Empirical Mode Decomposition Based Multiscale Analysis of Physiological Signal

Young-Seok Choi

Abstract—We presents a refined multiscale Shannon entropy for analyzing electroencephalogram (EEG), which reflects the underlying dynamics of EEG over multiple scales. The rationale behind this method is that neurological signals such as EEG possess distinct dynamics over different spectral modes. To deal with the nonlinear and nonstationary nature of EEG, the recently developed empirical mode decomposition (EMD) is incorporated, allowing a decomposition of EEG into its inherent spectral components, referred to as intrinsic mode functions (IMFs). By calculating the Shannon entropy of IMFs in a time-dependent manner and summing them over adaptive multiple scales, it results in an adaptive subscale entropy measure of EEG. Simulation and experimental results show that the proposed entropy properly reveals the dynamical changes over multiple scales.

Keywords—EEG, subscale entropy, Empirical mode decomposition, Intrinsic mode function.

I. INTRODUCTION

ELECTROENCEPHALOGRAM (EEG) is indicative of the electric activity of the brain, whose waveform contains useful information about the states of the brain. Recently, EEG has been exploited in connection with functional brain mechanisms as a potential tool for the identification of brain disorders [1]. However, visual inspection cannot monitor the subtle information embedded in EEG. Thus, the need for objective measures gives rise to the development of quantitative EEG measure to uncover neurological states [2][3]. Until recently, quantitative EEG measures based on entropy have shown promising results for monitoring and detecting brain rhythm [4][5]. Among those, the Shannon entropy [6] has been widely used due to its simplicity. However, EEG signals is representative of the interacting mechanisms between numerous neurons across multiple temporal and spatial scales, resulting in dynamical changes in both time and frequency associated with the states of the brain [1]. Thus, the Shannon entropy which is calculated using gross EEG signals may not properly reveal the underlying dynamics of EEG over multiple scales.

Here, a new entropy measure which exploits scale-dependent dynamics of EEG is presented. In order to extract the scale-dependent oscillatory components of EEG, the empirical mode decomposition (EMD) which is a data-driven decomposition tool, is incorporated because of its suitability to a nonstationary time-series [7]. It decomposes a time-series into different spectral components inherent in the time-series, namely intrinsic mode functions (IMFs). Thus, due to the potential of the EMD, it has been gradually used to analyze nonstationary physiological signals [8][9]. Next, calculating the Shannon entropy of IMFs in a manner of a time-dependent scheme and summing over data-driven multiple scales, it leads to a scale-dependent quantitative measure of EEG, termed a data-driven subscale entropy.

Through simulation and experimental studies, the proposed data-driven subscale entropy has shown its effectiveness in terms of sensitivity for reflecting the dynamical changes over scales.

II. DATA-DRIVEN SUBSCALE ENTROPY

A. Empirical Mode Decomposition of EEG

Recently, Huang et al. [7] have developed a data-driven decomposition method, thus being suited for nonlinear and nonstaionary time series. In an iterative manner, termed a sifting process, EMD extracts the highest frequency oscillation (finest temporal scale) from the underlying time series, being considered as an IMF. The remaining part after the extraction contains lower frequency oscillatory components. The resulting IMFs represent the oscillatory patterns at different scale. This gives rise to the following major feature of EMD: EMD results in basis functions which are derived from the time-series in self-originated way, whereas other conventional methods such as Fourier and wavelet analyses rely on the use of pre-defined basis functions.

Let $s(i)$ denote the raw sampled EEG signal. Then,the EMD consists of the following steps:

1) Identify all the local maxima and minima of $s(i)$.
2) Interpolate between local maxima and minima respectively, getting an upper envelope $e_u(i)$ and a lower envelope $e_l(i)$.
3) Compute the mean between $e_u(i)$ and $e_l(i)$, i.e., $\mu(i) = [e_u(i) + e_l(i)]/2$.
4) Subtract the mean from the original signal $d(i) = s(i) - \mu(i)$.
5) Repeat steps 1–4 until $d(i)$ satisfies the above two criteria to be an IMF. If $d(i)$ satisfies conditions, it becomes the first intrinsic mode function that contains the finest temporal scale in the signal. Also denote as $d_1(i)$.
6) Compute the residue $r_1(i) = s(i) - d_1(i)$.
7) Iterate through steps 1–6 with $s(i)$ instead of $s(i)$ until the residue satisfies some stopping criterion. A commonly used stopping criterion is the sum of difference.

Y.-S. Choi is with the Department of Electronic Engineering, Gangneung-Wonju National University, 7 Jukheon-gil, Gangneung, 210-702 Republic of Korea (phone: +82-33-640-2429; fax: +82-33-646-0740; e-mail:yschoi@gwnu.ac.kr).
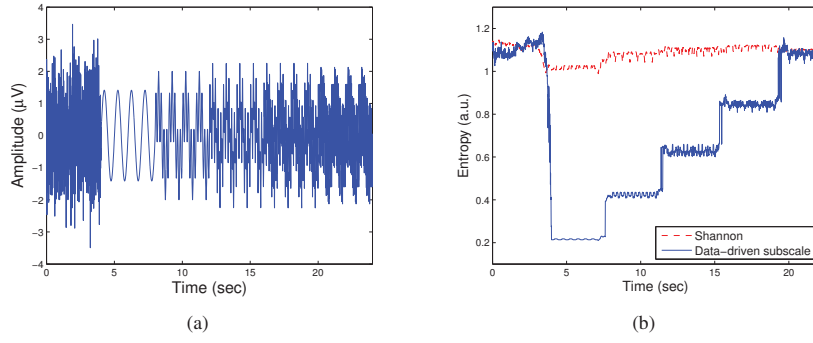
Fig. 1. Time evolution of the Shannon entropy and the data-driven subscale entropy for synthetic signal with time varying frequency components. (a) Synthetic signal in time domain. (b) Comparison of the Shannon entropy and data-driven subscale entropy.

Through the sifting process, the raw EEG signal $s(i)$ is decomposed as follows:

$$s(i) = \sum_{k=1}^{K} d_k(i) + r_K(i), \qquad (1)$$

where K is the number of all extracted intrinsic mode functions, $d_k(i)$ is the kth IMF, and $r_K(i)$ is the final residue. The last residue $r_K(i)$ can be considered as the last IMF, and thus (1) can be rewritten as $s(i) = \sum_{k=1}^{K+1} d_k(i)$.

B. Data-driven Subscale Entropy

Next, the distribution of the time-varying individual oscillatory components obtained in (1), i.e., $d_k(i)$, are utilized to evaluate the adaptive subscale entropy. To deal with continuously acquired signals, EEG recording is divided into a number of segments using a sliding temporal window, leading to a time dependent entropy measure [10]. For a given $\{s(i) : i = 1, \cdots, N\}$, a sliding temporal window $w \leq N$ and a sliding interval $\Delta \leq w$ are defined. Then, the nth sliding window of the raw EEG signal are defined by

$$\mathbf{s}_n(i) = \{s(i); i = 1 + n\Delta, \dots, w + n\Delta\}, \qquad (2)$$

where $n = 0, 1, \cdots, [(N - w + 1)/\Delta]$ and $[x]$ denotes the integer part of x.

Then, EMD is incorporated to utilize the underlying time-varying oscillatory components in EEG. Let assume EEG is decomposed by a sifting process, yielding totally K IMFs and one residual which is considered as $(K+1)$th IMF. A set of IMFs, EMD$[\mathbf{s}_n(i)]$, is obtained from EEG in the sliding window $\mathbf{s}_n(i)$

$$\mathrm{EMD}[\mathbf{s}_n(i)] = [\mathbf{d}_n^1, \mathbf{d}_n^2, \dots, \mathbf{d}_n^{K+1}], \qquad (3)$$

where $\mathbf{d}_n^k = [d_k(i); i = 1 + n\Delta, \dots, w + n\Delta]$ for $k = 1, \dots, K + 1$ denote the kth IMF in the nth sliding window.

In order to compute the probability distributions of the IMFs, $\mathbf{d}_n^k$ for $k = 1, \dots, K+1$ are partitioned into M disjoint intervals $\{I_m, m = 1, \dots, M\}$ spanning the range between the minimum and maximum values of IMF with $l_l = \min\{\mathbf{d}_n^k\}$ and $l_M = \max\{\mathbf{d}_n^k\}$, where $l_1 < l_2 < \dots < l_M$, which is as follows:

$$\mathrm{EMD}[\mathbf{s}_n(i)] = [\mathbf{d}_n^1, \mathbf{d}_n^2, \dots, \mathbf{d}_n^{K+1}] = \cup_{m=1}^{M} I_m, \qquad (4)$$

Then, $p_n^k(m)$ is the probability that the IMF belongs to the interval I_m in kth IMF $\mathbf{d}_n^k$. It is computed as a ratio of number of samples of $\mathbf{d}_n^k$ within I_m and the total sample number of $\mathbf{d}_n^k$.

By sliding the window w, a data-driven subscale entropy (DSE) of Shannon framework in the kth scale is defined as

$$\mathrm{DSE}^k(n) = -\sum_{m=1}^{M} p_n^k(n) \log(p_n^k(n)), \qquad (5)$$

where $k = 1, \dots, K+1$, $0 \leq p_n^k(m) \leq 1$, and $\sum_{m=1}^{M} p_n^k(m) = 1$. Finally, the data-driven subscale entropies in each scale are summed over all scales, leading to the adaptive subscale entropy

$$\mathrm{DSE}(n) = \sum_{k=1}^{K+1} \mathrm{DSE}^k(n). \qquad (6)$$

III. RESULTS

A. Simulation

To verify the capability of the proposed adaptive subscale entropy, a synthesized signal consisting of Gaussian distribution and multiple sinusoidal components is used, which is shown in Fig. 1(a). For the first 4 sec, the synthetic signal has Gaussian distribution. Following periods of the synthetic signal has different number of sinusoids in time-dependent manner as follows: From 4 to 8 sec, it begins with a single sinusoid of 1 Hz, followed by the addition of one more sinusoid with 5 Hz after 4 sec. From 12 to 16 sec, it consists of three sinusoids whose frequencies are 1, 5, and 10 Hz. During following 4 sec, it consists of four sinusoids whose frequencies are 1, 5, 10, and 20 Hz. During the last 4 sec, five sinusoids with 1, 5, 10, 20, and 40 Hz are included. From the perspective of entropy, it is expected that the more the number of sinusoidal components, the higher value of entropy. Fig. 1(b) depicts the results of the conventional Shannon entropy and the data-driven subscale entropy, respectively. In the figure, the Shannon entropy is almost constant regardless of the distribution and the number of sinusoidal components of the signal, while the data-driven subscale entropy has higher value in accordance with the increase of sinusoidal components and is discriminative with Gaussian distribution.
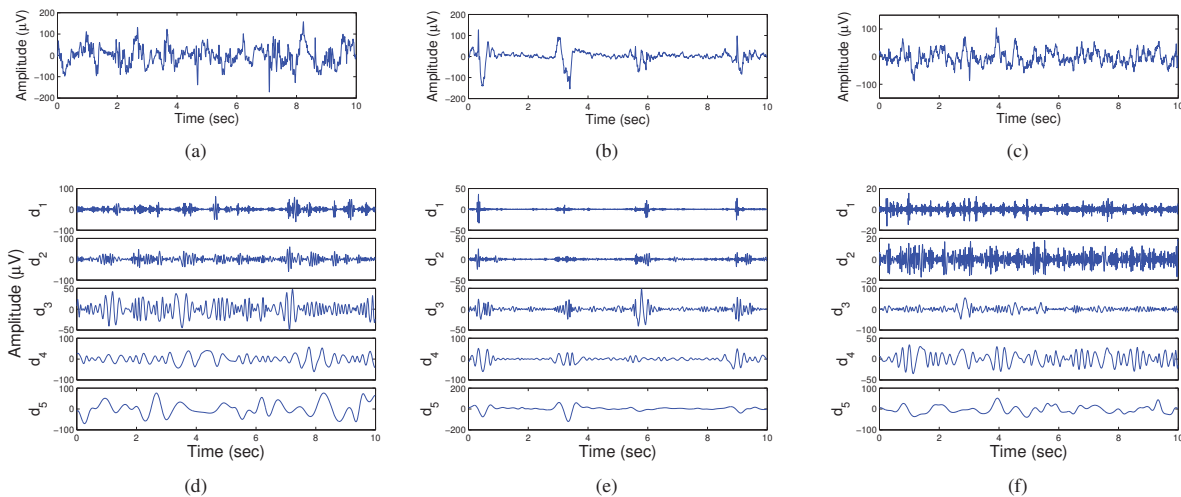
Fig. 2. Three EEG recordings chosen at different phase of brain injury and their EMD results. (a) Baseline EEG at 5 min, (b) Early recovery EEG at 50 min, and (c) Late recovery EEG at 180 min from the beginning of experiment, respectively. (d)–(f) IMFs of (a)–(c), respectively. Top traces denote IMFs with highest scale d_1 and bottom traces depict IMFs with lowest scale d_5.
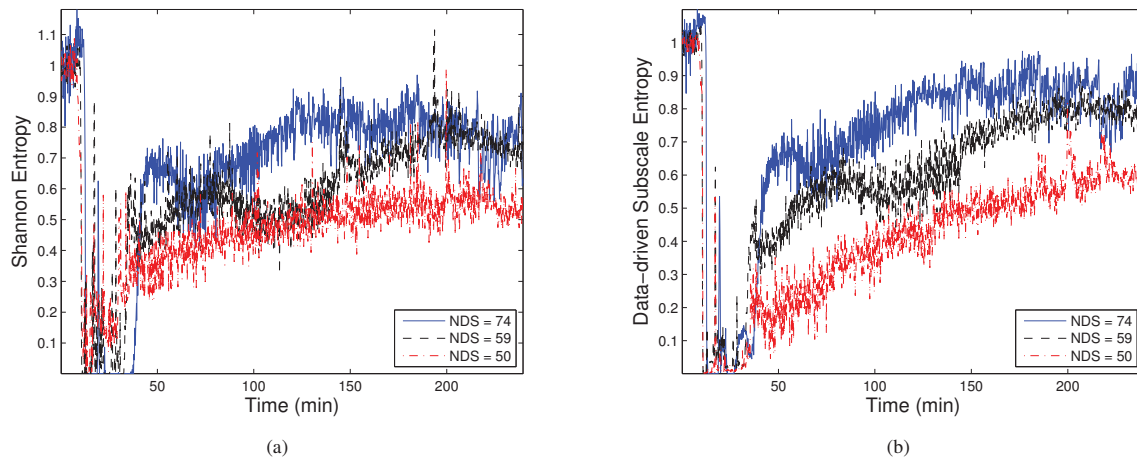

Fig. 3. Time evolutions of the entropy measures of EEG for three rats (NDS = 74, 59, and 50). (a) Shannon entropy. (b) Data-driven subscale entropy.

B. Experimental Study on EEG following Brain Injury

Next, this study investigate EEG signals from rats subject to hypoxic-ischemic brain injury due to cardiac arrest. The experimental model of brain injury by cardiac arrest has been approved by Animal Care and Use Committee of the Johns Hopkins Medical Institutions [11]. Nine adult male Wistar rats (300±25g) were used. Anesthesia was induced with 4% halothane in 50%:50% nitrous oxide:oxygen. A 10 min of baseline EEG was recorded including 5 min washout period to ensure that halothane did not influence the EEG. Subsequently, 7 min asphyxia was induced by stopping and disconnecting the ventilator and clamping the tracheal tube. The duration of cardiac arrest was determined by the mean arterial blood pressure being below 10 mmHg. Cardio Pulmonary Resuscitation (CPR) was carried out by chest compression until return of spontaneous circulation which was decided a spontaneous the mean arterial blood pressure greater than 60 mmHg.

The EEG signals were continuously recorded with DATAQ acquisition package (DATAQ Instruments INC., Akron, OH). All rats underwent neurological testing at 72 h from the beginning of recovery. Neurological deficit score (NDS) was used as the measure for comprehensive neurological outcome of rats. Since NDS is evaluated quantitatively, which ranges from 0 (worst) to 80 (best), is servers as an appropriate tool for relating entropy measures to neurological outcome.

Fig. 2(a)–2(c) show the EEG recordings of the experiment which were recorded at different stage as follows: at 5 min (Fig. 2(a)), 50 min (Fig. 2(b)), and 180 min (Fig. 2(c)) from the start of experiment, respectively. In the figures, the EEG recordings at different stage show distinct waveforms in both amplitude and frequency. To reveal the inherent oscillations of EEGs, the five IMFs of each EEG recording are presented in Figs. 2(d)–2(f), respectively. The first IMF, here, denoted

as d_1, has highest frequency and frequency component in the IMFs decreases along with the next IMFs.

For calculating the data-driven subscale entropy, the following parameters were used: sliding temporal window length of $w = 10$ sec, sliding interval of Δ=10 sec, and M=20. Figs. 3(a) and 3(b) show the time evolutions of the conventional Shannon entropy and the data-driven subscale entropy for three rats which have different NDS values, which implies distinct neurological outcome of rats. In both plots, after washout around 15 min, entropies of three rats dramatically fall to approximately zero, followed by rapid increase from 35-40 min. In the figures, the Shannon entropy values during recovery are not highly separable for different animals with good (79) to bad (50) NDS values. On the other hand, the data-driven subscale entropies in Fig. 3(b) are consistently separable for the three animals with different neurological deficit scores. This result indicates that the higher neurological score, the higher entropy value at the end of the four hour recovery period.

To assess with a larger sample, the Shannon and data-driven subscale entropies of nine rats including the previous three rats were calculated. To demonstrate the entire trend, entropies for each rat were averaged over selected intervals and the average of recovery phase (30-240 min from the start of experiment). The results reveal that the subscale entropy is more closely correlated to NDS than its counterpart.

IV. Conclusion

Here, we demonstrate successful use of a data-driven decomposition scheme, empirical mode decomposition, which captures the intrinsic oscillations contained in EEG. Each mode is shown to cover the clinical EEG frequency bands of interest. Due to the property of adaptive basis function derived from original signal itself, EMD is an effective tool for representing nonstationary signal such as EEG, whereas other conventional Fourier and wavelet based schemes need a pre-defined basis functions. In addition, since the IMFs obtained by EMD have a good de-correlating property, the resulting information measure is capable of separately assessing the clinical bands of interest without any need of external filter. Further, Shannon entropy evaluated from each IMF is able to reflect the degree of information of the corresponding oscillatory state. Utilizing a data-driven (adaptive) decomposition tool, i.e., EMD, it make possible to capture locally changing feature from fine to coarse scales of EEG. Following evaluation of entropy using probability distribution of IMFs at each subscale, it leads an effective quantitative measure of both spectral and temporal changes in EEGs. To the end, this study lays the foundation for applying this novel approach to clinical studies of EEG signals recorded during comparable episodes of brain injury.

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