

# Simulations of Cryogenic Cavitation of Low Temperature Fluids with Thermodynamics Effects

A. Alhelfi, B. Sunden

**Abstract**—Cavitation in cryogenic liquids is widely present in contemporary science. In the current study, we re-examine a previously validated acoustic cavitation model which was developed for a gas bubble in liquid water. Furthermore, simulations of cryogenic fluids including the thermal effect, the effect of acoustic pressure amplitude and the frequency of sound field on the bubble dynamics are presented. A gas bubble (Helium) in liquids Nitrogen, Oxygen and Hydrogen in an acoustic field at ambient pressure and low temperature is investigated numerically. The results reveal that the oscillation of the bubble in liquid Hydrogen fluctuates more than in liquids Oxygen and Nitrogen. The oscillation of the bubble in liquids Oxygen and Nitrogen is approximately similar.

**Keywords**—Cryogenic liquids, cavitation, rocket engineering, ultrasound.

## I. INTRODUCTION

RECENTLY much attention has been paid to studies on wide applications of liquid hydrogen, oxygen and nitrogen which are commonly produced by liquefaction of these gases at low temperatures. Such liquids are known as cryogenic liquids [1]. Cryogenic liquids are attractive and of great interest in various engineering applications such as aerospace systems, nuclear physics and commercial launch vehicle which became the largest consumer of low temperature liquids. A key design issue is related to rocket fuel propellants because the by-products are clean and the power/gallon ratio is high [2]. Among different types of low temperature liquids, cryogenic hydrogen is a significant and important liquid because it is considered to be the most practical fuel in various engineering fields such as new passenger cars and aviation. Cryogenic hydrogen is considered as the fuel of the future.

Cavitation phenomenon due to pulsating pressure fields in cryogenic fluids is the most characteristic phenomenon. It produces significant thermal effects which subsequently change the fluid characteristics. Studying and assessing thermodynamics effects are essential for the design of liquid rocket turbomachinery systems that pump cryogenic liquids [3].

Cryogenic liquids, such as hydrogen and oxygen are commonly used as a high performance rocket engine

propellant. In order to supply and pressurize a large amount of propellant, a gas turbine driven turbopump is used as shown in Fig. 1, and a radial impeller is usually used as the main pump. The rotational speed of the turbopump should be high to minimize the pump size and mass, therefore, cavitation appearance cannot be avoidable [4].

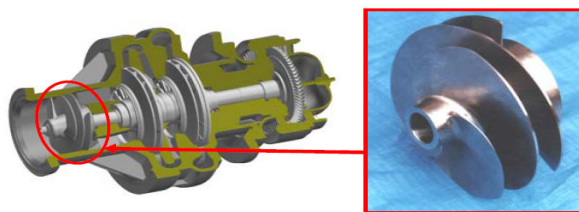


Fig. 1 A turbopump for a rocket engine and its inducer

In the open literature, several experimental techniques have been tested and various numerical models have been developed to characterize the cavitating flows in traditional fluids such as water. However, there are few studies about the cryogenic cavitation [5], [6]. Cavitation in liquid water takes place when the local ambient pressure of the liquid falls below the vapour pressure of that liquid at its local ambient temperature. It arises at considerable pressure variations. During the cavitation, bubbles or cavity produce high power and this phenomenon has caught the attention of scientists and researchers and it is still the focus of many research works [7]. In cryogenic fluids cavitation is developed easily with minor pressure pulsations. The reasons are that the low temperature liquids have smaller values of kinematic viscosity and surface tension than liquid water, which leads to decreasing cavitation resistance.

The characteristics of low temperature liquids are illustrated in Table I [8].

TABLE I  
CHARACTERISTICS OF CRYOGENIC LIQUIDS

	Helium	Hydrogen	Nitrogen	Oxygen
Melting temperature (K)	2.2	13.8	63.1	54.4
Boiling temperature (K)	4.2	20.4	77.3	90.2
Critical temperature (K)	5.2	33.2	126.2	154.6

Table I presents data corresponding to the equilibrium states on the curve of phase equilibrium. The boiling temperatures of all cryogenic fluids are usually lower than the ambient temperature when they are used under normal conditions. Even at the freezing point of water, 273 K, in real cryogenic systems, low-temperature liquids are either in the boiling or

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near-boiling state [1].

## II. PROBLEM STATEMENT AND MATHEMATICAL FORMULATION

The complete problem of a gas bubble undergoing nonlinear radial pulsations in a sound field is a complex problem, as its exact solution requires a consideration of the conservation equations in the liquid and in the gas coupled by suitable interface conditions.

Several forces affect the bubble dynamics in acoustic cavitation phenomenon as illustrated in Fig. 2.

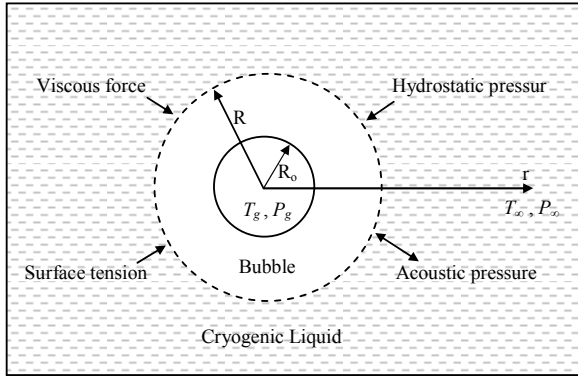


Fig. 2 Schematic representation of the forces acting on the surface of bubble in a cryogenic liquid with an ultrasonic field

A force balance on the bubble surface should always be considered in describing cavitation dynamics [9], [10], and it leads to the total pressure for the content as represented in (1):

$$P_g = P_{st} + P_{vk} + P_o + P_a(t) = \frac{2\sigma}{R} + 4\mu \frac{\dot{R}}{R} + P_o + P_a(t) \quad (1)$$

where  $(R)$  is the bubble radius,  $(\dot{R})$  is the bubble surface velocity,  $\sigma$  is the surface tension,  $\mu$  is the liquid viscosity,  $P_g$  is the gas pressure in the bubble,  $P_o$  is the hydrostatic pressure,  $P_{st}$  is the surface tension,  $P_{vk}$  is the viscous force,  $t$  is the time and  $P_a$  is the time dependent acoustic pressure and given by:

$$P_a(t) = P_a \sin \omega t \quad (2)$$

where  $\omega$  is the angular frequency and  $P_a$  is the amplitude of the acoustic pressure.

The pressure at the bubble surface ( $P_{LW}$ ) is written by taking the surface tension and the liquid viscosity into account:

$$P_{LW} = P_g - \frac{2\sigma}{R} - 4\mu \frac{\dot{R}}{R} \quad (3)$$

At infinity the pressure far from the bubble is denoted by ( $P_\infty$ ), and is given by,

$$P_\infty = P_o - P_a \sin \omega t \quad (4)$$

The most common approach to understand the radial motion

of a bubble within a viscous and compressible liquid was invented by Keller and Kolodner [11] and led to the Keller-Kolodner equation. More details regarding derivation of this equation can be found in [9], [10].

$$R\ddot{R}\left(1 - \frac{\dot{R}}{c}\right) + \frac{3}{2}\dot{R}^2\left(1 - \frac{\dot{R}}{3c}\right) = \left(1 + \frac{\dot{R}}{c}\right)H + \frac{R}{c}\frac{dH}{dt} \quad (5)$$

where  $H$  is the liquid enthalpy.

$$H(r, t) = \int_{p_\infty}^{p_L(r, t)} \frac{dP_L}{\rho_L} \quad (6)$$

The gas velocity distribution inside the bubble in terms of the temperature gradient is given as [9], [10],

$$u(r, t) = \frac{1}{\gamma P} \left[ (\gamma - 1)\lambda \left( \frac{\partial T}{\partial r} \right)_R - \frac{1}{3}r\dot{P} \right] \quad (7)$$

By applying the velocity boundary condition ( $u = \dot{R}$  at  $r=R$ ) for “(7),” the time dependent pressure term is obtained as,

$$\dot{P} = \frac{3}{R} \left[ (\gamma - 1)\lambda \left( \frac{\partial T}{\partial r} \right)_R - \gamma P \dot{R} \right] \quad (8)$$

The energy equation inside the bubble with spherical symmetry is written explicitly in the following form [9], [10],

$$\frac{\gamma}{\gamma - 1} \frac{P}{T} \left[ \frac{\partial T}{\partial t} + \frac{1}{\gamma P} \left\{ (\gamma - 1)\lambda \frac{\partial T}{\partial r} - \frac{1}{3}r\dot{P} \right\} \frac{\partial T}{\partial r} \right] - \dot{P} = \nabla \cdot (\lambda \nabla T) \quad (9)$$

where  $r$  is the radial distance from the center of the bubble,  $T$  and  $P$  are the gas temperature and pressure respectively,  $\dot{P}$  refers to the first derivative of pressure with respect to the time,  $\gamma$  is the specific heat ratio,  $\lambda$  is the thermal conductivity of the gas inside the bubble.

To include the thermodynamic effects on bubble dynamics, the liquid temperature on the external side of the bubble surface is assumed varied during oscillations.

$$T_g(R, t) = T_L(R, t) \quad (10)$$

The heat flux at the bubble surface at  $r = R$  is given as,

$$k_g \frac{\partial T_g}{\partial r} = k_L \frac{\partial T_L}{\partial r} \quad (11)$$

In this case the heat transfer between the bubble and the surrounding liquid is considered, so the energy equation of the liquid is required, and is given as [9], [10],

$$\frac{\partial T_L}{\partial t} + \frac{R^2 \dot{R}}{r^2} \frac{\partial T_L}{\partial r} = \frac{D_L}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial T_L}{\partial r} \right) \quad (12)$$

where  $D_L$  is the liquid thermal diffusivity, and  $T_L$  is the

temperature of the liquid at a distance  $r$  from the bubble center.

III. NUMERICAL APPROACH

A periodic solution is performed by numerical calculation for a gas (Helium) bubble with an equilibrium radius  $R_o$  of 25  $\mu\text{m}$  driven by an acoustic field with a frequency of 50 kHz and amplitude of 1 bar, in liquids (Nitrogen, Hydrogen and Oxygen). Initial temperature of all liquids is taken between the melting and evaporating point (70 K for liquid Nitrogen and Oxygen, 16 K for liquid Hydrogen). The calculation is started from time  $t=0 \mu\text{s}$  with the initial condition that  $R=R_o, \dot{R}=0$  for a gas bubble in liquids with physical properties described in Table II.

TABLE II  
LIQUIDS PHYSICAL PROPERTIES

Physical quantities	Liquid Nitrogen	Liquid Hydrogen	Liquid Oxygen
$C$ (m/s)	1092	1049	1272
$C_p$ (kJ/kg.K)	2.024	7.539	1.668
$\lambda$ (W/m.K)	0.1402	0.1153	0.1894
$\mu$ (N.s/m <sup>2</sup> )	0.2167	0.020	0.368
$\rho$ (kg/m <sup>3</sup> )	840.9	75.19	1236
$\sigma$ (N/m)	10.52	2.52	18.268

$C$  = speed of sound in the liquid,  $C_p$  = liquid specific heat,  $\lambda$  = the thermal conductivity,  $\mu$  = viscosity,  $\rho$  = density,  $\sigma$  = surface tension.

IV. RESULTS AND DISCUSSION

The effect of the acoustic pressure amplitude on the bubble radius is shown in Figs. 3-5.

From Figs. 3-5, it is clearly illustrated that the acoustic pressure amplitude has a great influence on the bubble dynamics. It is observed that the bubble starts to grow immediately and begins to oscillate in a nonlinear behaviour due to the action of the periodic acoustic pressure. Subsequently, the bubble can oscillate with low amplitude on a very short time scale. Two values of the acoustic pressure 1 bar and 0.7 bar are used in the comparison. It is clear that the value of the maximum bubble radius decreases with the decrease of the acoustic pressure amplitude. The main reason is that decreasing the acoustic pressure amplitude will transform the transient cavitation to stable cavitation.

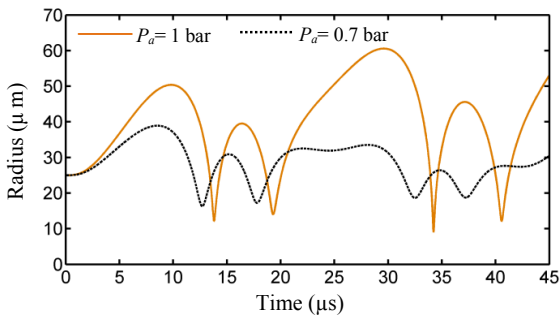


Fig. 3 Effect of acoustic pressure amplitude on radius-time behavior of liquid Nitrogen

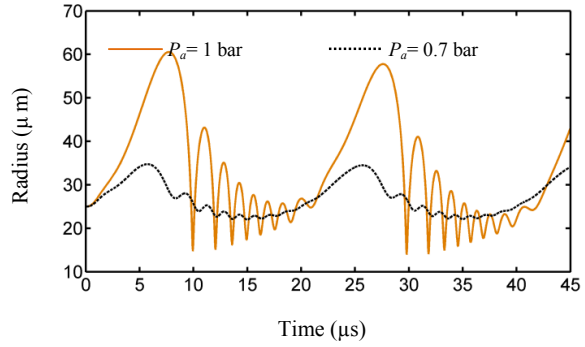


Fig. 4 Effect of acoustic pressure amplitude on radius-time behavior of liquid Hydrogen

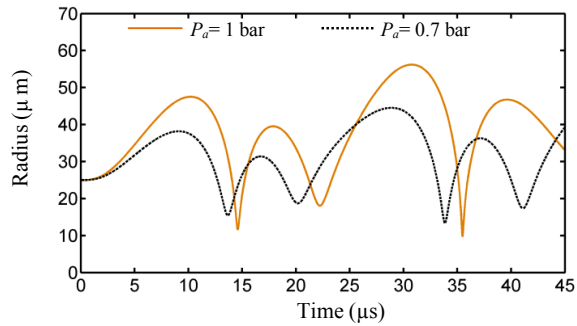


Fig. 5 Effect of acoustic pressure amplitude on radius-time behavior of liquid Oxygen

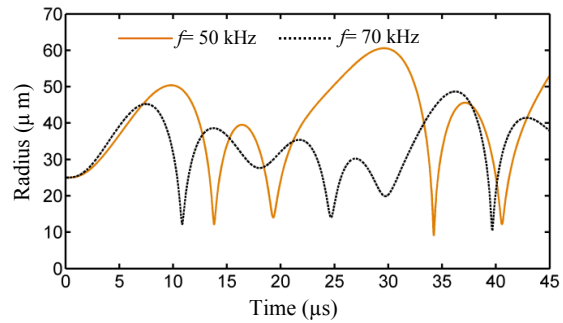


Fig. 6 Effect of acoustic frequency on radius-time behavior of liquid Nitrogen

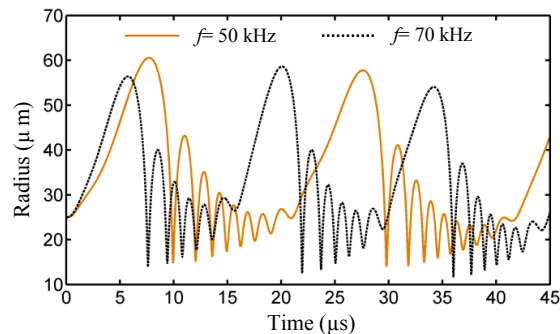


Fig. 7 Effect of acoustic frequency on radius-time behavior of liquid Hydrogen

The effect of the acoustic frequency on the bubble radius is shown in Figs. 6-8. Two frequencies (50 kHz and 70 kHz) are

used in the comparison.

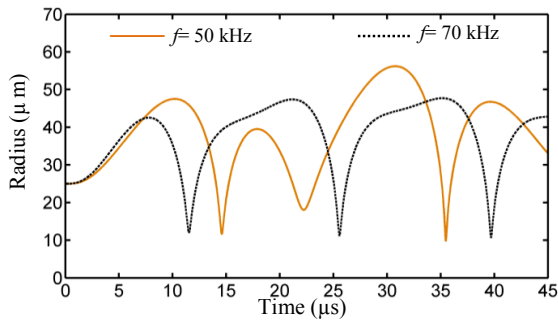


Fig. 8 Effect of acoustic frequency on radius-time behavior in liquid Oxygen

From Figs. 6-8 it can be noted that with an increase of the acoustic frequency, the number of cycles will increase due to increased number of the cycles of the acoustic pressure field.

Further more this leads to a decrease in the maximum bubble radius because there is not enough time for the bubble to grow.

The main components in studies of the cavitation phenomenon are the temperature and pressure fields, so the pressure-time variation and temperature-time variation are given in Figs. 9 and 10.

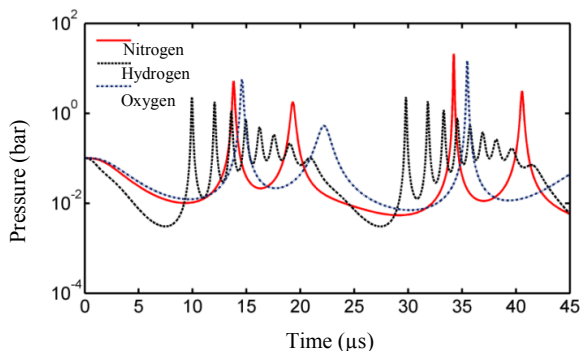


Fig. 9 The pressure at bubble center as a function of time with a logarithmic scale of the vertical axis

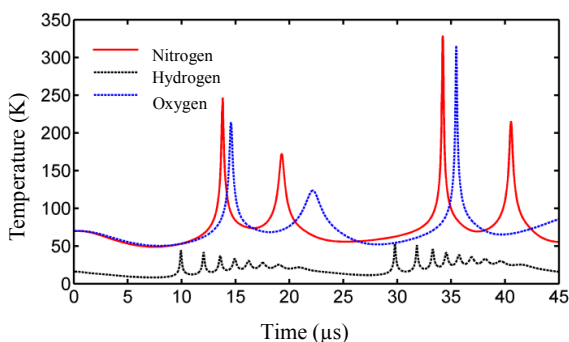


Fig. 10 The temperature at bubble center as a function of time with a logarithmic scale of the vertical axis

Maximum temperatures and pressures are predicted during

compression at the bubble center and at minimum bubble radius. In liquid nitrogen the values of pressure and temperature are 20.94 bar and 331.5 K, respectively. In liquid hydrogen pressure and temperature inside the bubble at minimum bubble radius are 2.25 bar and 54.3 K, respectively, while in liquid oxygen pressure and temperature inside the bubble, reach the maximum values of 14.78 bar and 318.6 K, respectively.

## V. CONCLUSION

In the present study, a numerical simulation method of cavitation in cryogenic liquids is established. A situation of a gas bubble (Helium) in the liquids Nitrogen, Oxygen and Hydrogen in an acoustic field at ambient pressure and low temperature is investigated. The computational model takes into account the thermal equilibrium assumption between the gas and liquid in the cavitating region.

The effects of the acoustic pressure amplitude and frequency on the bubble dynamics are presented.

The following conclusions were drawn:

- The present cavitation model shows that the oscillation of the bubble in liquid Hydrogen is more fluctuating than for liquids Oxygen and Nitrogen.
- The oscillation of the bubble in liquids Oxygen and Nitrogen is approximately similar.
- The change of the acoustic pressure amplitude results in a change of the kind of cavitation (stable and transit cavitation).
- Increasing the frequency leads to an increasing number of cycles.
- Considerable increases of temperature and pressure fields resulted from the collapse of microsize bubbles.
- Liquid properties affect the cavitation and bubble characteristics.

## ACKNOWLEDGMENTS

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