

# Fault Detection and Isolation in Attitude Control Subsystem of Spacecraft Formation Flying Using Extended Kalman Filters

S. Ghasemi, K. Khorasani

**Abstract**—In this paper, the problem of fault detection and isolation in the attitude control subsystem of spacecraft formation flying is considered. In order to design the fault detection method, an extended Kalman filter is utilized which is a nonlinear stochastic state estimation method. Three fault detection architectures, namely, centralized, decentralized, and semi-decentralized are designed based on the extended Kalman filters. Moreover, the residual generation and threshold selection techniques are proposed for these architectures.

**Keywords**—Formation flight of satellites, extended Kalman filter, fault detection and isolation, actuator fault.

## I. INTRODUCTION

THERE are several advantages in satellite formation flying concept. The ability to make a formation more robust by eliminating single point failure is one of the most important advantages of formation flying. This multiple spacecraft approach will also impose less requirements and limitations on launch vehicles and thereby reducing the mission cost. Higher reliability and redundancy, higher resolution, simpler design and faster built time are other advantages of using multiple smaller satellites over a single large satellite [1], [2].

It is well-known that the efficiency and reliability of the formation can be degraded as a consequence of occurrence of a fault in the actuators of the satellites. Therefore, autonomous, real-time and on-line fault detection and isolation (FDI) strategies are required in order to diagnose faults before they can cause severe damages and lead to catastrophic failures in the entire networked formation system.

Fault detection and isolation in single satellite has been investigated with various methods in the literature (e.g. [3]-[7]). However there is not as much as research fault detection for formation flight of satellites. In [8] a hierarchical fault diagnosis decomposition framework is developed for satellite formation flight through a component dependency model using Bayesian Network structure. In [9] the overlapping block-diagonal state space representation of a hierarchical large-scale system is transformed into constrained –state

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block-diagonal state space model, and then a constrained – state distributed Kalman filter is proposed to estimate the states of the model.

These methods are proposed for hierarchical formations and they cannot be generalized for all types of formations. A nonlinear observer combining second order sliding mode and wavelet networks applied to a multiple satellite formation flying system in [10]. However this approach does not have the distribution characteristic and is a localized method.

In this work, a model based method is presented for the problem of actuator fault detection in formation flight of satellites. The fault detection and isolation problem in formation flying of spacecraft is investigated by designing three different FDI architectures, namely, Decentralized, Centralized, and Semi-Decentralized, to analyze and represent the advantages and disadvantages of each architecture versus the others. Extended Kalman filter has been chosen as the fault detection technique because of its model-based and nonlinear characteristics.

## II. METHODOLOGY DESCRIPTION

### A. Satellite Attitude Model

Reference [11] discusses the satellite's attitude model in details. Satellite attitude dynamics relies on "rigid body" dynamics and its orientation behavior can be explained on this basis. A rigid body has six degrees of freedom, which three of them are rotational parameters. In order to describe satellite attitude dynamics, the Euler parameters  $(\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4)$  can be used which can be defined as

$$\vec{q} = i\epsilon_1 + j\epsilon_2 + k\epsilon_3 + \epsilon_4 \quad (1)$$

$$\epsilon_1^2 + \epsilon_2^2 + \epsilon_3^2 + \epsilon_4^2 = 1 \quad (2)$$

Then the attitude kinematics and dynamics equations of a satellite relative to the inertial frame  $F_0$  respectively are

$$\dot{\vec{q}} = \begin{bmatrix} \dot{\epsilon} \\ \dot{\eta} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} \epsilon^\times + \eta I_{3 \times 3} \\ -\epsilon^\top \end{bmatrix} \omega \quad (4)$$

$$J \dot{\omega} = -\omega \times (J\omega) + \tau \quad (5)$$

$\omega = [\omega_1, \omega_2, \omega_3]^T$  is the angular velocity of the satellite relative to the inertial frame, and  $J$  is the inertia tensor

$$J = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{yx} & I_{yy} & -I_{yz} \\ -I_{xz} & -I_{yz} & I_z \end{bmatrix} \quad (6)$$

and also

$$f(x) = [f_1(x)^T, f_2(x)^T]^T$$

The spacecraft that is considered in this research is assumed to be symmetric with respect to the plane  $x = 0$ . Therefore, we will have  $I_{xy} = I_{xz} = I_{yz} = 0$ . Now the inertia tensor can be written as

$$J = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} \quad (7)$$

In the equation [5],  $\tau$  is the control torque, and the cross-product operator is defined by

$$\varepsilon^* = \begin{bmatrix} 0 & -\varepsilon_3 & \varepsilon_2 \\ \varepsilon_3 & 0 & -\varepsilon_1 \\ -\varepsilon_2 & \varepsilon_1 & 0 \end{bmatrix} \quad (8)$$

where

$$f_1(x) = \begin{bmatrix} -\frac{1}{2}(\bar{\omega}_2 \bar{\varepsilon}_3 - \bar{\omega}_3 \bar{\varepsilon}_2) + \frac{1}{2} \eta \bar{\omega}_1 \\ -\frac{1}{2}(\bar{\omega}_3 \bar{\varepsilon}_1 - \bar{\omega}_1 \bar{\varepsilon}_3) + \frac{1}{2} \eta \bar{\omega}_2 \\ -\frac{1}{2}(\bar{\omega}_1 \bar{\varepsilon}_2 - \bar{\omega}_2 \bar{\varepsilon}_1) + \frac{1}{2} \eta \bar{\omega}_3 \end{bmatrix} \quad (12)$$

$$f_2(x) = \begin{bmatrix} k_4 \bar{\omega}_2 \bar{\omega}_3 \\ k_5 \bar{\omega}_1 \bar{\omega}_3 \\ k_6 \bar{\omega}_1 \bar{\omega}_2 \end{bmatrix} \quad (13)$$

$$k_4 = \frac{I_y - I_z}{I_x}, k_5 = \frac{I_z - I_x}{I_y}, k_6 = \frac{I_x - I_y}{I_z} \quad (14)$$

and

$$\eta = \sqrt{1 - \bar{\varepsilon}_1^2 - \bar{\varepsilon}_2^2 - \bar{\varepsilon}_3^2} \quad (15)$$

Equations (4) and (5) can be combined and described in one matrix which is

$$\begin{bmatrix} \dot{\varepsilon}_1 \\ \dot{\varepsilon}_2 \\ \dot{\varepsilon}_3 \\ \dot{\eta} \\ \dot{\omega}_1 \\ \dot{\omega}_2 \\ \dot{\omega}_3 \end{bmatrix} = \begin{bmatrix} -\frac{1}{2}(\bar{\omega}_2 \bar{\varepsilon}_3 - \bar{\omega}_3 \bar{\varepsilon}_2) + \frac{1}{2} \eta \bar{\omega}_1 \\ -\frac{1}{2}(\bar{\omega}_3 \bar{\varepsilon}_1 - \bar{\omega}_1 \bar{\varepsilon}_3) + \frac{1}{2} \eta \bar{\omega}_2 \\ -\frac{1}{2}(\bar{\omega}_1 \bar{\varepsilon}_2 - \bar{\omega}_2 \bar{\varepsilon}_1) + \frac{1}{2} \eta \bar{\omega}_3 \\ -\frac{1}{2}(\varepsilon_1 \omega_1 + \varepsilon_2 \omega_2 + \varepsilon_3 \omega_3) \\ \frac{I_y - I_z}{I_x} \bar{\omega}_2 \bar{\omega}_3 + \frac{1}{I_x} \tau_1 \\ \frac{I_z - I_x}{I_y} \bar{\omega}_1 \bar{\omega}_3 + \frac{1}{I_y} \tau_2 \\ \frac{I_x - I_y}{I_z} \bar{\omega}_1 \bar{\omega}_2 + \frac{1}{I_z} \tau_3 \end{bmatrix} \quad (9)$$

The output matrix C in (8) is based on the measurement considered for the satellites. For the attitude measurement, we have

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

and for the system with angular velocity measurement, it can be written as

$$C = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

For the purpose of detecting the actuator fault of satellite with extended Kalman filter, we write this nonlinear dynamics as following

$$\begin{aligned} \dot{x}(t) &= f(x(t)) + Bu(t) + w(t) \\ y(t) &= Cx(t) + v(t) \end{aligned} \quad (10)$$

where  $x = [\varepsilon^T, \omega^T]^T$  is the state vector,  $y = [\omega]^T$  is the output vector,  $u = [u_1, u_2, u_3]^T$  is the control torque  $\tau$  that produces in three actuators of three different axes,  $w(t) \in \mathfrak{R}^6$  and  $v(t) \in \mathfrak{R}^6$  are white Gaussian noise with zero mean and covariance respectively  $Q(t)$  and  $R(t)$ ,  $C = I_{6 \times 6}$ ,  $B = [0_{3 \times 3}, \bar{k}]^T$  where  $k = \text{diag}(k_1, k_2, k_3)$  with

$$k_1 = \frac{1}{I_x}, k_2 = \frac{1}{I_y}, k_3 = \frac{1}{I_z} \quad (11)$$

As it is shown in the above equations,  $\eta$  is not an independent parameter and it can be obtained by using the other Euler parameters. Therefore, we have eliminated it from the states of (9).

### B. Fault Modeling

The system with actuator faults modeled by Loss of Effectiveness faults can be written as

$$\dot{x}(t) = f(x(t)) + B^f(t)u(t) \quad (16)$$

where  $B^f \in \mathfrak{R}^{n \times m}$  is the post fault control matrix and is related to the nominal control matrix  $B$  with

$$B^f(t) = B - \Delta B = B - B\Gamma(t) \quad (17)$$

where

$$\Gamma(t) = \begin{bmatrix} \gamma_1(t) & 0 & \cdots & 0 \\ 0 & \gamma_2(t) & \ddots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & \gamma_m(t) \end{bmatrix} \quad (18)$$

The  $\gamma_i = 0$  denotes the healthy  $i$ -th actuator and the  $\gamma_i = 1$  indicates complete failure in the  $i$ -th actuator. In general, the  $0 < \gamma_i < 1$  shows the partial loss in the control effectiveness of the  $i$ -th actuator.

The signal  $u_i^m$  is the actuation produced by the  $i$ -th actuator. For a healthy actuator we have

$$u_i^m = u_i \quad (19)$$

and for a faulty actuator we have

$$u_i^m = (1 - \gamma_i)u_i \quad (20)$$

which implies  $\gamma \times 100\%$  reduction in the actuation effectiveness.

By substituting (17) into (16), we obtain

$$\dot{x}(t) = f(x(t)) + B(I - \Gamma(t))u(t) \quad (21)$$

An alternative representation of this equation is formulated by

$$\dot{x}(t) = f(x(t)) + Bu(t) - [b_1\gamma_1 \quad b_2\gamma_2 \quad \cdots \quad b_m\gamma_m] \begin{bmatrix} u_1 \\ \vdots \\ u_m \end{bmatrix} \quad (22)$$

which can be written in a more compact form with

$$\dot{x}(t) = f(x(t)) + Bu(t) - \sum_{i=1}^m b_i \gamma_i u_i \quad (23)$$

This equation will be used in this paper as the description of a faulty spacecraft attitude control subsystem.

### C. Decentralized Architecture

In decentralized architecture, each spacecraft has its own FDI unit and there is no communication among them for the purpose of fault detection. Therefore, spacecraft  $\#i$  detects the actuator faults which exist in the formation only based on the information of its own output  $y_i$  and control signal  $u_i$ . The system for this architecture has the representation

$$\begin{aligned} \dot{x}_i(t) &= f(x_i(t)) + Bu_i(t) + \sum_{k=1}^3 b_k \rho_{ik}(t) \\ y_i(t) &= Cx_i(t) \end{aligned} \quad (24)$$

The state  $x_i \in R^6$ , the output  $y_i \in R^3$ , the control input  $u_i \in R^3$ , the dynamic model  $f(x_i)$ , the control input matrix  $B$ ,

the output matrix  $C$ , and the actuator fault mode  $\rho_{ik} = -\gamma_{ik}u_{ik}$  describe the mathematical model for the decentralized architecture of spacecraft  $\#i$ . The vector  $b_k$  is the  $k$ -th column of the control input matrix  $B$ .

### D. Semi-Decentralized Architecture

In the semi-decentralized architecture, the FDI unit for each spacecraft receives the output measurement and control input information of its neighboring spacecraft. This information gives the FDI unit the capability to detect and isolate the actuator faults of its neighboring spacecraft not only based on the effect of  $u_i = \{g(x_i, x_j) : j \in N_i\}$ , but also using the direct output measurement and control signal information received from them.

The neighboring set of spacecraft  $\#i$  is shown with  $N_i = \{j \in V(S) \mid j \sim i\}$ . The set that includes spacecraft  $\#i$  is denoted by  $\bar{N}_i = N_i \cup \{i\}$ , and the notation  $|\bar{N}_i|$  symbolizes the number of spacecraft in this set.

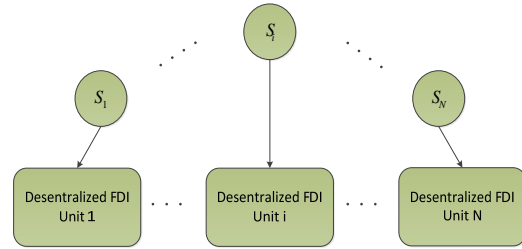


Fig. 1 Decentralized FDI architecture

The system for the semi-decentralized architecture has the representation

$$\begin{aligned} \dot{x}_{\bar{N}_i}(t) &= f(x_{\bar{N}_i}(t)) + B^{|\bar{N}_i|}u_{\bar{N}_i}(t) + \sum_{k=1}^{|\bar{N}_i|} \sum_{j=1}^3 \bar{b}_{kj} \rho_{kj} \\ y_{\bar{N}_i}(t) &= C^{|\bar{N}_i|}x_{\bar{N}_i}(t) \end{aligned} \quad (25)$$

where  $B^{|\bar{N}_i|} = I^{|\bar{N}_i|} \otimes B$ ,  $C^{|\bar{N}_i|} = I^{|\bar{N}_i|} \otimes C$ ,  $\otimes$  denotes the Kronecker product,  $I^{|\bar{N}_i|}$  is an  $|\bar{N}_i| \times |\bar{N}_i|$  identity matrix, and  $\bar{b}_{kj}$  is the  $(k-1) \times 3 + j$ -th column of  $B^{|\bar{N}_i|}$ , and

$$x = \begin{bmatrix} x_1 \\ \vdots \\ x_{\bar{N}_i} \end{bmatrix}, f(x) = \begin{bmatrix} f(x_1) \\ \vdots \\ f(x_{\bar{N}_i}) \end{bmatrix} \quad (26)$$

### E. Centralized Architecture

In the centralized architecture, one FD center is considered for the fault detection and isolation of the whole formation. All spacecraft send their state and control input information to this FD center, as shown in Fig. 3. Considering a formation with  $N$  spacecraft, the system for the centralized architecture has the representation

$$\begin{aligned}\dot{x}(t) &= f(x) + B^N u(t) + \sum_{i=1}^N \sum_{k=1}^3 \bar{b}_{ik} \rho_{ik} \\ z(t) &= C^N x(t)\end{aligned}\quad (27)$$

where  $B^N = I^N \otimes B$ ,  $C^N = I^N \otimes C$ , and  $\otimes$  denotes the Kronecker product,  $I^N$  is an  $N \times N$  identity matrix, and  $\bar{b}_{ik}$  is the  $(i-1) \times 3 + k$ -th column of  $B^N$ , and

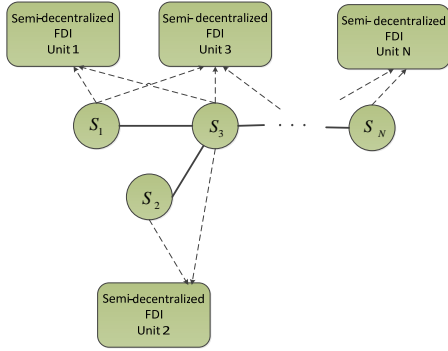


Fig. 2 Semi-decentralized FDI architecture

$$x = \begin{bmatrix} x_1 \\ \vdots \\ x_N \end{bmatrix}, f(x) = \begin{bmatrix} f(x_1) \\ \vdots \\ f(x_N) \end{bmatrix}, u = \begin{bmatrix} u_1 \\ \vdots \\ u_N \end{bmatrix}, y = \begin{bmatrix} y_1 \\ \vdots \\ y_N \end{bmatrix}\quad (28)$$

The state  $x_i \in R^6$ , the output  $y_i \in R^3$ , the control input  $u_i \in R^3$ , and the dynamic model  $f(x_i)$ , with  $i=1, \dots, N$  denotes the information received from the  $i$ -th spacecraft in a formation with  $N$  spacecraft.

The actuator fault mode  $\rho_{ik} = -\gamma_{ik} u_{ik}$  shows reduction of torque effectiveness in the  $k$ -th actuator of the  $i$ -th spacecraft, where  $u_{ik}$  is the  $k$ -th entry of  $u_i$  and  $\gamma_{ik}$  is the partial loss in the torque effectiveness of axis  $k$  of spacecraft  $\#i$ .

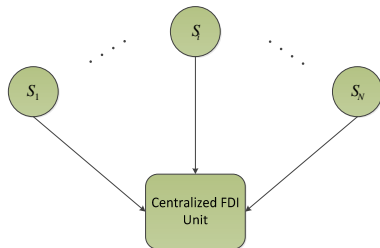


Fig. 3 Centralized FDI architecture

### III. FAULT DETECTION AND ISOLATION (FDI)

#### A. Fault Detection

Considering the decentralized architecture,  $N$  fault detection units are constructed to detect the actuator faults of all spacecraft. Assume the following system as the

decentralized architecture for the spacecraft formation in the presence of additive noise

$$\begin{aligned}\dot{x}_i(t) &= f(x_i(t)) + B u_i(t) + \sum_{j=1}^3 b_j \rho_{ij}(t) + w_i(t) \\ y_i(t) &= C x_i(t) + v_i(t)\end{aligned}\quad (29)$$

where  $w_i(t) \in \mathfrak{R}^6$  and  $v_i(t) \in \mathfrak{R}^3$  are white Gaussian noise with zero mean and covariance  $Q^i(t) \in \mathfrak{R}^{6 \times 6}$  and  $R^i(t) \in \mathfrak{R}^{3 \times 3}$ .

#### 1.State Estimation

Predict and update equations for the continuous-time extended Kalman filtering of a decentralized architecture are described by the following equations.

Updated State Estimate:

$$\dot{\hat{x}}_i(t) = f(\hat{x}_i(t)) + B u_i(t) + K^i(t)(y_i(t) - C \hat{x}_i(t))\quad (30)$$

Differential Riccati Equation:

$$\begin{aligned}\dot{P}^i(t) &= F^i(t) P^i(t) + P^i(t) (F^i(t))^T \\ &\quad - P^i(t) C^T (R^i(t))^{-1} C (t) P^i(t) + Q^i(t)\end{aligned}\quad (31)$$

Kalman Gain:

$$K^i(t) = P^i(t) C (R^i(t))^{-1} C^T\quad (32)$$

where the observation matrix  $F(t)$  is defined by

$$F^i(t) = \frac{\partial f}{\partial x} \Big|_{x_i(t), u_i(t)}\quad (33)$$

The initial state  $\hat{x}_i(t_0)$  is a random vector with known mean  $\mu_i(t_0) = E[x_i(t_0)]$  and covariance  $P^i(t_0) = E[(x_i(t_0) - \mu_i(t_0))(x_i(t_0) - \mu_i(t_0))^T]$ .

#### 2.Residual Generation

By applying the decentralized EKF estimator to the system of (29), the estimated state vector  $\hat{x}_i$  is obtained. The residual  $e_i(t) = y_i(t) - C \hat{x}_i(t)$  is a comparison between the actual outputs and estimated outputs of the  $i$ -th spacecraft. The residual vector  $e_i(t)$  is a vector with three residuals:

$$e_i(t) = \begin{bmatrix} e_{i1}(t) \\ e_{i2}(t) \\ e_{i3}(t) \end{bmatrix}\quad (34)$$

For the spacecraft system with attitude measurement, the residual  $e_i(t)$  is given by

$$e_i(t) = \tilde{q}_i(t) = q_i(t) - \hat{q}_i(t)\quad (35)$$

$$e_{ij}(t) \Big|_{j=1,2,3} = \tilde{q}_{ij}(t) = q_{ij}(t) - \hat{q}_{ij}(t)\quad (36)$$

and for the spacecraft system with angular velocity measurement, the residual  $e_i(t)$  is given by

$$e_i(t) = \tilde{\omega}_i(t) = \omega_i(t) - \hat{\omega}_i(t) \quad (37)$$

$$e_{ij}(t)|_{j=1:3} = \tilde{\omega}_{ij}(t) = \omega_{ij}(t) - \hat{\omega}_{ij}(t) \quad (38)$$

The norm of the residual vector of spacecraft in the decentralized architecture is chosen as the residual evaluation function vector  $J_i(t)$  according to

$$J_i(t) = \begin{bmatrix} J_{i1}(t) \\ J_{i2}(t) \\ J_{i3}(t) \end{bmatrix} = \begin{bmatrix} |e_{i1}(t)| \\ |e_{i2}(t)| \\ |e_{i3}(t)| \end{bmatrix} \quad (39)$$

where  $J_{ij}(t)$ ,  $i \in 1:N$  and  $j \in 1:3$ , is the  $j$ -th residual evaluation function of the  $i$ -th spacecraft.

A fault in the  $i$ -th spacecraft can be detected by comparing the mean value of the residual evaluation function  $J_{ij}$ , namely  $d_{ij}$ , with a threshold function  $T_{ij}$ . According to the test given below, if  $d_{ij}$  surpasses the threshold, the occurrence of fault is declared in one of the actuators of spacecraft #  $i$ :

$$\begin{aligned} d_{ij}(m) \leq T_{ij} & \quad \text{if } \{ \rho_{ij}(m) = 0 \mid j \in 1, 2, 3 \} \\ d_{ij}(m) > T_{ij} & \quad \text{if } \{ \rho_{ij}(m) \neq 0 \mid j \in 1, 2, 3 \} \end{aligned} \quad (40)$$

The mean value of the residual evaluation function over the time window length of  $M$  can be obtained from:

$$d_{ij}(m) = \frac{1}{M} \sum_{n=m-M+1}^m J_{ij}(n) \quad (41)$$

where  $m$  is the sample number, and  $M$  is the window length. The value for the window length  $M$ , and the decision threshold  $T_{ij}$  must be determined in such a way that a trade-off is made between the probability of the false alarms and the probability of the missed alarms.

### 3. Threshold Selection

The threshold is selected as the sum of the mean and standard deviation of the norm of residual evaluation function.

By considering the worst case analysis of the residual evaluation functions corresponding to the healthy operation of the satellites that are subject to the measurement noise, the threshold for the  $j$ -th residual evaluation function of the  $i$ -th decentralized FDI unit is defined by

$$T_{ij} = \text{mean}(|J_{ij}(t)|) + \sqrt{\text{var}(J_{ij}(t))} \quad (42)$$

### B. Fault Isolation

Assume  $J_i(t) \in R^3$  as the residual evaluation function obtained from the  $i$ -th FDI unit of the decentralized

architecture, and the  $j$ -th entry of this vector is defined by  $J_{ij}(t) = |e_{ij}(t)|$ , where  $e_{ij}$  is the  $j$ -th innovation sequence of the  $i$ -th spacecraft. Using arrays of this matrix we build our residual structure set.

The residual evaluation function obtained for the angular velocity measurements is given by

$$J_i(t) = \begin{bmatrix} J_{i1}(t) \\ J_{i2}(t) \\ J_{i3}(t) \end{bmatrix} = \begin{bmatrix} |e_{i1}(t)| \\ |e_{i2}(t)| \\ |e_{i3}(t)| \end{bmatrix} = \begin{bmatrix} |\omega_{i1} - \hat{\omega}_{i1}| \\ |\omega_{i2} - \hat{\omega}_{i2}| \\ |\omega_{i3} - \hat{\omega}_{i3}| \end{bmatrix} \quad (43)$$

and the residual evaluation function obtained for the attitude measurements is given by

$$J_i(t) = \begin{bmatrix} J_{i1}(t) \\ J_{i2}(t) \\ J_{i3}(t) \end{bmatrix} = \begin{bmatrix} |e_{i1}(t)| \\ |e_{i2}(t)| \\ |e_{i3}(t)| \end{bmatrix} = \begin{bmatrix} |q_{i1} - \hat{q}_{i1}| \\ |q_{i2} - \hat{q}_{i2}| \\ |q_{i3} - \hat{q}_{i3}| \end{bmatrix} \quad (44)$$

The mean of  $J_i(t)$  over window length  $M$  is obtained by

$$g_{ij}(m) = \frac{1}{M} \sum_{n=m-M+1}^m J_{ij}(n) \quad (45)$$

where  $m$  is the sample number, and  $M$  is the window length.

In light of this illustration, it is convenient to introduce some notations. Fault  $f_{ij}$  is the occurrence of fault in the  $j$ -th actuator of spacecraft #  $i$ . Our isolation method is based on this idea that the occurrence of fault  $f_{ij}$  causes  $g_{ij}$  to surpass the threshold  $T_{ij}$ . In this situation, the indicator  $r_{ij}$  changes from zero to one and fault in the actuator #  $j$  of spacecraft #  $i$

$$\begin{aligned} g_{ij}(m) > T_{ij} & \Rightarrow r_{ij}(m) = 1 \\ g_{ij}(m) < T_{ij} & \Rightarrow r_{ij}(m) = 0 \end{aligned} \quad (46)$$

The threshold  $T_{ij}$  is selected as the sum of the mean and standard deviation of  $g_{ij}$ . By considering the worst case analysis of  $g_{ij}$  corresponding to the healthy operation of spacecraft that are subject to measurement noise, the thresholds are defined by

$$T_{ij} = \text{mean}(|g_{ij}(n)|) + \sqrt{\text{var}(g_{ij}(n))} \quad (47)$$

## IV. CONCLUSION

In this work, a model based method is presented for the problem of actuator fault detection in formation flight of satellites. The fault detection and isolation problem in formation flying of spacecraft is investigated by designing three different FDI architectures, namely, Decentralized, Centralized, and Semi-Decentralized, to analyze and represent the advantages and disadvantages of each architecture versus the others. Extended Kalman filter has been chosen as the fault

detection technique because of its model-based and nonlinear characteristics.

## REFERENCES

- [1] Scharf, D.P.; Hadaegh, F.Y.; Ploen, S.R.; "A survey of spacecraft formation flying guidance and control (part 1): guidance," *American Control Conference, 2003. Proceedings of the 2003*, vol.2, no., pp. 1733- 1739, Jun 2003.
- [2] Scharf, D.P.; Hadaegh, F.Y.; Ploen, S.R.; "A survey of spacecraft formation flying guidance and control. Part II: control," *American Control Conference, 2004. Proceedings of the 2004*, vol.4, no., pp.2976-2985 vol.4, June 30 2004-July 2 2004.
- [3] Jia Qing-xian; Zhang Ying-chun; Guan Yu; Wu Li-na; "Robust nonlinear unknown input observer-based fault diagnosis for satellite attitude control system," *Intelligent Control and Information Processing 2011 2nd International Conference on* , vol.1, no., pp.345-350, 2011.
- [4] Jiang, T.; Khorasani, K.; Tafazoli, S.; "Parameter Estimation-Based Fault Detection, Isolation and Recovery for Nonlinear Satellite Models," *Control Systems Technology, IEEE Transactions on* , vol.16, no.4, pp.799-808, July 2008.
- [5] Talebi, H.A.; Khorasani, K.; Tafazoli, S.; "A Recurrent Neural-Network-Based Sensor and Actuator Fault Detection and Isolation for Nonlinear Systems With Application to the Satellite's Attitude Control Subsystem," *Neural Networks, IEEE Transactions on* , vol.20, no.1, pp.45-60, Jan. 2009.
- [6] Wang, J.; Jian, B.; Shi P.; "Adaptive Observer Based Fault Diagnosis for Satellite Attitude Control Systems," *International Journal of Innovative Computing, Information and Control*, vol.4, no.8, Aug. 2008.
- [7] Zhao-hui Cen; Jiao-long Wei; Jiang Rui; Liu Xiong; "Real-time fault diagnosis of satellite attitude control system based on sliding-window wavelet and DRNN," *Control and Decision Conference (CCDC), 2010 Chinese* , vol., no., pp.1218-1222, 26-28 May 2010.
- [8] A. Barua and K. Khorasani, "Hierarchical Fault Diagnosis and Health Monitoring in Satellites Formation Flight," *IEEE Transactions on Systems, Man and Cybernetics - Part C*, Vol. 41, No. 2, pp. 223-239, March 2011.
- [9] Azizi, S.M.; Khorasani, K.; "A distributed Kalman filter for actuator fault estimation of deep space formation flying satellites," *Systems Conference, 2009 3rd Annual IEEE*, vol., no., pp.354-359, March 2009.
- [10] Qing Wu; Saif, M.; "Robust Fault Detection and Diagnosis for a Multiple Satellite Formation Flying System Using Second Order Sliding Mode and Wavelet Networks," *American Control Conference, 2007. ACC '07*, vol., no., pp.426-431, 9-13 July 2007.
- [11] P. C. Hughes, *Spacecraft attitude Dynamics*, John Wiley & Sons Inc., 1986.