

Inversion of Electrical Resistivity Data: A Review

Shrey Sharma, Gunjan Kumar Verma

Abstract—High density electrical prospecting has been widely used in groundwater investigation, civil engineering and environmental survey. For efficient inversion, the forward modeling routine, sensitivity calculation, and inversion algorithm must be efficient. This paper attempts to provide a brief summary of the past and ongoing developments of the method. It includes reviews of the procedures used for data acquisition, processing and inversion of electrical resistivity data based on compilation of academic literature. In recent times there had been a significant evolution in field survey designs and data inversion techniques for the resistivity method. In general 2-D inversion for resistivity data is carried out using the linearized least-square method with the local optimization technique. Multi-electrode and multi-channel systems have made it possible to conduct large 2-D, 3-D and even 4-D surveys efficiently to resolve complex geological structures that were not possible with traditional 1-D surveys. 3-D surveys play an increasingly important role in very complex areas where 2-D models suffer from artifacts due to off-line structures. Continued developments in computation technology, as well as fast data inversion techniques and software, have made it possible to use optimization techniques to obtain model parameters to a higher accuracy. A brief discussion on the limitations of the electrical resistivity method has also been presented.

Keywords—Resistivity, inversion, optimization.

I. INTRODUCTION

THE inverse problem in resistivity interpretation was described first when Slichter reported a method of interpretation of resistivity data over a layered earth using Hankel's Fourier-Bessel inversion formula [55]. It gives a unique solution if the resistivity is a continuous function of electrode spacing. In practice, resistivity measurements are limited to a small number of readings taken at discrete electrode spacing. Thus a unique resistivity response does not exist [8]. Vozoff used this method on field and synthetic data generated for three- and four-layer models [63]. Zohdy proposed a method of direct resistivity-interpretation which is valid for noisy data as well [71]. However, none of these earlier investigations deal with existence, uniqueness, construction and stability, which are important concerns and must be dealt with within any inverse problem. Backus and Gilbert introduced a linear inverse theory for geophysical problems [1]. They thoroughly discussed model resolution, least-squares fit of the data and solution uniqueness. The method is valid even for noisy or insufficient data, and they quantified the trade-off between resolution and stability for solutions to inverse problems. Following Backus and Gilbert's

work, generalized linear inverse theory was described by Jackson in terms of linear algebra [33], [34].

Since the forward problem of electrical soundings for stratified media was solved by means of the linear filter theory many articles have appeared dealing with automatic and numerical interpretation [24]. This approach looks for a fit between the experimental and the theoretical data in a least-squares sense, either in the resistivity transform domain or in the apparent resistivity domain. Different error functions have been proposed and different minimization procedures considered. The best known are the steepest descent method [63], [4], [46] and the methods based on the algorithm of the generalized inverse matrix [32]. Due to the non-linear nature of this problem, an iterative procedure improves results. In recent times due to increase in computational capacity a number of optimization methods have been used in this context.

However, some problems dealing with the interpretation of electrical soundings have not been solved yet for example: equivalence in models [46], [51]. Therefore, when a solution is obtained it may not correspond to the geological reality, or worse, a priori parameters or constraints included in the initial model may be modified in the final solution.

II. THEORY: BASIC PRINCIPLES OF THE RESISTIVITY METHOD

The relationship between the electrical resistivity, current and the electrical potential is governed by Ohm's law. To calculate the potential in a continuous medium, the form of Ohm's Law, combined with the conservation of current, as given by Poisson's equation is used. The potential due to a point current source located at x_s is given by:

$$\nabla \cdot \frac{1}{\rho(x, y, z)} \nabla \Phi(x, y, z) = -\frac{\partial j_c}{\partial t} \delta(x_s) \quad (1)$$

where ρ is the resistivity, Φ is the potential and j_c is the charge density. The potential at any point on the surface or within the medium can be calculated if the resistivity distribution is known. For 2-D and 3-D models, analytical methods are used for simple structures such as a cylinder or sphere in a homogeneous medium [64]. Boundary and analytical element methods [57] can also be used for more general structures but are usually limited to models where the subsurface is divided into a relatively small number of regions. For modeling of field data, the finite-difference and associated finite volume methods [18], [19] and the finite-element method [7] are more commonly used. By using a sufficiently fine mesh and the proper boundary conditions, an accurate solution for the potential over complex distributions of resistivity can be obtained.

Shrey Sharma is with Indian Institute of Technology Kharagpur- India (corresponding author to provide phone: (+91)-8293657041; e-mail: shrey@iitkgp.ac.in).

Gunjan Kumar Verma is with Indian Institute of Technology Kharagpur.

The purpose of the resistivity method is to calculate the electrical resistivity of the subsurface, which is an unknown quantity. The basic data from a resistivity survey are the positions of the current and potential electrodes, the current (I) injected into the ground and the resulting voltage difference (V) between the potential electrodes. The current and voltage measurements are then converted into an apparent resistivity (ρ_a) value using:

$$\rho_a = k \frac{\Delta V}{I} \quad (2)$$

where k is the geometric factor that depends on the configuration of the current and potential electrodes [46]. Equation (2) represents the simplest form of the inverse problem and assumes that the earth is homogeneous for each combination of current and potential measurements. The suitability of an array depends on many factors; among which are its sensitivity to the target of interest, signal-to-noise ratio, depth of investigation, lateral data coverage and more recently the efficiency of using it in a multichannel system. The advantages and disadvantages of the different arrays are discussed in various papers such as in [15], [12].

III. DATA ACQUISITION – DIFFERENT TYPES OF SURVEYS

A. One Dimensional Systems

From the 1920's to the late 1980's there were essentially two surveying techniques used- Lateral profiling and vertical sounding. In a profiling survey, the distances between the electrodes were kept fixed and the four electrodes were moved along the survey line. In the sounding method the center point of the electrodes array remained fixed but the spacing between the electrodes was increased to obtain information about the deeper sections of the subsurface [59].

The modern application of resistivity processing includes inverse modeling. A commonly used method for sounding data inversion is the damped least-squares method [30] [31], based on:

$$(J^T J + \lambda I) = J^T \Delta g \quad (3)$$

where the discrepancy vector g contains the difference between the logarithms of the measured and the calculated apparent resistivity values and q is a vector consisting of the deviation of the estimated model parameters from the true model. Here, the model parameters are the logarithms of the resistivity and thickness of the model layers. J is the Jacobian matrix of partial derivatives of apparent resistivity with respect to the model parameters. λ is a damping or regularization factor that stabilizes the ill-condition Jacobian matrix usually encountered for geophysical problems.

The major drawback of the sounding method is the assumption that there are no lateral changes in the resistivity. It is useful in geological situations where this is approximately true, but gives inaccurate results where there are significant

lateral changes. The effect of lateral variations on the sounding data can be reduced by using the offset Wenner method [2] but for more accurate results the lateral changes must be directly incorporated into the interpretation model.

B. Multi-Electrode Systems

Pelton et. al. developed an algorithm for inversion of 2D resistivity and induced polarization (IP) data using a method to calculate solutions of the forward problem for the range of anticipated model parameters (i.e. depth of overburden, thickness and width of anomalous body, resistivity of anomalous body, resistivity of the host rock and center of the body) [49]. It uses spline interpolation between the various stored forward models. Smith and Vozoff proposed a 2D resistivity inversion using a finite difference technique [56]. Their schemes are similar and suitable for complex 2D models, but do not incorporate the effects of topography on resistivity data in the inversion scheme. A terrain correction has been advocated by Holcombe and Jiracek but it may not be possible for a complex 2D model to completely separate the resistivity anomalies because of topography from those associated with subsurface features by using a simple correction [29]. Tong and Yang developed an algorithm for 2D resistivity inversion where topography is considered in the model, allowing for the direct inversion without applying external corrections [60].

Shima developed an algorithm to invert resistivity data gathered over complex 2D structures, formulating the forward problem by the alpha centers method, and the inverse problem by steepest descent and Gauss-Newton methods [54]. His field investigations show that the method has good application for resistivity surveys in steep, mountainous areas.

Sen, Bhattacharya and Stoffa developed an algorithm for nonlinear inversion of resistivity data. This method does not require a good starting model but is computationally more expensive. They used the heat bath algorithm of simulated annealing in which the mean square error (difference between observed and synthetic data) is used as the energy function. The resulting correlation and covariance matrices indicated how the model parameters affect one another and are very useful in relating geology to the resulting resistivity values [53].

Daily and Owen describe a tomographic inversion scheme to image the resistivity distributions between boreholes from current and voltage data measured along the boreholes [17]. They used a finite element Newton-Raphson algorithm developed by [69]. Their test results indicate that the accuracy of image reconstruction and spatial resolution mainly depends on data errors.

Fry and Neuman introduced a technique to image subsurface features for resistivity problems using an impedance-computed tomography algorithm [45]. The algorithm employs the solution of the Poisson equation without assuming straight line behavior of the current flow paths.

TABLE I
CHARACTERISTICS OF DIFFERENT 2D ARRAYS CONFIGURATIONS TYPES

		Wenner	Wenner-Schlumberger	Dipole-dipole	Pole-Pole	Pole-dipole
1	Sensitivity of the array horizontal structures	****	**	*	**	**
2	Sensitivity of the array vertical structures	*	**	****	**	*
3	Depth of Investigation	*	**	***	****	***
4	Horizontal Data coverage	*	**	***	****	***
5	Signal Strength	****	***	*	****	**

Here each star represents degree of effectiveness of the method (4 stars being most effective)

A 2-D model that consists of a large number of rectangular cells is commonly used to interpret the data [41]. The resistivity of the cells is allowed to vary in the vertical and one horizontal direction, but the size and position of the cells are fixed. Again, different numerical methods can be used to calculate the potential values for the 2-D forward model. Inverse methods are then used to back calculate the resistivity that gave rise to the measured potential measurements. Starting from a simple initial model (usually a homogeneous half-space), an optimization method is used to iteratively change the resistivity of the model cells to minimize the difference between the measured and calculated apparent resistivity values. As an example, the following equation includes a model smoothness constraint to the least-squares optimization method:

$$(J^T + \lambda F)\Delta q_k = J^T \Delta g_k - \lambda F q_{k-1} \quad (4)$$

where $F = \alpha_x C_x^T C_x + \alpha_z C_z^T C_z$; C_x and C_z are the roughness filter matrices in the horizontal (x) and vertical (z) directions and α_x and α_z are the respective relative weights of the roughness filters. k represents the iteration number. One common form of the roughness filter is the first-order difference matrix, but the elements of the matrices can be modified to introduce other desired characteristics into the inversion model [20], [21]. Joint inversion algorithms using other geophysical or geological data to constrain the model have also been implemented to help produce models that are consistent with known information [5].

A 3-D resistivity survey and interpretation model should give the most accurate results as all geological structures are 3-D in nature. Although at present it has not reached the same level of usage as 2-D surveys, it is increasingly more widely used in complex areas for many environmental and engineering problems [9], [10]. This method uses reciprocity for efficient evaluations of the partial derivatives of apparent resistivity with respect to model resistivity. Loke et al. developed techniques to reduce the time needed to carry out 3D resistivity surveys with a moderate number (25 to 100) of electrodes by arranging them in a square grid and using pole-pole array for potential measurements [41]. They concluded that the number of measurements required could be reduced to about one-third of the maximum possible number without seriously degrading the resolution of the resulting inversion model by making measurements along the horizontal, vertical and 45° diagonal rows of electrodes passing through the current electrode. New data acquisition like multiple gradient

array have been designed for the multi-channelled systems [16].

Many of the early 3-D surveys used the pole-pole array over rather small grids (up to about 20 by 20 electrodes) with measurements in different directions [39]. The use of other arrays, such as the dipole-dipole and Wenner-Schlumberger, is now becoming more common in surveys that involve thousands of electrode positions.

In 4-D surveys, the change of resistivity in both space and time is measured. Measurements are repeated at different times using the same 2-D survey line or 3-D survey grid. A number of techniques have been proposed for the inversion of time-lapse data [44]. Techniques that incorporate regularization in space and time have been proposed to reduce inversion artifacts that may lead a misinterpretation of geophysical monitoring data. To alleviate this problem, Karaoulis et al. proposed an algorithm for inverting time-lapse resistivity monitoring data using 4D active time constrained resistivity inversion [36].

IV. OPTIMIZATION TECHNIQUES

Traditional inversion method is the most commonly used procedure for resistivity inversion, which usually takes the linearization of the problem and accomplish it by iterations. However, its accuracy is often dependent on the initial model, which can make the inversion trapped in local optima, even cause a bad result. Non-linear method is a feasible way to eliminate the dependence on the initial model [52]. However, for large problems such as 3D resistivity inversion with inversion parameters exceeding a thousand, main challenges of non-linear method are premature and quite low search efficiency.

A. Genetic Algorithms

Usually, a Genetic Algorithm for solving a particular problem has five major components:

1. Genetic representation of the problem
2. Tournament selection scheme
3. Crossover and mutation operators
4. Objective function
5. Termination criteria

The GA method has strong global search capability; however, for large inversion problem the main problem is low efficiency, which may cause bad solution. This problem may be solved using the mutation direction control method which is essentially a joint algorithm, in which the linearization method is embedded in GA because of its high local search capability. This joint algorithm is quite good at controlling

search direction and improving inversion efficiency, making the GA feasible for 3D resistivity inversion. Synthetic and practical examples have demonstrated that with GA method one can eliminate the dependence on initial model, and obtain excellent and high-quality inversion results. For a detailed discussion on the theory on this method the reader may refer to [25], [26], [35] and [67].

B. Artificial Neural Networks

Artificial neural network (ANN) with characteristics of learning and non-linear approximation is widely used in interpretation of geophysical data, such as pattern classification, rock recognition in well logs and inversions of gravity, seismic and electromagnetic data [58], [61]. However, most of geophysical inversions using NN are limited to one-dimensional models with a small number of parameters. El-Qady and Ushijima studied NN approach to solve 2-D resistivity inverse problems, but similarly a small number of resistivity data and model parameters in their approach make it difficult for the interpretation of complex models [23]. So the problems of large networks with complicated input/output patterns should be tackled in the application of NN approaches to 2-D geophysical problems.

Xu H L et. al. concluded that proper selection of the network, training samples and learning paradigms, the network can be well designed and trained to perform the large

inversion of 2-D resistivity data[68]. Results show that the NN non-linear inversion method can reconstruct the fine structures of the subsurface model and is better than the conventional 2-D inversion method.

C. Simulated Annealing

The Simulated Annealing (SA) method bears analogy with the physical annealing process in which a solid in a heat bath is warmed by increasing the temperature. This process is followed by slow cooling until the global minimum energy state is reached where it forms a crystal. Mathematically this involves drawing samples from a probability density function that is proportional to $\exp(-E(m)/T)$, where $E(m)$ is the energy function for a model m and T is the parameter called temperature. It can be shown that as the temperature is slowly reduced, then in the limit as T goes to zero, the minimum energy state (model) becomes overwhelmingly probable. The Metropolis [43] and the heat bath algorithms [50] are computer algorithms that do nearly the same thing without computing $E(m)$ at each point in model space.

Following the work of Kirkpatrick et al., the simulated annealing (SA) method has become very popular in multi parameter optimization problems including those of geophysical inversion [37]. A fairly detailed account of the subject including proof of asymptotic convergence is given by van [62].

TABLE II
COMPARISON BETWEEN VARIOUS OPTIMIZATION SCHEMES

Method	Neural Network	Simulated Annealing	Genetic Algorithm
Advantages	It can obtain an arbitrary close approximation to any continuous function, be it associated with a direct or an inverse problem. Once trained, the network can obtain the result of inversion rapidly from the network output while the observed data are input as the test data set.	It allows for considerable flexibility in model definition and parameterization and seeks a global rather than a local minimum in a misfit function. It has the added advantage in that it can be used to determine uncertainties. It is preferable over gradient methods because very good solution can be obtained even with poor starting models.	It is intrinsically parallel - Performs well in problems for which the fitness landscape is complex - ones where the fitness function is discontinuous, noisy, changes over time, or has many local optima. Another area in which genetic algorithms excel is their ability to manipulate many parameters simultaneously.
Limitations	Good enough training samples are required to guarantee over constrained training procedure. In cases where input is complicated it is very difficult to select training samples.	Very high computation cost is associated with the process. The issue of nonlinear uncertainty estimation can only be addressed using importance sampling. However most algorithms as implemented do not do an importance sampling.	The problem of how to write the fitness function must be carefully considered so that higher fitness is attainable and actually does equate to a better solution for the given problem. One well-known problem that can occur with a GA is premature convergence.

A further improvement was proposed by Chunduru et al. who used a Cauchy-like distribution, which is also a function of a control parameter called temperature [11]. The advantage of using such a scheme is that at high temperatures, the algorithm allows for searches far beyond the current position, while at low temperatures, it looks for improvement in the close vicinity of the current model. We have used the mean square error between the synthetics and original data as the error function to be minimized. The synthetic response for 2-D models was obtained by finite-difference modeling, and cubic splines were used to parameterize the model space to get smooth images of the subsurface and to reduce computational cost.

V. FIGURES

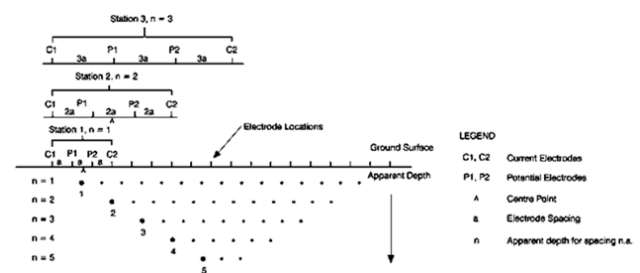


Fig. 1 Schematic layout for a 2d resistivity survey .By making measurements with different spacing at different locations along the cable, a two dimensional profile of the subsurface is obtained.

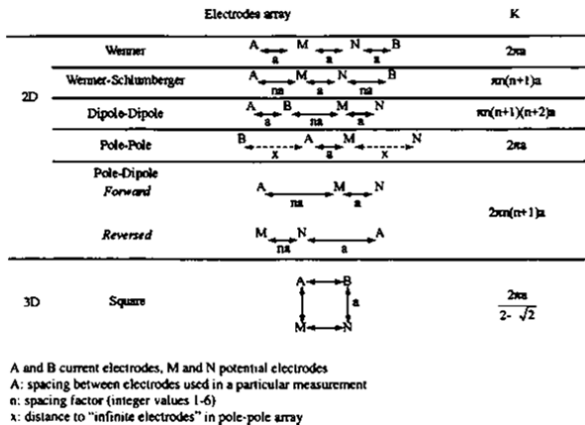


Fig. 2 Some commonly used electrode arrays and their geometric factors

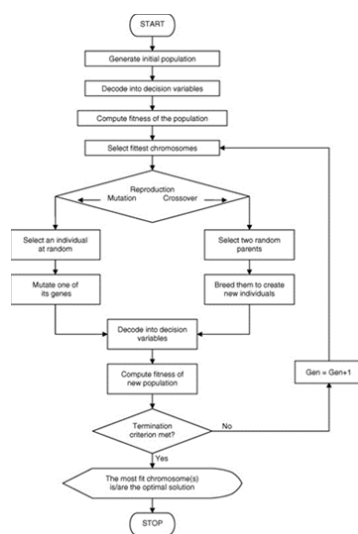


Fig. 3 A flowchart showing general steps involved a Genetic Algorithm Simulation

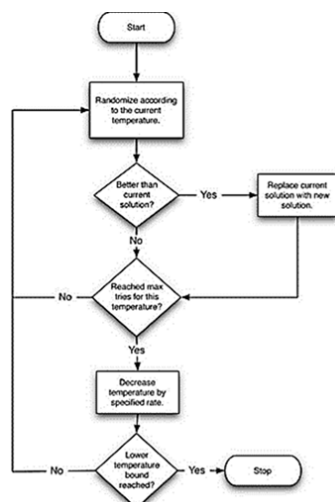


Fig. 4 General steps involved in a Simulated Annealing algorithm

VI. LIMITATIONS OF THE RESISTIVITY METHOD

A. Non-Uniqueness and Resolution of Data

The inverse resistivity problem has a unique solution for 1-D, 2-D and 3-D resistivity distributions within a boundary [22], but only under strict conditions where the voltage and current distributions are known continuously and precisely over the boundary for a complete set of current injection patterns. In practice, resistivity inversion is ill-posed and non-unique due to only being able to use a finite number of electrodes covering part of the surface. This implies that more than one resistivity model will produce responses consistent with the observed data to the limits of the data accuracy [51]. Regularization is used, often in the form of a smoothing to enforce uniqueness without sacrificing too much resolution. However regions of the inverted image where the model resolution is lower will depend more strongly on the type of constraint used [47], [48]. The inverse images will therefore only accurately reflect the true subsurface resistivity if the regularization constraints are realistic.

B. Data Quality

Measured data, and the resulting resistivity images are subject to error from a variety of sources including that introduced by the measurement device, poor electrode contact (usually identified through high contact resistances) or electrode polarization, and other indeterminate external effects [38]. These issues are addressed through the appropriate selection and conditioning of electrodes to reduce contact resistance, by using appropriate filters (including reciprocal error analysis) prior to inversion [70], and through employing measurement sequences that reduce the influence of electrode polarization. In practice, polarization can be reduced by ensuring adequate time between using an electrode to pass current and measure potential.

C. Electrode Position and Survey Design

For difficult ground conditions such as steep or heavily vegetated areas, it can be difficult to accurately position electrodes. Moreover monitoring unstable ground from land sliding, electrode positions can shift resulting in systematic data error which cannot be reduced through reciprocal error filtering [70]. Approaches to reduce the impact of these effects include the selection of measurement array geometries that are less sensitive to positional errors [65], and in the case of moving electrodes, to estimate electrode position using a position inversion routine [66].

For very long 2-D survey lines, and for 3-D imaging grids, it can be impractical to undertake measurements in a single deployment due to the long cable lengths required. 3-D surface surveys are often undertaken using a network of lines, where a single line is incrementally migrated across the surface to build up a measurement set comprising data from multiple lines [3]. Where a single line orientation is used, linear features parallel to the line direction can be poorly resolved and banding or herring-bone effects can be present in the model [40]. Mitigation measures include: roll-along (or multiple line) data acquisition methodologies [13], [14];

orthogonal line directions and line separations of no more than two electrode spacing; and appropriate inversion settings (e.g. horizontal diagonal roughness filters) [20].

D.Effect of Anisotropy

Typical geological causes of anisotropy include fracturing, jointing, layering and rock fabric (e.g. strong alignment of grains slates and shales). Detection of anisotropy has typically been undertaken using azimuthal [6] and square array [42] techniques. However, the most effective approach is perhaps surface to borehole imaging arrays, which unlike surface arrays, are sensitive to anisotropy associated with vertical or sub-vertical axes of symmetry, such as horizontal layering [28]. The influence of even moderately anisotropic media on the results of resistivity inversion that assumes isotropic conditions can produce significant distortions in electrical. Anisotropic resistivity inversion schemes have been developed [27]; but they are limited by number of factors, including the requirement to solve for additional parameters that exacerbates the problem of non-uniqueness, and the need for subsurface electrodes to effectively detect anisotropy associated with horizontal layering.

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