

B. Estimation of the Systems

In this part we proceed to estimate systems of economic equations presented in Section II for the United State of America. Since the Federal Reserve virtually controls over the US monetary policy [6]-[8], our rule is based on short run interest rate. Table I report the GMM estimation of the models after substitute the Taylor rule in (1) and rearrange it[6].

Overall, the results in the Table I express that the estimations of the models under the guidelines which we choose for conduct of monetary policy-current, backward, and forward looking of Taylor rules-are statistically acceptable and mostly supported due to economic theory. Now we are able to use the estimated coefficients to simulate the output gap as the most important economic variables in economic under different types of Taylor rule. There is a strong link between output gap, monetary policy instrument and economic objective such as inflation and unemployment. Hence, the output gap provides a very useful way of thinking about economic problems. If we find any chaotic behavior in output gap, the degree of predictability will reduce at least in the long-run. Before we employed our chaos test we remove the linear structure from the simulated time series by use of the following $ARMA(p, q)$ model:

$$\Psi(L) y_t = \Phi(L) \varepsilon_t \qquad \varepsilon_t \sim \text{iid}(\mu, \sigma^2) \tag{7}$$

where $\Psi(L)$ and $\Phi(L)$ are the polynomial lag operators $1 + \varphi_1 L + \varphi_2 L^2 + \cdots + \varphi_p L^p$ and $1 + \phi_1 L + \phi_2 L^2 + \cdots + \phi_q L^q$, respectively. Table II shows the suitable $ARMA$ filter for each simulated series.

[3]Generalized Method of Movements
[4]Hodrick-Prescott
[5]Autoregressive Moving Average
[6]We employed the Engle-Granger Cointegration test. The results confirm the long run relation between the variables. On the other words, there is no worry about the spurious regression.

TABLE I
ESTIMATED COEFFICIENTS OF THE MODELS WITH DIFFERENT LOOKING AT RULE

Eq.	Variables	Coefficient of the Models		
		Current	Backward	Forward
(1)	$E_t y_{t+1}$	1.882442 (5.046081)	-	3.097058 (5.784395)
	π_t	62.825335 (5.875213)	81.54970 (7.000959)	-
	y_{t-1}	-	-0.008034 (-1.476188)	-
	$E_t \pi_{t+1}$	54.53382 (3.908904)	100.4598 (5.955918)	16.79534 (0.816888)
	e_t	-19.34149 (-1.433388)	120.9017 (1.949958)	-38.37506 (-0.798567)
(2)	$E_t \pi_{t+1}$	0.964386 (12.32815)	0.923307 (12.16909)	0.956582 (11.09098)
	y_t	0.012122 (9.468353)	0.009997 (8.613003)	0.011065 (9.949469)
(3)	γ_0	0.010111 (0.716701)	1.523532 (7.877441)	1.434390 (12.40819)
	$E_t \pi^*_{t+1} - E_t \pi_{t+1}$	0.975191 (56.74047)	0.009505 (0.986973)	0.008021 (0.806884)

Numbers in Parentheses show the *t-statistics.*

Our results in Table II show that estimated coefficients of $ARMA$ model are all properties of well-estimated time series model. Now we can use the filtered series to detect the chaos by LLE.

TABLE II
ESTIMATED COEFFICIENTS OF ARMA MODEL

Coefficients	Current Looking $ARMA(5,2)$	Backward Looking $ARMA(2,1)$	Forward Looking $ARMA(5,2)$
$AR(1)$	1.330759 (27.63187)	0.311799 (3.546513)	1.362366 (25.66763)
$AR(2)$	0.961197 (11.13247)	0.319392 (3.672517)	0.932270 (9.084171)
$AR(3)$	-1.146411 (-14.77265)	-	-1.241849 (-15.18380)
$AR(4)$	-1.023058 (-12.14352)	-	-0.868339 (-8.522717)
$AR(5)$	0.876359 (17.91875)	-	0.814495 (15.03110)
$MA(1)$	1.679701 (97.89077)	0.971673 (45.22173)	1.722994 (1218470)
$MA(2)$	0.947806 (58.33616)	-	0.950983 (74.14994)

Numbers in Parentheses show the *t-statistics.*

C. The LLE Chaos Test

One of the widely used methods to find the existence of chaotic behavior in the time series data based on the sensitivity to initial condition is the LLE. This method measure the average of divergence (convergence) between a reference (y_0) and a perturbed trajectory $(y_0 + \Delta y_0)$. The separation between two trajectories is Δy_0 that shows an infinitesimally small perturbation. During the time, this perturbation from the initial condition can make a new perturbation trajectory i.e. Δy, that can be shown as a function of time and the reference orbit, i.e. $\Delta y(y_0, t)$.

Definition: The Lyapunov exponent λ is defined as:

$$\lambda = \lim_{n\to\infty} \frac{1}{n} \ln \frac{|\Delta y(y_0,t)|}{|\Delta y_0|} \quad (8)$$

The following theorem describes the value of the Lyapunov exponent [13]:

Theorem: If at least one of the average Lyapunov exponents is positive, then the system is chaotic, if the average Lyapunov exponents is negative, then the orbit is periodic and when the average Lyapunov exponents is zero, a bifurcation occurs.

There are many methods available for determining LLE, but here we employed BenSaïda's algorithm [4], [5]. The null hypothesis of the test is $H_0: \lambda \geq 0$, and its rejection provides a strong evidence of no chaotic Dynamics [5]. In fact, the more chaotic the system, the higher the value of the LLE [13] as it can be seen in Table III.

TABLE III
CHAOS RESULT TESTS

Taylor Rule	(L, m, q)	λ	$P-Value$	$Accepted\ Hypothesis$
Current	(1, 6, 3)	0.0121	0.0623	H_0
Backward	(3, 5, 5)	0.0342	0.0646	H_0
Forward	(1, 6, 4)	0.0508	0.0923	H_0

The results of Table III represent the existence of chaos in our simple monetary dynamical systems. Our results indicate that using rational or adaptive expectation hypotheses in the monetary policy rules increase the complexity of the behavior of the system and lead to the more chaotic behavior.

IV. SUMMARY AND CONCLUSION

The objective of this paper is to detect the chaotic behavior in a simple monetary economic system. We used the quarterly data from 1980:1-2014:2 of the United State of America to estimate the coefficients of the system. We employed the LLE that is the most widely used method for diagnosing the existence of chaos in time series data. The results suggest the existence of chaotic behavior in all different monetary systems. By comparing these systems we fund that forward looking Taylor rule which is based on the rational expectation hypothesis shows the highest value of the LLE than the others. In addition, the LLE for the current looking Taylor rule is smallest among the all. Therefore, using expectation especially the rational one in the monetary policy rule can increase complexity of the system and leads to more chaotic behavior.

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