Particle Concentration Distribution under Idling Conditions in a Residential Underground Garage

Yu Zhao, Shinsuke Kato, Jianing Zhao

Abstract—Particles exhausted from cars have adverse impacts on human health. The study developed a three-dimensional particle dispersion numerical model including particle coagulation to simulate the particle concentration distribution under idling conditions in a residential underground garage. The simulation results demonstrate that particle disperses much faster in the vertical direction than that in horizontal direction. The enhancement of particle dispersion in the vertical direction due to the increase of cars with engine running is much stronger than that in the car exhaust direction. Particle dispersion from each pair of adjacent cars has little influence on each other in the study. Average particle concentration after 120 seconds exhaust is 1.8-4.5 times higher than the initial total particles at ambient environment. Particle pollution in the residential underground garage is severe.

Keywords—Dispersion, Idling conditions, Particle concentration, Residential underground garage.

I. INTRODUCTION

PARTICLE emitted from vehicles is one of the most important particle course. important particle sources in urban environment and the adverse impacts of particles from vehicle exhaust on human health is of particular interest for decades. Increase of some cardiovascular and respiratory diseases has been demonstrated to be in association with particle pollution [1]-[3]. Due to the severe health hazards, the prediction of particle concentration and the reduction of particle pollution should be of major importance.

The majority of new residential areas have been equipped with underground garage in China. Idling conditions is quite common in underground garage. These idling conditions have been shown to be associated with high exposure to air pollutants due to the lack of turbulent dispersion normally created by the wake of moving vehicles [4], [5]. Particle dispersion is also enhanced due to thermal buoyancy induced by temperature between vehicle tailpipe and ambient environment, which is more than 10°C in most cases [6], [7]. In the high temperature difference combined with enclosed environment, characteristics of particle dispersion is complicated and not yet fully understood. Consequently, a better understanding of characteristics of vehicle exhaust

Yu Zhao is with the Harbin Institute of Technology, Harbin, Heilongjiang Province 150090 China. He is now with Kato Lab and Ooka Lab, the University of Tokyo, Tokyo 153-8505 Japan as a special research student from Oct. 2013 to Sep. 2014 (e-mail: zy19870104@163.com).

Shinsuke Kato is with the Kato Lab and Ooka Lab, the University of Tokyo.

Tokyo 153-8505 Japan (e-mail: kato@iis.u-tokyo.ac.jp).

Jianing Zhao is with the Harbin Institute of Technology, Harbin, Heilongjiang Province 150090 China (phone: +86-451-86282123; fax: +86-451-86282123; e-mail: zhaojn@hit.edu.cn).

particles is essentially required.

There are substantial literatures describing the methods for predicting vehicle exhausts in the atmosphere. Reference [8] proposed a simple approach to describe urban scales: length scale such as street (less than 100~200m), neighborhood (up to 1 or 2 km), city (up to 10 or 20 km) and regional (up to 100 or 200 km) scales. The length scale of vehicle wake where the mixing and dilution of pollutants occurs faster than at any other scale is much smaller than urban scales and could be divided into two distinguished flow regions: near wake which is normally considered to be up to a distance of about 10-15 times the vehicle height, far wake which is a region beyond the near wake [9]-[12]. Due to the complexity of flow vortex and turbulence in the near wake region of vehicle, characteristic of particle dispersion in the near wake is quite different from the far wake and urban scales. Classical theory dispersion model that has been proved to predict particle dispersion successfully could not be simply employed to evaluate particle dispersion in the near wake region of vehicles. In recent years, researchers pay their attention to the development of particle dispersion model in the near wake region and CFD based models using RANS or LES techniques that integrates particle dynamics models have been recently used for particle dispersion process in the near wake of vehicle [13], [14]. Nevertheless, most studies were conducted at the open space such as on road environment. Few researchers have focused on particle dispersion under idle condition in the enclosed environment that is the real situation in underground garage. Due to narrow parking spaces and enclosures in underground garage, characteristics of particle dispersion in the near wake region of vehicle might be different from other open spaces.

The aim of the study is to analyze particle concentration distribution under idling conditions in a residential underground garage combining a Realizable k-& model and particle transport model. Particle size distribution was treated by nine categories based on the particle distribution measured by [7]. Three-dimensional model was validated by comparing the simulated results with measured data from [15]. Three cases that included different cars with engine running under 120 seconds unsteady state after vehicle engine started were simulated and investigated in the study.

II. METHODOLOGY

A. Turbulent Flow Model

The k- ε model has been successfully applied to simulate the airflow and energy fields in enclosed environments [16]. This study adopted Reynolds averaged Navier-Stokes (RANS) equation with an Realizable k- ε model. More details of the

Realizable k- ε model could be found in [17].

B. Particle Transport Model

The aim of this study is to predict the airflow field, the particle concentration distribution and the particle diameter variation simultaneously. Before the simulation, the following assumptions have been made to simplify the particle movement problem and make the numerical simulation efficient.

- (a) Resuspension is neglected in the study.
- (b) The effect of particles on the turbulent flow is negligible due to low particle volume fraction in airflow.
- (c) All particles are in spherical shape. The particle density is constant.
- (d) No mass loss occurs during the collision of particles.

The interaction between the air and particle can be treated as one-way coupling from air to particle. For particle dynamic analysis, particle diameter is the key parameter for determining the particle dynamics such as deposition, coagulation and condensation. Some published studies have shown that the majority of particles exhausted by petroleum vehicle are in the size range less than 0.4µm [7]. As the car with gasoline engine in [7] measures up to the Chinese National Active Standard on vehicle exhaust control, particle number concentrations measured under idle conditions in [7] were employed as particles exhausted from tailpipe in the study.

The general transport equation describing the particle concentration variation is difficult to be solved mainly because of its dependence on particle size. As a compromise, several techniques have been developed to overcome the shortcoming. Among these methods, the sectional method has been widely accepted as an effective way to simplify the general transport equation [18]-[20]. The sectional method divides the particle size into several discrete categories. At each category, the particle size is considered to be a constant. In the present study, the particle size is divided into 9 categories. The total number concentration is proportionally distributed into each category according to the particle size distribution. The respective particle diameter and number concentration in each category is shown in Fig. 1.

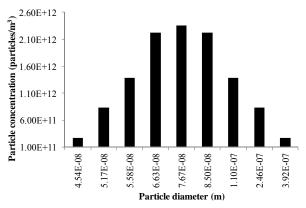


Fig. 1 Particle diameter and number concentration in the study

The average air velocity component u_j in the three directions are computed by solving the RANS equations with the

Realizable k- ε model. In addition, the velocity of particles undergoing gravitational settling V_{TSi} is mostly determined by particle size and it could be calculated based on (1):

$$V_{TSi} = \frac{\rho_P d_{pi}^2 g C_{ci}}{18 \,\mu} \tag{1}$$

where ρ_p represents particle density (kg/m³), d_{pi} is the particle diameter in each particle category (m), g is the gravitational acceleration (m/s²), C_{ci} is the Cunningham correction factor, which is shown in (2), μ is the kinetic viscosity of air (kg·m¹·s¹):

$$C_{ci} = 1 + \frac{\lambda}{d_{pi}} [2.514 + 0.800 \exp(-0.55 \frac{d_{pi}}{\lambda})]$$
 (2)

where λ is air mean free path (m), which could be calculated by (3):

$$\lambda = \frac{kT}{\sqrt{2}\pi d^2 p} \tag{3}$$

where k is Boltzmann's constant, 1.3807×10^{-23} (J/K), T is the ambient temperature (K), d is the air molecule diameter (m), p is the air pressure (Pa).

The Brownian diffusion coefficient is the constant of proportionality that relates the flux of particles and the concentration gradient due to Brownian motion. This relationship is called Fick first law of diffusion and the diffusion coefficient in each particle category D_i could be given by (4):

$$D_i = \frac{kTC_{ci}}{3\pi\mu d_{pi}} \tag{4}$$

The Hinze-Tchen equation is employed for calculating the particle turbulent diffusion coefficient, which is shown in (5):

$$\frac{v_{pi}}{v_{ti}} = (1 + \frac{\tau_{pi}}{\tau_{ti}})^{-1} \tag{5}$$

where v_{ti} is the air turbulent diffusion coefficient in each particle category (m²/s), τ_{pi} is the particle relaxation time in each category (s), τ_{ti} is the turbulent fluctuation time (s).

Reference [21] proved mathematically that for nano-particles, τ_{pi} is at least an order of magnitude smaller than τ_{ti} in a homogeneously turbulent flow field, which means v_{pi} is equal to v_{ti} in the study.

C. Particle Coagulation

The theory of particle coagulation in the study is thermal coagulation due to Brownian collision. As Brownian diffusion increases with the decrease of particle size, it should be an important factor influencing the fine and ultrafine particle concentration distribution. The thermal coagulation equation employed in the study is shown in (6):

$$S_{ci} = \sum_{m,j=1}^{i-1} K_{mj} C_m C_j - C_i \sum_{a=1}^{n} K_{ai} C_a$$
 (6)

where S_{ci} represents variation of particle number concentration in each particle category due to coagulation (m⁻³·s⁻¹), K_{mj} and K_{ai} are the coagulation coefficient (m³·s⁻¹), C_m , C_j , C_a and C_i are particle concentration in different particle categories (m⁻³).

The coagulation coefficient K_{ij} employed in the study is given by (7) [22]:

$$K_{ijB} = \frac{2kT}{3\mu} \left(\frac{C_c(i)}{v_i^{1/3}} + \frac{C_c(j)}{v_j^{1/3}} \right) \left(v_i^{1/3} + v_j^{1/3} \right) \tag{7}$$

where v_i and v_j are the volume of a particle in Category i and Category j, respectively (m³). $C_c(i)$ and $C_c(j)$ represents the Cunningham correction factor in Category i and Category j, respectively.

The first part in the right side of (8) is the particle increase due to pairs of particles smaller than d_{i-1} collide and become the particles in the range between d_{i-1} and d_i . As no mass loss occur during the process of particle collision and coagulation. Particle diameter after collision d_{pk} could be calculated by (8) and (9). The possibility of particles smaller than d_{i-1} change to the particle size range between d_{i-1} and d_i should be analyzed based on (9):

$$\frac{\pi}{6}\rho_{p}d_{pi}^{3} + \frac{\pi}{6}\rho_{p}d_{pj}^{3} = \frac{\pi}{6}\rho_{p}d_{pk}^{3}$$
 (8)

$$d_{pk} = \sqrt[3]{d_{pi}^3 + d_{pj}^3} \tag{9}$$

III. MODEL PROCEDURE

The model was a parking area with 6 cars at the middle of a residential underground garage in Harbin, China. The model geometry was a rectangular box of 9.8m, 17.5m and 4.2m in X, Y, and Z direction, respectively. 6 gasoline-powered cars with the same dimension of $4.5 \text{m}(L) \times 1.8 \text{m}(W) \times 1.3 \text{m}(H)$ was placed inside the parking spaces that abuts on 4 columns in the model. The same exhaust tailpipe exit with a diameter of 0.05m was in the right end of each studied car and the height of the tailpipe above the ground was 0.3m. Detailed description of model geometry could be found in Fig. 2.

TABLE I
INITIAL AND BOUNDARY VALUES IN THE MODEL

INTEREMEDIA DOCUMENT VILLOUS IN THE MOBEL		
Parameters	Values	
Vehicle exhaust temperature (K)	303.15	
Vehicle emitted velocity (m/s)	1.56	
Vehicle exhaust particles in each bin (m ⁻³)	Shown in Fig. 1	
Total exhaust particles at the tailpipe (m ⁻³)	1.18×10^{13}	
Initial ambient temperature (K)	281.56	
Initial ambient velocity (m/s)	0.07 in x direction and 0 in y and z directions	
Initial total particles at ambient environment (m ⁻³)	2.45×10^{10}	
Initial particles in each bin at ambient environment (m ⁻³)	2.72×10 ⁹	
•		

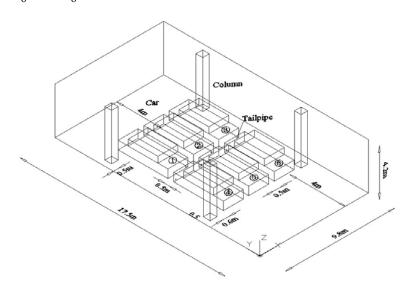


Fig. 2 Model geometry in the study: Number 1-6 in the figure represent car number in the model

The commercial CFD software STAR CCM $^+$ 8.02 was employed to perform the simulation. Approximately 1,430,000 hexahedron meshes were established using STAR CCM $^+$. The Realizable k- ε model and Passive Scalar Models with Field Functions were developed to simulate air flow, particle dispersion, and coagulation.

Initial and boundary conditions in the model were the same

as our previous research on particle coagulation [23]. No-slip conditions were set for car surfaces, column, ceiling and floor. The tailpipe of car with and without engine running was defined as velocity inlet boundary condition and wall, respectively. The walls that surround the model geometry in vertical direction were set to flow-slip outlet condition. As total particles emitted from vehicle shown in Fig. 1 was 1.18×10^{13}

m⁻³, an order of magnitude higher than ambient total particle number concentration (2.45×10¹⁰ m⁻³), ambient particle concentration had little effect on the particle concentration distribution. The initial particle number concentration in each category was simplified to be equal and one ninth of ambient total particle number concentration. Initial and boundary values in the model were given by Table I.

TABLE II
DESCRIPTION OF MODEL CASES SETUP

Case Number	Cars with engine running	Cars with engine stalling
Case 1	1 and 4 in Fig. 2	2, 3, 5, and 6 in Fig. 2
Case 2	1, 2, 4, and 5 in Fig. 2	3 and 6 in Fig. 2
Case 3	1-6 in Fig. 2	None

The developed particle dispersion model with coagulation has been validated through comparing with experimental results from [15]. Model verification in detail was described in [23]. Fig. 3 shows comparison of particle concentration between experimental and simulated value. The simulated results reflect that the numerical model can predict well the dispersion of particle number concentration in the near field region of the exhaust plume.

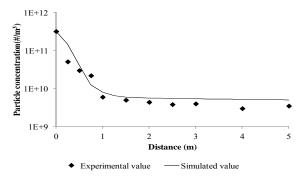


Fig. 3 Comparison of particle concentration between simulated and experimental results

Three cases for different vehicles with engine running under idling conditions were simulated in the study. Detailed description could be found in Table II. Initial values for three cases and boundary values at tailpipe for each vehicle with engine running were the same and equal to the values given by Table I. Particles exhausted from the tailpipe of car with engine running were constant during the simulation. Particle values in each category for cars with engine stalling were set to 0 in the study. The simulation were undergone under unsteady state conditions and lasted 120s after vehicle engine started.

IV. RESULTS AND DISCUSSION

Fig. 4 shows particle concentration distribution between Car 1 and Car 4 at the vertical tailpipe cross-section of Car 1 for different cases at the 120 second. In Case 1, particles dispersed obviously along the back of Car 4 in the vertical direction due to the narrow spaces between Car 1 and Car 4 as the barrier to particle dispersion in the horizontal direction. The particle diffusion length in the upward direction is much longer than that in the downward direction. That could be attributed to an

upward movement induced by thermal buoyancy due to temperature difference between the tailpipe and ambient environment. Particle concentration behind the car decreases rapidly to 10% of the tailpipe at a narrow area in the vertical direction. The length that particles decrease rapidly in the vertical conditions increases from about 3 times to 4 times of the tailpipe diameter along the centerline of the tailpipe. In the area near the back of Car 4, particle concentration decreases to 10% of the tailpipe at a almost 1m long in the vertical direction.

A more prominent upward tendency of particle dispersion was observed at both Case 2 and Case 3 compared with Case 1. The results could be caused by the enhancement of thermal buoyancy induced by more energy sources due to more cars with engine running in Case 2 and Case 3. Particle concentration distribution near the tailpipe is almost the same in three cases. However, in the area higher than the back of Car 1 and Car 4, particle decreases to 10% of the tailpipe at a much larger region in Case 2 and 3 compared with that in Case 1. In Case 2, the width of contour 1.1×10^{12} particles/m³ higher than the back of Car 1 and Car 4 is almost equal to the width of narrow parking space between Car 1 and Car 4. In Case 3, the essentially same contour 1.2×10^{12} particles/m³ becomes wider in the car exhaust direction.

The simulation results indicate that particle disperses intensely in an upward direction due to the narrow spaces in the residential underground garage and temperature difference between the tailpipe and ambient environment. The area between each pair of parked cars, especially in the vertical direction, is the most severe particle pollution region in the residential underground garage. The increase of cars with engine running could encourage the particle dispersion in both the vertical direction and car exhaust direction. The enhancement of particle dispersion in the vertical direction is much stronger than that in the car exhaust direction.

Fig. 5 shows particle concentration distribution between Car 1 and Car 4 at the horizontal tailpipe cross-section for different cases at the 120 second. Particle concentration at the horizontal tailpipe cross-section is distributed symmetrically and the axis of symmetrical distribution is the centerline of Car 1 and Car 4 in three cases. Due to the influence of the faces meeting at the corner, the same contour toward the axis of symmetry distributes farther than that at the corner of two faces. The accumulation of particles at the corner of Car 1 and Car 4 in three cases is very obvious in the study. An approximate symmetry of the particle accumulation at the corner in the horizontal direction is observed. As no other cars adjacent to the corner in the side of Car 4 tailpipe influence particle dispersion at the left corner of particle accumulation in the model, the symmetry distribution indicate that particle dispersion from each pair of adjacent cars such as Car 1 and Car 5 has little influence on each other under conditions in the

Particle dispersion toward the axis of symmetry distribution in the horizontal direction enhances slightly from 1.41×10^{12} particles/m³ to 1.61×10^{12} particles/m³ as the cars with engine running increase in the model. The results demonstrate that the effect of the adjacent Car 2, 3, 5, and 6 on particle dispersion

between Car 1 and 4 is quite limited. Particle decreases to 10% of the tailpipe in a narrower region at the horizontal direction compared with that at the vertical direction. The difference of dispersion length that particle decreasing to 10% of the tailpipe between the vertical direction and the horizontal direction in

Case 2 and 3 is larger than that in Case 1. The results indicate that thermal buoyancy had great influence on particle dispersion in the study. Particle disperses much faster in the vertical direction than that in the horizontal direction.

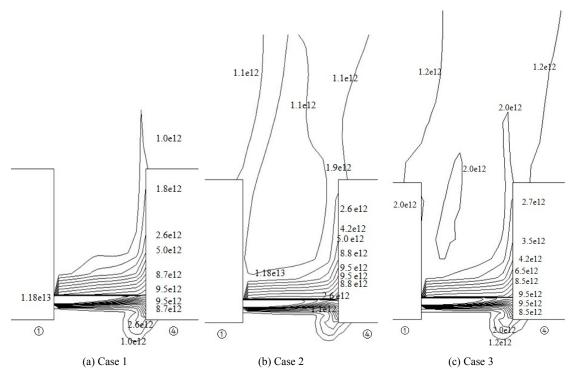
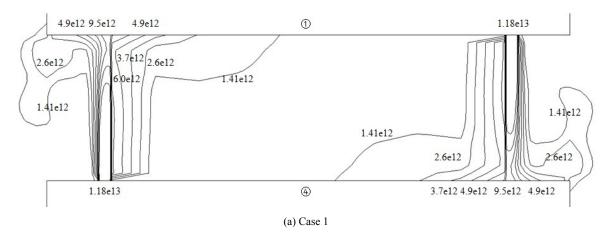


Fig. 4 Particle concentration distribution between Car 1 and Car 4 at the vertical tailpipe cross-section of Car 1 in different cases at the 120 second: Particle concentration in the figure is the sum of particle concentration in each category shown in Fig. 1. The unit in the figure is particles/m³. No. 1 and 4 represent Car 1 and Car 4, respectively



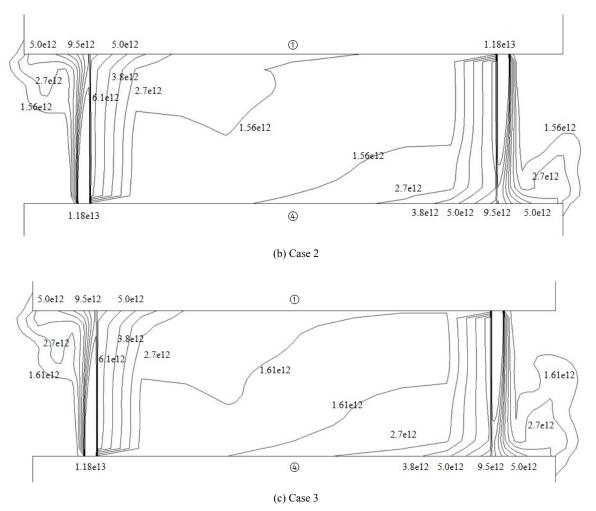


Fig. 5 Particle concentration distribution between Car 1 and Car 4 at the horizontal tailpipe cross-section in different cases at the 120 second. Particle concentration in the figure is the sum of particle concentration in each category shown in Fig. 1. The unit in the figure is particles/m³. No.

1 and 4 represent Car 1 and Car 4, respectively

As little Air Quality Standard has been issued on particle number concentration limits at indoor environment, initial total particles at ambient environment that is given by Table II was employed as an index to evaluate the particle pollution in the study. Fig. 6 shows the ratio between average particle concentration in the model and initial total particles at ambient environment in different cases at the 120 second. After 120 seconds exhaust, average particle concentration in the model is almost 1.8-4.5 times higher than the initial total particles at ambient environment in Case 1, 2, and 3. Since particle concentration increased very fast in quite a short time-scale exhaust, the simulation results reflect that particle pollution in the residential underground garage is severe.

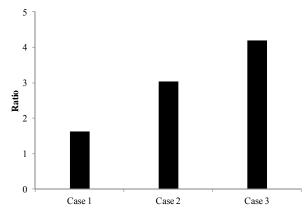


Fig. 6 The ratio between average particle concentration in the model and initial total particles at ambient environment in different cases at the 120 second. Particle concentration of each cell in the model is the sum of particle concentration in each category shown in Fig. 1

V.CONCLUSION

The study employed the Realizable k- ε model and Passive Scalar Models with Field Functions to simulate the particle concentration under idling conditions in a residential underground garage. Particle concentration distribution between Car 1 and Car 4 in both the vertical direction and the horizontal direction was analyzed in the study. Particle pollution in the residential underground garage was also evaluated by comparing with the initial total particle concentration at ambient environment. Some important conclusions were observed through the simulation and analysis.

Particle disperses much faster in the vertical direction than that in horizontal direction. A more prominent upward tendency of particle dispersion was observed when the number of vehicles under idling conditions is numerous. Average particle concentration after 120 seconds exhaust is almost 1.8-4.5 times higher than the initial total particles at ambient environment in three cases.

The cases in the study were undergone under the special conditions. To reflect the particle concentration distribution at the real environment in the residential underground garage, particle dispersion under the random car parking conditions will be considered in our future research.

ACKNOWLEDGMENT

The authors wish to acknowledge Kato Lab and Ooka Lab, Environmental Tech. for Urban Architecture, Institute of Industrial Science, The University of Tokyo, Tokyo, Japan for providing the advanced calculators to carry out the numerical simulation. The authors also wish to thank Yan Wang and Kai Zhu at the School of Municipal and Environmental Engineering, Harbin Institute of Technology in China for their assistances with the boundary and initial conditions measurement in the simulation.

REFERENCES

- Klot S. Von et al., "Increased asthma medication use in association with ambient fine and ultrafine particles," *European Respiratory Journal*, vol. 20, no. 3, pp. 691–702, Sep. 2002.
- [2] Nicolai T. et al., "Urban traffic and pollutant exposure related to respiratory outcomes and atopy in a large sample of children," European Respiratory Journal, vol. 21, no. 6, pp. 956–963. Jan. 2003.
- Respiratory Journal, vol. 21, no. 6, pp. 956–963, Jan. 2003.

 [3] Annette Peters et al., "Exposure to Traffic and the Onset of Myocardial Infarction," The new England Journal of Medicine, vol. 351, no. 17, pp. 1721–1730. Oct. 2004.
- [4] Pengzhi Jiang et al., "Simulation of the Evolution of Particle Size Distributions in a Vehicle Exhaust Plume with Unconfined Dilution by Ambient Air," Journal of the Air & Waste Management Association, vol. 55, no. 4, pp. 437–445, Apr. 2005.
- [5] Y. H. Liu, Z. He, and T. L. Chan, "Three-Dimensional Simulation of Exhaust Particle Dispersion and Concentration Fields in the Near-Wake Area of the Studied Ground Vehicle," *Aerosol Science and Technology*, vol. 45, no. 8, pp. 1019–1030, Aug. 2011.
- [6] D. B. Kittelson, W. F. Watts, and J. P. Johnson, "On-road and Laboratory evaluation of combustion aerosols—Part 1: Summary of diesel engine results," *Journal of Aerosol Science*, vol. 37, no. 8, pp. 913–936, Aug. 2006

- [7] Gao Ji-dong et al., "Size Distributions of Exhaust Particulates from a Passenger Car with Gasoline Engine(Translation Journals style)," *Journal* of Combustion Science and Technology, vol. 13, no. 3, pp. 248-252, Jun. 2007.
- [8] R. E. Britter, and S. R. Hanna, "Flow and dispersion in urban areas," Annu. Rev. Fluid Mech, vol. 35, pp. 469-496, Jan. 2003.
- [9] Wolf-Heinrich Hucho, and Gino Sovran, "Aerodynamics of road vehicles," *Annu. Rev. Fluid Mech.*, vol. 25, pp. 485-537, Jan. 2003.
- [10] C. J. Baker, "Flow and dispersion in ground vehicle wakes," *Journal of Fluids and Structures*, vol. 15, no. 7, pp. 1031-1060, Oct. 2007.
- [11] Matteo Carpentieri, Prashant Kumar, and Alan Robins, "An overview of experimental results and dispersion modelling of nanoparticles in the wake of moving vehicles," *Environmental Pollution*, vol. 159, no. 3, pp. 685-693, Mar. 2011.
- [12] Prashant Kumar, Matthias Ketzel, Sotiris Vardoulakis, Lissa Pirjola, and Rex Britter, "Dynamics and dispersion modelling of nanoparticles from road traffic in the urban atmospheric environment-A review," *Journal of Aerosol Science*, vol. 42, no. 9, pp. 580-603, Sep. 2011.
- [13] Dong-Hee Kim, Mridul Gautam, and Dinesh Gera, "Modeling Nucleation and Coagulation Modes in the Formation of Particulate Matter inside a Turbulent Exhaust Plume of a Diesel Engine," *Journal of Colloid and Interface Science*, vol. 249, no. 1, pp. 96-103, May. 2012.
- [14] Z.Q. Yin, J.Z. Lin, K. Zhou, and T.L. Chan, "Numerical Simulation of the Formation of Pollutant Nanoparticles in the Exhaust Twin-jet Plume of A Moving Car," *International Journal of Nonlinear Sciences and Numerical Simulation*, vol. 8, no. 4, pp. 535–544, Dec. 2007.
- [15] Z. Ning, C. S. Cheung, Y. Liu, M. A. Liu, and W. T. Hung, "Experimental and numerical study of the dispersion of motor vehicle pollutants under idle condition," Atmospheric Environment vol. 39, no. 40, pp. 7880-7893, Dec. 2005.
- [16] Z. Zhang, and Q. Chen, "Comparison of the Eulerian and Lagrangian methods for predicting particle transport in enclosed spaces," *Atmospheric Environment*, vol. 41, no. 25, pp. 5236–5248, Aug. 2007.
- Atmospheric Environment, vol. 41, no. 25, pp. 5236–5248, Aug. 2007.
 [17] Tsan-Hsing Shih et al., "A new k-ε eddy viscosity model for high Reynolds number turbulent flows," Computers & Fluids, vol. 24, no. 3, pp. 227-238, Mar. 1995.
- [18] Scott E. Miller, and Sean C. Garrick, "Nanoparticle Coagulation in a Planar Jet. Aerosol Science and Technology, vol. 38, no. 1, pp. 79–89, Jan. 2004
- [19] S.C. Garrick, K.E.J. Lehtinen, and Zachariah, "Nanoparticle coagulation via a Navier–Stokes/nodal methodology: Evolution of the particle field," *Journal of Aerosol Science*, vol. 37, no. 5, pp. 555–576, May. 2006.
- [20] D. Mukherjee, A. Prakash, and M.R. Zachariah, "Implementation of a discrete nodal model to probe the effect of size-dependent surface tension on nanoparticle formation and growth," *Journal of Aerosol Science*, vol. 37, no. 10, pp. 1388-1399, Oct. 2006.
- [21] Long Fan, Nong Xu, Xiyong Ke, and Hanchang Shi, "Numerical simulation of secondary sedimentation tank for urban wastewater," *Journal of the Chinese Institute of Chemical Engineers*, vol. 38, no. 5-6, pp. 425–433, Sep-Nov. 2007.
- [22] D. S. Kim, S. H. Park, Y. M. Song, D. H. Kim, and K. W. Lee, "Brownian coagulation of polydisperse aerosols in the transition regime," *Journal of Aerosol Science*, vol. 34, no. 7, pp. 859-868, Jul. 2003.
- [23] Yu Zhao, Shinsuke Kato, and Jianing Zhao, "Effect of coagulation on particle dispersion in an underground parking lot," *Building and Environment*, submitted for publication.