

# Synthetic Daily Flow Duration Curves for the Çoruh River Basin, Turkey

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**Abstract**—The flow duration curve (FDC) is an informative method that represents the flow regime's properties for a river basin. Therefore, the FDC is widely used for water resource projects such as hydropower, water supply, irrigation and water quality management. The primary purpose of this study is to obtain synthetic daily flow duration curves for Çoruh Basin, Turkey. For this aim, we firstly developed univariate auto-regressive moving average (ARMA) models for daily flows of 9 stations located in Çoruh basin and then these models were used to generate 100 synthetic flow series each having same size as historical series. Secondly, flow duration curves of each synthetic series were drawn and the flow values exceeded 10, 50 and 95% of the time and 95% confidence limit of these flows were calculated. As a result, flood, mean and low flows potential of Çoruh basin will comprehensively be represented.

**Keywords**—ARMA models, Çoruh basin, flow duration curve, Turkey.

## I. INTRODUCTION

AN understanding of how much water is flowing down a river is fundamental to hydrology. Of particular interest for both flood and low flow hydrology is the question of how representative a certain flow is. This can be addressed by looking at the frequency of daily flows and some statistics that can be derived from the frequency analysis. The culmination of the frequency analysis is a flow duration curve [1]. The flow duration curve shows the percentage of time a daily, weekly, monthly or yearly streamflow is equaled or exceed over a historical period for a particular river basin. However, the lengths of historical data for location of interest are generally not adequate for reliable estimation of these percentages. Therefore, hydrologists often apply stochastic based models to generate synthetic flow series that preserves statistical properties of the historical flow series. For this purpose, auto-regressive (AR) and auto-regressive moving average (ARMA) models have been widely used hydrologic time series modeling [2]-[7].

In this study, we aim to determine high, mean and low flows potential of the Çoruh basin, Turkey. In the first phase of the study, auto-regressive (AR) and auto-regressive moving average (ARMA) models are used for stochastic modeling of daily streamflow of nine gauging stations in the basin.

In the second phase of the study, using the best AR and ARMA models that defined by Akaike Information Criterion

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(AIC), 100 synthetic daily streamflow series are generated. In the final phase of the study, flow duration curve of each synthetic series is drawn and confidence limit of the high, mean and low flows is estimated.

## II. MATERIAL AND METHODS

### A. Autoregressive Moving Average Models (ARMA)

Autoregressive moving average models (ARMA) are linear stochastic models which are used to model dependent stochastic components of a time series. ARMA models can be expressed as:

$$\phi(B)y_t = \theta(B)\varepsilon_t \quad (1)$$

where  $\phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \dots - \phi_p B^p$  is the autoregressive operator;  $p$  is the number of autoregressive terms;  $\theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \dots - \theta_q B^q$  is the moving average operator;  $q$  is the number of moving average terms;  $\varepsilon_t$  is the random component (residuals) of the model and  $B$  is the backward operator (defined as  $B^m y_t = y_{t-m}$ ). Logarithmic and square roots transformations are commonly used to obtain normal variables before modeling and also periodicities of the streamflow series need to be eliminated to be able to use deseasonalized models. The observed series  $x_{ij}$  is deseasonalized by:

$$y_{ij} = \frac{x_{ij} - \bar{x}_j}{s_j} \quad (2)$$

where  $\bar{x}_j$  and  $s_j$  are the respective streamflow mean and standard deviation for  $j^{\text{th}}$  day ( $j = 1, 2, \dots, 365$ ). In the case of using daily streamflow values, (1) can be rewritten in open form as:

$$y_t = \phi_1 y_{t-1} + \dots + \phi_p y_{t-p} + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \dots - \theta_q \varepsilon_{t-q} \quad (3)$$

where  $t$  is calculated from  $t = 365(i - 1) + j$ . This equation is known as the mathematical expression of ARMA ( $p, q$ ) model of order  $p$  and  $q$ . A total of  $p+q+2$  parameters must be evaluated from observed data. When several ARMA models are fitted to a given data series a goodness of fit test is required to find the best selection. Akaike's Information Criterion (AIC), Bayesian Information Criterion (BIC), Kolmogorov-Smirnov (KS) and Chi-square ( $\chi^2$ ) tests are commonly used for this purpose. In this study, Akaike Information Criteria (AIC) is applied and the model with minimum AIC value is accepted the best among alternative models. It is expressed as:

$$AIC = N \ln(\sigma_{\varepsilon}^2) + 2(p + q) \quad (4)$$

where N is total number of observations and  $\sigma_{\varepsilon}^2$  is the variance of residual terms.

TABLE I  
SUMMARY INFORMATION OF THE GAUGE STATIONS USED IN THE STUDY

| Station Number | Approx. Altitude (m) | Obser. Period | Mean Flow (m <sup>3</sup> /s) | Standard Deviation (m <sup>3</sup> /s) | Skewness (C <sub>s</sub> ) |
|----------------|----------------------|---------------|-------------------------------|--|----------------------------|
| 2304           | 1545                 | 1962–2004     | 15.83                         | 19.16                                  | 2.86                       |
| 2320           | 1365                 | 1971–2004     | 28.66                         | 34.90                                  | 2.24                       |
| 2316           | 1170                 | 1965–2004     | 38.49                         | 46.57                                  | 2.24                       |
| 2325           | 1129                 | 1974–2004     | 7.03                          | 10.19                                  | 5.01                       |
| 2321           | 705                  | 1972–2004     | 13.79                         | 14.16                                  | 1.67                       |
| 2305           | 654                  | 1963–2004     | 69.20                         | 78.23                                  | 2.02                       |
| 2323           | 580                  | 1965–2004     | 33.44                         | 39.46                                  | 3.39                       |
| 2322           | 201                  | 1972–2000     | 158.88                        | 171.65                                 | 2.08                       |
| 2315           | 57                   | 1965–2001     | 207.15                        | 205.91                                 | 2.22                       |

### B. Flow Duration Curves (FDCs)

Flow duration curve (FDC) is a tool that represent the relationship the magnitude and the frequency of daily, monthly, weekly or yearly time interval of stream flows for a particular river basin. This curve can provide an estimate of the percentage of time the stream flow was equaled or exceeded over a historical period [8]. The selection of the time interval depends on the purpose of the study. As the time interval increases, the range of the curve decreases. While daily flow rates of small storms are useful for the pondage studies in a runoff river power development plant, monthly flow rates for a number of years are useful in power development plants from a large storage reservoir [9]. There are the three most important statistics derived from flow duration curves. These are;

- The flow value that is exceeded 10 per cent of the time ( $Q_{10}$ ). The value can be useful for analysis of high flows and flooding.
- The flow value that is exceeded 50 per cent of the time ( $Q_{50}$ ). The value is the median flow value.
- The flow value that is exceeded 90 per cent of the time ( $Q_{90}$ ). The value can be useful for analysis of low flow and droughts. An example of flow duration curve is shown Fig. 1.

### III. STUDY AREA

The Çoruh River is the longest river of the East Black Sea region and is of high economic importance to Turkey because of it is largely undeveloped but has economically exploitable hydropower potential. The Çoruh Basin Development Project Dam and Hydropower Plants in operation by energy viewpoint constitute the 1.23% of the total hydroelectric potential of Turkey and 3.47% of the total hydropower potential in operation [10]. There are numerous dams and hydropower plants projects in the Çoruh river basin planned or developed by the Basin Development projects or private sector companies. Approximately 91% of the basin's drainage area is in Turkey, and Georgia's share amounts to 9%. In this study, based on the record lengths of the historical data, its continuity

and suitability we selected 9 gauging stations located in the basin and used the daily streamflow data compiled from the stations for analysis. Fig. 2 shows the location map of the study area and distribution of gauging stations and Table I summarizes basic statistics and information about stations.

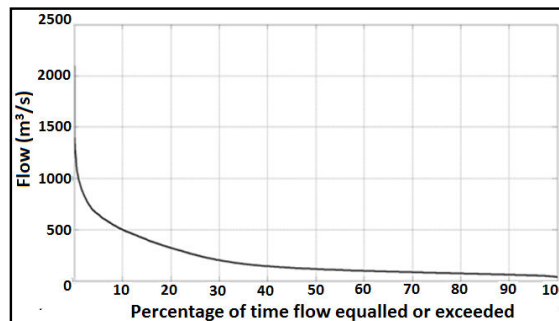


Fig. 1 An example of flow duration curve based on daily flows for the Çoruh river

All stations are operated by Turkish General Directorate of Electrical Power Resources and Development Administration (EIE).

### IV. RESULTS

#### A. Univariate ARMA Models for Daily Streamflow

In this part of the study, we aim to develop univariate ARMA models for daily streamflow of the nine stations located in the Çoruh basin, Turkey. For this purpose, before developing ARMA models, we applied to the logarithmic transformation and then the periodicity in the means and variances were removed by standardizing the data in order to obtain normally distributed and standardized data series. After then, various ARMA models were fitted to the standardized series and based on Akaike Information Criteria (AIC) the best ARMA models were determined for each station. Table II represents the best ARMA models with values of their parameters. Using the best ARMA models, 100 sets of synthetic streamflow series each having same length as the historical time series were generated for all stations. For comparison of the models' performance, certain relevant statistics were computed from both historical and synthetic series and the compared.

The statistic considered here were daily mean, daily standard deviation and autocorrelation. The results indicated that these synthetic series have satisfactorily preserved statistical characteristics of historical stream flows (see Figs. 3-5). Detailed information about univariate ARMA modeling for daily streamflow in the Çoruh basin can be found in [11].

#### B. Synthetic Flow Duration Curves

After the most suitable ARMA models were developed and synthetic daily streamflow series were generated we then obtained flow duration curve of each synthetic series. A MATLAB code was written to estimate the flow values exceeded 10 ( $Q_{10}$ ), 50 ( $Q_{50}$ ) and 95 ( $Q_{95}$ ) % of the time. Then, lower and upper limits of these flow values were estimated

within 95% confidence intervals. Table III represents the estimated limits values. These values indicate potential of low, mean and high flows for rivers in the Çoruh basin.

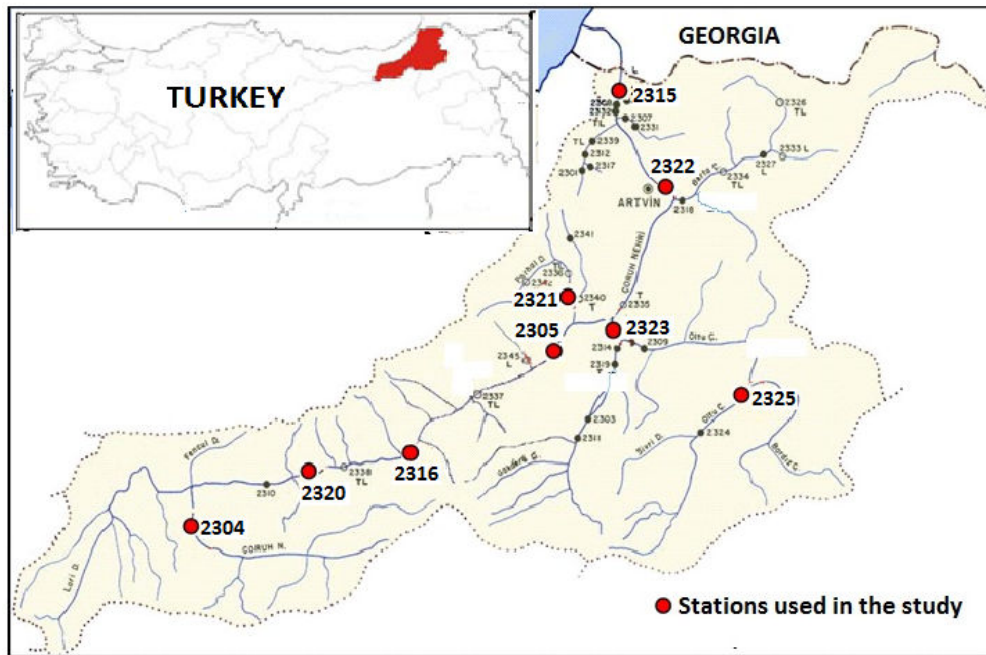


Fig. 2 Map of study area and gauge stations used in the study

TABLE II  
THE BEST ARMA MODELS FOR DAILY STREAMFLOWS OF RIVERS IN ÇORUH BASIN, TURKEY

| Station Num. | The Best ARMA Models | $\phi_1$ | $\phi_2$ | $\phi_3$ | $\phi_4$ | $\theta_1$ | $\theta_2$ | $\theta_3$ |
|--------------|----------------------|----------|----------|----------|----------|------------|------------|------------|
| 2304         | ARMA(2,2)            | 1.93     | -0.93    | -        | -        | -0.73      | -0.23      | -          |
| 2305         | ARMA(3,2)            | 2.08     | -1.33    | 0.24     | -        | -0.72      | -0.17      | -          |
| 2315         | ARMA(2,2)            | 1.82     | -0.82    | -        | -        | -0.61      | -0.31      | -          |
| 2316         | ARMA(3,2)            | 1.99     | -1.16    | 0.17     | -        | -0.69      | -0.19      | -          |
| 2320         | ARMA(4,2)            | 1.14     | 0.68     | -1.13    | 0.31     | 0.16       | -0.83      | -          |
| 2321         | ARMA(2,2)            | 1.87     | -0.87    | -        | -        | -0.68      | -0.27      | -          |
| 2322         | ARMA(4,3)            | 2.89     | -2.97    | 1.27     | -0.18    | -1.57      | 0.44       | 0.13       |
| 2323         | ARMA(2,2)            | 1.82     | -0.82    | -        | -        | -0.70      | -0.18      | -          |
| 2325         | ARMA(2,0)            | 1.15     | -0.19    | -        | -        | -          | -          | -          |

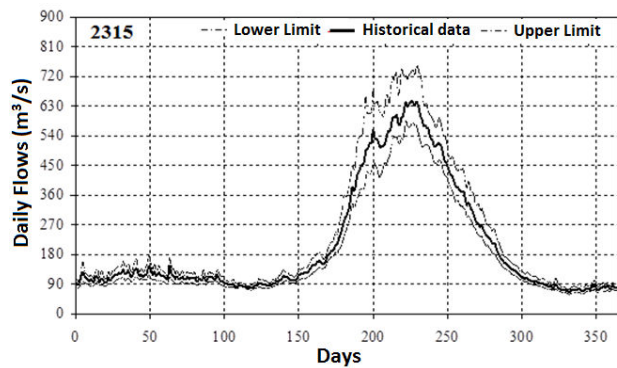


Fig. 3 Mean daily flows generated by using ARMA (2,2) model for 2315 gauging station with 95% confidence interval

TABLE III  
LOWER AND UPPER LIMITS OF Q<sub>10</sub>, Q<sub>50</sub> AND Q<sub>95</sub> FLOWS (M<sup>3</sup>/S)

| Station Num. | Q <sub>10</sub> |             | Q <sub>50</sub> |             | Q <sub>95</sub> |             |
|--------------|-----------------|-------------|-----------------|-------------|-----------------|-------------|
|              | Lower Limit     | Upper Limit | Lower Limit     | Upper Limit | Lower Limit     | Upper Limit |
| 2304         | 36.45           | 46.83       | 7.38            | 8.42        | 3.68            | 4.23        |
| 2305         | 170.06          | 204.19      | 28.95           | 32.87       | 14.25           | 16.12       |
| 2315         | 454.68          | 531.15      | 111.33          | 123.18      | 50.99           | 57.68       |
| 2316         | 98.18           | 111.15      | 15.30           | 16.36       | 9.43            | 10.05       |
| 2320         | 74.41           | 82.82       | 11.53           | 12.29       | 7.02            | 7.54        |
| 2321         | 33.54           | 37.99       | 6.49            | 7.24        | 2.69            | 2.99        |
| 2322         | 385.50          | 436.08      | 71.38           | 76.56       | 44.01           | 47.46       |
| 2323         | 70.94           | 84.57       | 17.10           | 18.42       | 9.92            | 11.08       |
| 2325         | 14.16           | 17.59       | 3.57            | 3.85        | 0.68            | 0.94        |

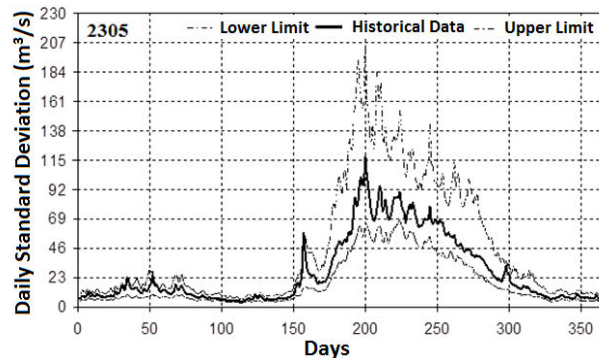


Fig. 4 Mean daily standard deviations generated by using ARMA (3,2) model for 2305 gauging station with 95% confidence interval

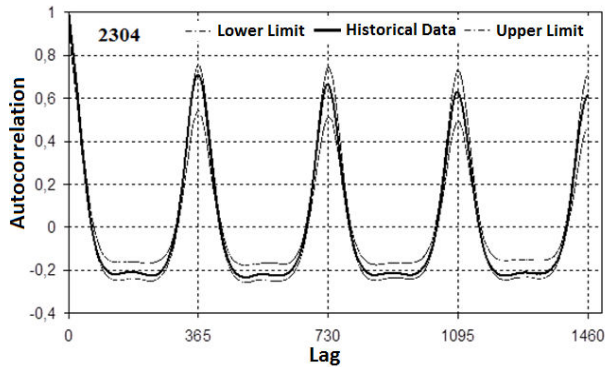


Fig. 5 Correlograms generated by using ARMA (2,2) model for 2304 auing station with 95% confidence

#### V. CONCLUSION

The main conclusions drawn from the study are summarized as;

- The most suitable univariate autoregressive moving average (ARMA) models were developed for daily streamflows of nine gauge stations located critical part of the Çoruh basin, Turkey.
- Using the ARMA models, synthetic streamflow series were generated each having same length as historical data series.
- Flow duration curve of each series were drawn.
- Low, mean and high flow characteristics of Çoruh River were comprehensively analyzed.

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