

# Nonlinear Modeling of the PEMFC Based On NNARX Approach

Shan-Jen Cheng, Te-Jen Chang, Kuang-Hsiung Tan, Shou-Ling Kuo

**Abstract**—Polymer Electrolyte Membrane Fuel Cell (PEMFC) is such a time-vary nonlinear dynamic system. The traditional linear modeling approach is hard to estimate structure correctly of PEMFC system. From this reason, this paper presents a nonlinear modeling of the PEMFC using Neural Network Auto-regressive model with eXogenous inputs (NNARX) approach. The multilayer perceptron (MLP) network is applied to evaluate the structure of the NNARX model of PEMFC. The validity and accuracy of NNARX model are tested by one step ahead relating output voltage to input current from measured experimental of PEMFC. The results show that the obtained nonlinear NNARX model can efficiently approximate the dynamic mode of the PEMFC and model output and system measured output consistently.

**Keywords**—PEMFC, neural network, nonlinear identification, NNARX.

## I. INTRODUCTION

FUEL CELL (FC) technologies development and commercialization motivation is concerned with increasing environment and resource issues. Polymer Electrolyte Membrane Fuel Cell (PEMFC), as a renewable energy source, is one of the most promising fuel cells due to their compact modular, high efficiently and good stability. Because of its advantage, PEMFC is demanded as a dependable power sources for many application such as distributed power generation and automobile [1], [2].

PEMFC is an extremely complex nonlinear multi-input and multi-output and coupled dynamic system. The performance of PEMFC can be represented by a current-voltage relation that is influenced by levels of internal influential parameters such as gas flow channel design, relative humidity ratio, operation temperature or pressure, stoichiometric flow rate, and others. All these parameters have strong impacts on PEMFC performance, and are related to each other by nonlinear behaviors. The inner working processes are accompanied with liquid, vapor, gas-mixed transportation, heat conduction and electrochemical dynamic reaction. For such kind of nonlinear system of PEMFC, yet there is no standardized procedure neither to estimate a matching mode structure not to select a

suitable types of models. During the last several decades, various mechanism models of PEMFC, based on mass, energy and momentum conservation laws, has received much attention in an attempt to better understand the phenomena occurring within the cell, and a variety of mechanism models have been established in previous research [3], [4]. In open literatures, these models characteristics focused on FC operating condition such as temperature effects, reaction gas transportation phenomena, heat management, etc. Each parameter with according to the operating conditions will exert different effects to improve the performance and define quantitative determination whether the effects of operating factors are necessary on the PEMFC. These models are very useful for analyzing the transient characteristic, but they are too complicated to be used for control system design.

For the purpose of dynamic control of real system in future work, precise dynamic characteristic model of the PEMFC are necessary. However, no matter what kind of models, there must be some errors between the models and real performance of the PEMFC because assumptions and approximations are made in modeling for computing simplify. In order to improve the accuracy of mechanism models and make the models reflect the actual PEMFC performance better, it is necessary to mode the structure of the models using nonlinear model approach. Most dynamic systems can be better described by nonlinear models, which are able to present the whole behavior of the system during the all operating condition [5]-[7]. Motivated by this need, an attention has been paid to identification of nonlinear dynamical systems. The nonlinear dynamic systems behavior has made the employ of Artificial Neural Network (ANN) for the modeling task in recent decades [8], [9]. In addition, all the numerical studies have proven the multilayer perceptron (MLP) neural networks match very well for nonlinear system identification.

In this work, a nonlinear model approach, consisting of a Neural Network Auto-regressive model with eXogenous inputs (NNARX) approach is adopted to model the nonlinear dynamic of the PEMFC. The paper organized as follows: Section II gives a description of NNARX model approach. Section III presents the results of modeling of PEMFC based on NNARX approach. Section IIII is the conclusion. The proposed nonlinear modeling of the PEMFC based NNARX approach procedure is graphically summarized in Fig. 1.

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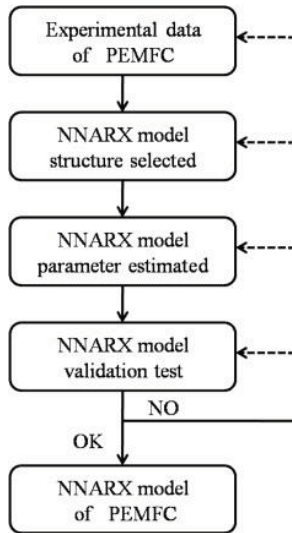


Fig. 1 Nonlinear model of PEMFC procedure

## II. NEURAL NETWORK AUTO-REGRESSIVE MODEL WITH EXOGENOUS INPUTS (NNARX) MODELING APPROACH

Modeling is an important issue in the process of parameter estimation. Auto-regressive eXogenous models have been employed extensively to represent the relationship of the system output with the system input in the presence of noise in many linear systems. In the process of parameters estimation, the Levenberg-Marquardt (LM) algorithm is usually used in neural networks (NN) method. In order to meet a closer approximation to the real system, nonlinear ARX models are used, which are modeled by means of NN. The Multilayer perceptron (MLP) network is one of the most studied members in the NN. The primary of MLP neural network reason is its ability to model simple as well as complex functional relationships. The LM algorithm minimizes the mean-square error of the prediction errors for the nonlinear ARX model, which is as particular case of a nonlinear neural network ARX model (NNARX), as described in after [10]-[13].

### A. NNARMAX Model

A general linear system ARX empirical model can be described by:

$$A(q^{-1})y(k) = B(q^{-1})u(k) + e(k) \quad (1)$$

where  $y(k)$  denotes the system output or autoregressive (AR) factor;  $u(k)$  is the system input or exogenous (X) factor,  $e$  is the white noise or disturbance and  $q^{-1}$  negative shift operator. The polynomials  $A(q)$  and  $B(q)$  are given by:

$$\begin{aligned} A(q) &= 1 + a_1 q^{-1} + \dots + a_{na} q^{-na} \\ B(q) &= b_0 + b_1 q^{-1} + \dots + b_{nb} q^{-nb} \end{aligned} \quad (2)$$

The model of system corresponding predictor:

$$\hat{y}(k|\theta) = -a_1 y(k-1) - \dots - a_{na} y(k-n_a) + b_1 u(k-n_k) + \dots + b_{nb} u(k-n_k-n_b+1) \quad (3)$$

is thus based regression vector:

$$\vartheta(k) = [y(k-1) \dots y(k-n_a) u(k-n_k) \dots u(k-n_k-n_b+1)] \quad (4)$$

where  $n_a$  is number of output poles;  $n_b$  is the number of input zeros and  $n_k$  is the system time delay. In order to estimate the parameter of nonlinear ARX model structure, the NN can be done. The neural network version of ARX model structure is defined as Neural Network ARX (NNARX). The NNARX model structure is presented in Fig. 2. The relationship between input-output structures of NNARX mode can be shown by

$$y(k) = g[\varphi(k), \theta] + e(k) \quad (5)$$

The one-step-ahead (OSA) prediction of the NNARX model structure is defined by

$$\hat{y}(k|\theta) = g[\varphi(k), \theta] \quad (6)$$

where  $g$  is the function realized by the multilayer perceptron neural network method.

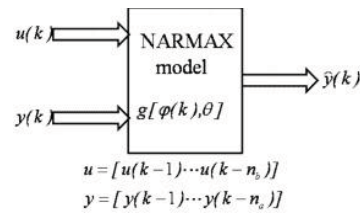


Fig. 2 NNARX model structure approach

### B. Multilayer Perceptron (MLP) Network

The multilayer perceptron (MLP) network is one of most used of the NN family; because of its enable simply represent complex function. The class of MLP NN meted with three layers: an input, an output and hidden layer. In the hidden layer ( $j$ ) of each neuron, the sums up of input data  $x_i$  after weighting them with strengths of the respective connections  $w_{ji}$  from the input layer and computed output  $y_j$  as a function of the sum:

$$y_j = f\left(\sum_{i=1}^q w_{ji} x_i\right) \quad (7)$$

where the function  $f(\cdot)$  can be linear, threshold, sigmoid, hyperbolic tangent and radial basis. In this paper, hyperbolic tangent functions are considered for the neurons in the hidden layer and linear function for the output layer neurons, respectively. The output of the MLP presented:

$$\hat{y}_i(w, W) = F_i\left(\sum_{j=1}^q W_{ij} \cdot f_j\left(\sum_{i=1}^m w_{ji} X_i + w_{j0}\right) + W_{i0}\right) \quad (8)$$

where  $q$  is hidden neurons,  $w_{ji}$  between input and hidden neuron weighting,  $W_{ij}$  between hidden neuron and output weighting and  $m$  is input number. The weighting  $w$  and  $W$  of are the adjustable parameter of the network and determine through the training process. The sets of training inputs data  $u(t)$  and corresponding outputs  $y(t)$  defined

$$Z^N = \{[u(k), y(k)] \mid k = 1, \dots, N\} \quad (9)$$

The goal of training is to meet a mapping from the training data set to the set of possible weights  $Z^N \rightarrow \theta$ , so that the network will produce the close to the true outputs  $y(k)$ . The prediction error measurement is often described by a function required as the loss function. The general form can be depicted as

$$P_N(\theta, Z^N) = \frac{1}{2N} \sum_{k=1}^N \varepsilon^2(k, \theta) \quad (10)$$

In (10) is used to simplify differentiation when minimizing residual  $\varepsilon(k, \theta) = y(k) - \hat{y}(k, \theta)$ .  $Z^N$  is mean the training data set. The minimizing solution implements the Levenberg-Marquardt (LM) algorithm, due to its rapid convergence properties and robustness.

### C. The Levenberg-Marquardt (LM) Algorithm

The LM algorithm is the iterative numerical process in realizing solution. In this paper, NNARX model of PEMFC is obtained by LM algorithm. For minimal of sum of the squares  $\varepsilon(k, \theta)$ , the LM algorithm is used in optimizing the parameter vector  $\theta$ . The linear approximation to the  $i$ th residual of the LM algorithm at iteration is given by:

$$\hat{\varepsilon}(k, \theta) = \varepsilon(k, \theta^i) + [\varepsilon'(k, \theta^i)](\theta - \theta^i) \quad (11)$$

and the criterion id given by

$$L^i(\theta) = \frac{1}{2N} \sum_{k=1}^N \hat{\varepsilon}^2(k, \theta) \approx P_N(\theta, Z^N) \quad (12)$$

The gradient and Hessian for  $\theta = \theta^i$  are given by:

$$\begin{aligned} G(\theta^i) &= (L^i(\theta))^' \\ R(\theta^i) &= (L^i(\theta))'' \end{aligned} \quad (13)$$

The next iteration parameter vector is defined  $\theta^{i+1} = \theta^i + f^i$ . The search direction  $f^i$  is computed by

$$[R(\theta^i + \lambda^i I)]f^i = -G(\theta^i) \quad (14)$$

That damping factor  $\lambda$  is computed at each iteration. The

reduction of  $P_N(\theta, Z^N)$  is determined based on the ratio:

$$r^i = \frac{P_N(\theta^i, Z^N) - P_N(\theta^i + f^i, Z^N)}{P_N(\theta^i, Z^N) - L^i(\theta^i + f^i)} \quad (15)$$

The stopped criterion in iterative compute process of LM algorithm is satisfied convergence value.

## III. RESULT AND DISCUSSION

In these work, the identification process was presented by the widespread mathematical software package MATLAB, provided by the MathWorks Inc. [14]. The steady output voltage of power source of PEMFC is an important. PEMFC experimental data was recorded during various step load of current. The data were divided into two sets, one for training and remaining for validation. The input-output data is presented in Fig. 3. [15]

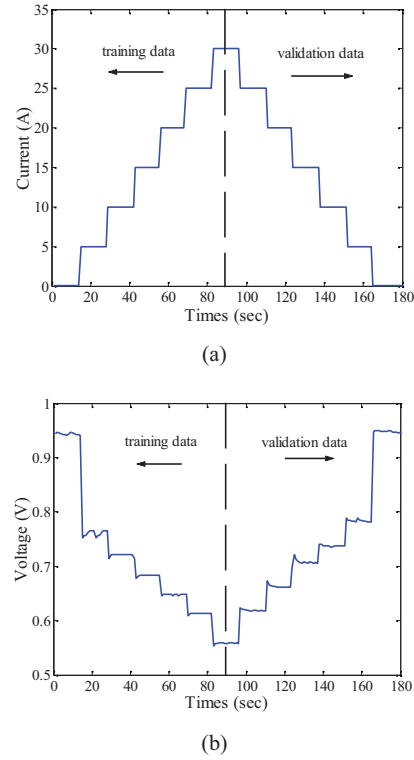


Fig. 3 Input and output experimental data of PEMFC

The NNARX model of PEMFC was employed for divided set data and it shows the modeling of second order nonlinear dynamic system of PEMFC. In order to model a good structure of PEMFC, the model structure parameter selection of system which we wish to employ is done firstly. In the process of model structure selection of NNARX model consists of two sub problems: a regressor parameter selection and MLP network architecture selection. As regressor structure selection is used two past inputs ( $n_b$ ) and two past outputs ( $n_a$ ) in this work as shown in Fig. 4.

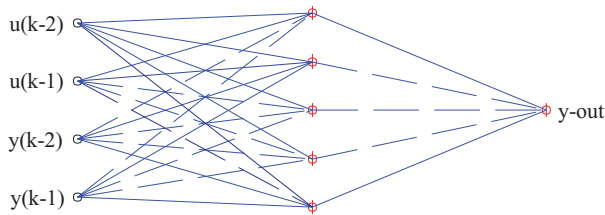


Fig. 4 Network architecture of NNARX model of PEMFC

Furthermore, the MLP architecture was selected five tanhhyperbolic (tanh) neurons in hidden layer, and a single linear neuron in output layer. Fig. 5 is the results of iteration criterion NNARX modeling which demonstrated a good training convergence value. The comparison of experimental data of PEMFC and NNARX model over training set data of patterns are presented in Fig. 6. From the residual plot in training process, the trained NNARX model output OSA is in good agreement with the experimental output of PEMFC.

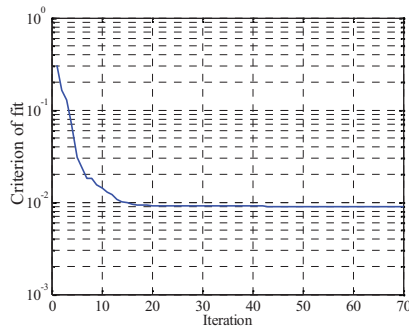


Fig. 5 Iterative criterion of fit with training process

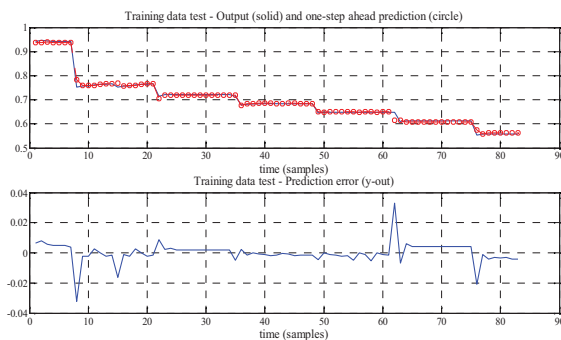


Fig. 6 Training test of NNARX model

Fig. 7 shows the experimental data of PEMFC and NNARX model OSA in validation procedure, the NNARX approach has ability to mode the real system data. The residual shows that mode prediction and experimental output is closer.

The residuals of autocorrelation and cross correlation with NNARX validation process plot in Fig. 8. The results show the residual pass the correlated coefficient of 95% confidence limited in autocorrelation process; the signification correlation between the input and residual was completely modeled in cross correlation process. That indicated NNARX model approach can fit accuracy of the PEMFC system.

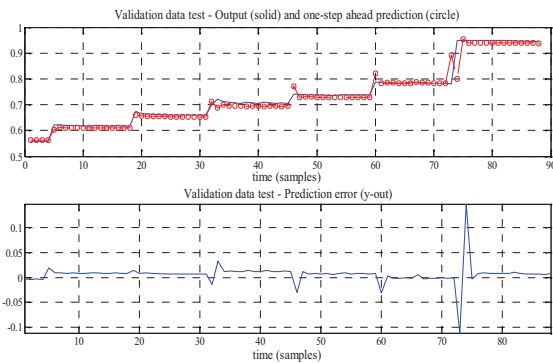


Fig. 7 Validation test of NNARX model

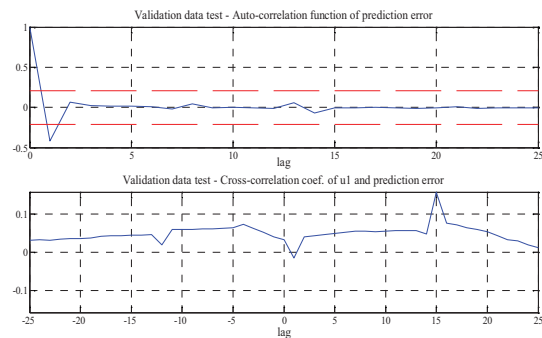


Fig. 8 Auto and cross correlation function of output prediction error

#### IV. CONCLUSION

In this paper, the nonlinear modeling of PEMFC is applied via Neural Network Auto-regressive model with eXogenous inputs (NNARX) approach. From the results of training process, the LM algorithm of MLP neural network can fit training iteration criterion convenience. In NNARX mode with small network architecture was founded to be adequate to model the PEMFC dynamic system. Applying the validation tests, the NNARX model could pass the residual test and cross correlation tests. This work demonstrates that the nonlinear modeling approach, employing the NNARX model, provides a very simple and yet highly accurate model of the PEMFC system.

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