Material Parameter Identification of Modified AbdelKarim-Ohno Model

M. Cermak, T. Karasek, J. Rojicek

Abstract—The key role in phenomenological modelling of cyclic plasticity is good understanding of stress-strain behaviour of given material. There are many models describing behaviour of materials using numerous parameters and constants. Combination of individual parameters in those material models significantly determines whether observed and predicted results are in compliance. Parameter identification techniques such as random gradient, genetic algorithm and sensitivity analysis are used for identification of parameters using numerical modelling and simulation. In this paper genetic algorithm and sensitivity analysis are used to study effect of 4 parameters of modified AbdelKarim-Ohno cyclic plasticity model. Results predicted by Finite Element (FE) simulation are compared with experimental data from biaxial ratcheting test with semi-elliptical loading path.

Keywords—Genetic algorithm, sensitivity analysis, inverse approach, finite element method, cyclic plasticity, ratcheting.

I. INTRODUCTION

EXPERIMENTAL measurement of any kind always bears certain level of uncertainty. The more complex the measured phenomena is the higher level of uncertainty brings to the measured values. Huge number of experiments would have to be performed to obtain good knowledge and understanding how different material parameters and their combination affect final result. Numerical modelling and simulation could help to reduce number of those experiments and also could give us better insight into this problematic.

In this paper, we focus on FE modelling of the phenomenon called ratcheting (cyclic creep). It can be described as the accumulation of plastic deformation in a component or specimen under cyclic loading. The ratcheting may occur in practice for instance in the rolling/sliding contact.

One of the first plasticity models, which can qualitatively capture ratcheting in numerical calculations, is Chaboche model [1]. Cyclic plasticity models have been extensively developed over the past three decades. The most popular kinematic hardening rules introduced into new constitutive theories are Ohno-Wang model II [2] and AbdelKarim-Ohno model [3].

The main aim of this contribution is comparison of various approaches to cyclic plasticity model calibration with emphasize on ratcheting. Two algorithms have been applied genetic algorithm and a sensitivity analysis to estimate 4 parameters of modified AbdelKarim-Ohno model [4] using a fatigue test conducted under non-proportional loading.

II. EXPERIMENTAL DATA

To compare effectiveness of two different algorithms for material parameters estimation a multi-axial cyclic test was realized on reconstructed electro-servo-hydraulic system INOVA 100kN/1000Nm at the VSB-Technical University of Ostrava [5].

The semi-elliptical loading path (Fig. 1) was gained by the symmetric tension/compression and simultaneous repeated torsion applied as harmonic function of time with 90° of phase shift. The frequency of loading was 0.1 Hz.

The test was proposed by McDowell [6] and simulates the stress-strain history in a point on a semi-infinite elastic-plastic half plane loaded by repeated Herzian pressure with Coulomb friction assumption. The case with axial stress magnitude of 625MPa and shear stress magnitude of 328 MPa was realized.

For experiment a tubular specimen made of Class C wheel steel was produced. The outer diameter was 12.5mm, while the inner diameter was 10 millimetres.

The shear strain and axial strain were measured simultaneously by extensometer EPSILON 3550 with the gauge length of 25 millimetres.

The stress-strain hysteresis loops evaluated for 20 cycles are presented at Figs. 1 and 2. It could be mentioned from Figs. 1 and 2 that the shear strain accumulation occurs cycle by cycle in the same direction as the torque is applied, because of a non-zero value of mean torque.
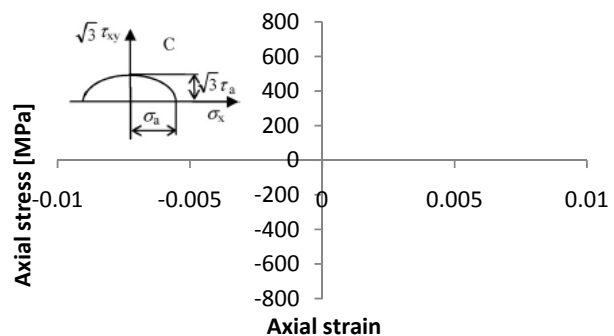

Fig. 1 Axial stress-strain hysteresis loops from biaxial fatigue test [7]

M. Cermak and T. Karasek are with the IT4Innovations, VSB - Technical University of Ostrava, 17. listopadu 15/2172, Ostrava, Czech Republic (e-mail: martin.cermak@vsb.cz, tomas.karasek@vsb.cz).

J Rojicek is with the Faculty of Mechanical Engineering, VSB - Technical University of Ostrava, 17. listopadu 15/2172, Ostrava, Czech Republic (e-mail: jaroslav.rojicek@vsb.cz).
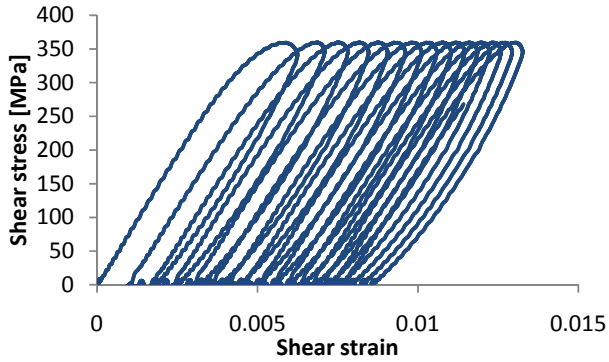
Fig. 2 Torsional stress-strain hysteresis loops from biaxial fatigue test [7]

III. Model Description

To describe material behaviour in our numerical models modified nonlinear AbdelKarim-Ohno model was used. Plastic behaviour is characterized by von Mises plasticity condition and could be described by

$$f = \sqrt{\frac{3}{2}(\mathbf{s}-\mathbf{a}):(\mathbf{s}-\mathbf{a})} - Y = 0, \qquad (1)$$

where $\mathbf{s}$ is the deviatoric part of stress tensor $\boldsymbol{\sigma}$, a is deviatoric part of kinematic tensor $\boldsymbol{\alpha}$ and Y is the isotropic variable corresponding to the radius of yield surface

$$Y = \sigma_Y + R, \qquad (2)$$

where σ_Y is the initial size of elastic region and R is the isotropic variable.

As it is well known, it is necessary to consider kinematic hardening rule for description of Bauschinger effect [8].

New kinematic hardening rule introduced specially for ratcheting with steady state was published by AbdelKarim and Ohno [3]

$$\boldsymbol{\alpha} = \sum_{i=1}^{M}\boldsymbol{\alpha}_i, \quad \mathbf{d}\boldsymbol{\alpha}_i = \frac{2}{3}C_i \mathbf{d}\boldsymbol{\varepsilon}_p - \mu_i\gamma_i\boldsymbol{\alpha}_i dp - \gamma_i H(f_i)\langle d\lambda_i\rangle\boldsymbol{\alpha}_i, (3)$$

where C_i, γ_i and μ_i are material parameters, $H(f_i)$ marks Heavisides step function ($H(f_i) = 1$, if $f_i = 0$ and $H(f_i) = 0$ if $f_i < 0$) whereas the function f_i is defined by

$$f_i = \frac{3}{2}\boldsymbol{\alpha}_i:\boldsymbol{\alpha}_i - \left(\frac{C_i}{\gamma_i}\right)^2 \qquad (4)$$

and

$$d\lambda_i = \mathbf{d}\boldsymbol{\varepsilon}_p:\frac{\boldsymbol{\alpha}_i}{C_i/\gamma_i} - \mu_i dp, \quad 0 \le \mu_i \le 1. \qquad (5)$$

The symbol $\langle x \rangle$ marks Macaulay's bracket ($\langle x \rangle = 0$, if $x < 0$ and $\langle x \rangle = x$, if $x > 0$). Parameters μ_i have a great meaning in the model. On the other hand, if $\mu_i = 0$ for all i, AbdelKarim-Ohno model corresponds to Ohno-Wang I model, which always predicts plastic shakedown (no ratcheting) under uniaxial loading [2]. Thus, parameters μ_i influence ratcheting

strain rate. The only one parameter $\mu = \mu_i$ is usually used for all i because of simplification.

AbdelKarim-Ohno cyclic plasticity model has some disadvantages too. It gives noncorrect results for multiaxial ratcheting when it is calibrated from uniaxial ratcheting test and vice versa. The second handicap is the possibility of simulations of ratcheting with steady state only if the parameter μ is constant during loading.

The transient effect in initial cycles, which occurred for some materials, can be described by evolution of parameter μ using

$$d\mu = \omega(\mu_\infty - \mu)dp, \qquad (6)$$

where μ_∞ is the target value of μ, ω is the evolution coefficient and the initial value of μ is μ_0. Next proposed modification of AbdelKarim-Ohno model is idea to express the parameters μ_i in following form

$$\mu_i = \mu \langle n: \frac{\alpha_i}{\alpha_i}\rangle^\chi, \qquad (7)$$

where

$$\bar{\alpha}_i = \sqrt{\frac{3}{2}\boldsymbol{\alpha}_i:\boldsymbol{\alpha}_i}, \quad n = \frac{\mathbf{d}\varepsilon_p}{dp}. \qquad (8)$$

The term in Macaulay's bracket is always less than 1 under nonproportional loading and equal to 1 under proportional loading (tension-compression, torsion and so on). Now it is clear, that choice of multiaxial parameter χ influence only ratcheting under nonproportional loading. Sometimes it is useful to introduce the evolution rule for multiaxial parameter too

$$d\chi = \omega(\chi_\infty - \chi)dp. \qquad (9)$$

For simplicity we consider zero value of the parameter χ. In this contribution, we introduce also cyclic hardening term in kinematic hardening rule by

$$\gamma_i = \bar{\gamma}_i \cdot \varphi(p), \qquad (10)$$

where

$$\varphi(p) = \varphi_\infty + (1 - \varphi_\infty)e^{-\omega_\varphi p}. \qquad (11)$$

Certain materials show additional hardening due to nonproportional loading. In such cases it is useful to use also a nonproportional parameter

$$A = 1 - \frac{(\mathbf{a}:\dot{\mathbf{a}})^2}{(\mathbf{a}:\mathbf{a})(\dot{\mathbf{a}}:\dot{\mathbf{a}})} \qquad (12)$$

in the nonlinear isotropic hardening rule

$$\dot{R} = b(Q - R)\dot{p}, \quad \dot{Q} = d \cdot A \cdot (Q_{AS}(A) - Q)\dot{p}, \qquad (13)$$

$$Q_{AS}(A) = \frac{gAQ_\infty}{gA+(1-A)}. \qquad (14)$$

The described modified AbdelKarim-Ohno model had to be coded into the FE software ANSYS as a user material

subroutine. We use 5 backstress parts in the model (M=5). All material parameters fixed in the analysis are stated in the Table I.

TABLE I
MATERIAL PARAMETERS OF MAKOC MODEL

$E = 205000 MPa; \mu = 0.3; \bar{y}_{1-5} = 2222,690,215,103,67;$
$C_{1-5} = 190234,86250,22463,7478,13810\ MPa;$
$d = 1; b = 10; g = 0.2; \mu_i = 0.12 \quad for\ all\ i$

We consider only these parameters for the optimization procedures:

C1 = σ_Y,

C2 = φ_∞,

C3 = ω_φ,

C4 = Q_∞ .

IV. METHODS DESCRIPTION

Several techniques for parameter identification such as random gradient, genetic algorithm, and sensitivity analysis are popular among researchers [9] [10]. In this paper genetics algorithm and sensitivity analysis are used and results from FE analysis are compared with experimental measurements.

Genetic algorithms are widely popular among researchers. Their advantage is that for well-defined problem number of iteration needed to reach global minimum could be quite low. How the genetic algorithm works could be briefly described by following steps:

In the first step a basic Group is created. Group is represented by the fixed number of Individuals where number of Individuals should be greater or at least equal than number of genes (number of parameters). Individuals in the Group are sorted by their Condition (estimated error). For each gene correlation based on the Condition in the Group is established.

In the next step new Individual is created. Property of Individual could be either random or could be results of crossbreeding. In the case of crossbreeding, properties of new Individual are based on properties of its parents.

First parent is randomly chosen from 3 best Individuals in the Group. Second parent is randomly selected from the rest of the Group. Properties (genes) of the new Individual are defined by properties (genes) of the parents.

Random properties are generated from values of the genes of best Individual in the Group. In this case higher scattering of values is used to overcome local minimum.

When new Individual is introduced to the Group, Individual with poorest quality of genes is removed from the Group.

This process is repeated until global minimum is found. In our case global minimum is defined as difference between measured and calculated values.

On the other hand sensitivity analysis due to high number of possible combinations, especially when large number of parameters and values of those parameters is used, could be seen as a brute force solution.

Number of all possible combinations for N number of parameters, and P values for each parameter could be calculated as

$$V_N^P = N^P. \qquad (15)$$

In our case when we have 4 parameters, and 5 values for each parameter number of all possible combinations is

$$V_5^4 = 5^4 = 625. \qquad (16)$$

The seeming disadvantage of this method i.e. high number of simulations could be surpassed by better understanding on how final results depends on input parameters and their combinations. Disadvantage of high number of simulations needed could be overcome by using supercomputers when many simulations could be run simultaneously and for large problems each simulation could be run in parallel.

V. NUMERICAL EXPERIMENTS

To carry out numerical experiments FE package ANSYS was used. Numerical model reproducing experimental set-up was created using only one axisymmetric structural shell element (SHELL 51). Since its axisymmetric element it has only two nodes with four degrees of freedom at each node: translations in the nodal x, y, and z directions and a rotation about the nodal z-axis. The element has plasticity, creep, swelling, stress stiffening, large deflection, and torsion capability. Shell element was modelled in such way that radial direction is coincident with x-axis of Cartesian coordinate system, axial direction with y-axis and tangential direction corresponds to the z-axis of Cartesian coordinate system.

As boundary conditions symmetry in axial direction i.e. displacements in y and z direction and rotational degree of freedom were set to zero. Nodal forces were applied according to experiment i.e. forces in y-direction and z-direction.

Material model used for numerical experiment is described in previous section. For sensitivity analysis values of parameters C1, C2, C3 and C4 are listed in Table II.

TABLE II
VALUES OF PARAMETERS FOR SENSITIVITY ANALYSIS

C1	450	475	500	525	550
C2	0.2	0.3	0.4	0.5	0.6
C3	10	30	50	70	90
C4	20	45	70	95	120

Since we could assume that simulation time for one calculation will be the same for both genetic algorithm and sensitivity analysis we use number of calculation need to find satisfactory results as a criterion for comparison of efficiency. Satisfactory results will be results for which difference between measured and calculated results is less than 1%

For genetics algorithm 670 number of iterations were needed to obtain combination of parameters which leads to solution satisfying our criteria described above. Fig. 3 shows comparison between numerical solution and experimental results. Maximal error is 0.73% and optimal values for input parameters are listed in Table III.

Sensitivity analysis needed 625 iterations as explained in previous chapter to test all possible combinations of the

parameters. The error for best combinations of the parameters is 1.5% and optimal value of input parameters are listed in Table III. Fig. 4 shows comparison between experimental results and results obtained from numerical simulation.
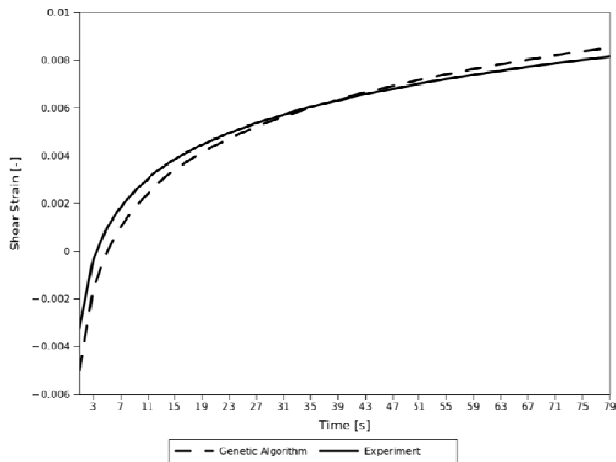

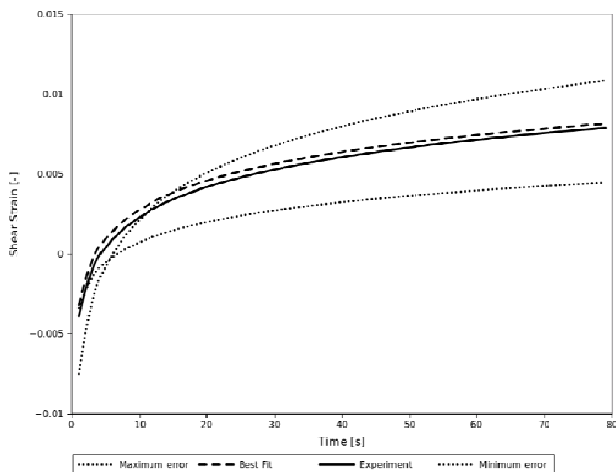Fig. 3 Comparison between numerical solution and experimental results (shear strain at peaks only)

Fig. 4 Comparison between experimental results and results obtained from numerical simulation (shear strain at peaks only)

TABLE III
COMPARISON OF THE BEST VALUE FOR GENETIC ALGORITHM AND SENSITIVITY ANALYSIS

PARAMETER	GEN. ALG.	SENS. ANAL.
C1	462.10 MPA	450.00 MPA
C2	0.2878	0.3000
C3	48.517	50.000
C4	50.291 MPA	45.000 MPA

VI. CONCLUSION

Results presented in previous section shows that both methods i.e. genetics algorithm and sensitivity analysis are able to produce results with required precision. Although sensitivity analysis did not meet criterion of difference between measured and calculated results it was only by

margin. This criterion could be easily fulfilled by running another analysis. We did not do this because for our purposes obtained results are satisfactory. It should be mentioned that number of iterations needed for genetic algorithm is higher than for sensitivity analysis which is fact which was not expected. On the other hand we need to mention that we consider only 4 parameters with 5 values. If we assume 9 parameters and the same number of values for each parameter, we get 1,953,125 number of iterations for sensitivity analysis and for genetic algorithm we can expected something around 1,000 iterations. Other examples and discussion we plan to present in the next articles. Based on results presented in this paper we could conclude that for small number of parameters sensitivity analysis is method which despite its seeming disadvantages could be equal alternative to methods using algorithms such as genetics algorithm.

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