

# The Application of Queuing Theory in Multi-Stage Production Lines

Hani Shafeek, Muhammed Marsudi

**Abstract**—The purpose of this work is examining the multi-product multi-stage in a battery production line. To improve the performances of an assembly production line by determine the efficiency of each workstation. Data collected from every workstation. The data are throughput rate, number of operator, and number of parts that arrive and leaves during part processing. Data for the number of parts that arrives and leaves are collected at least at the amount of ten samples to make the data is possible to be analyzed by Chi-Squared Goodness Test and queuing theory. Measures of this model served as the comparison with the standard data available in the company. Validation of the task time value resulted by comparing it with the task time value based on the company database. Some performance factors for the multi-product multi-stage in a battery production line in this work are shown.

The efficiency in each workstation was also shown. Total production time to produce each part can be determined by adding the total task time in each workstation. To reduce the queuing time and increase the efficiency based on the analysis any probably improvement should be done. One probably action is by increasing the number of operators how manually operate this workstation.

**Keywords**—Production line, manufacturing, performance measurement, queuing theory.

## I. INTRODUCTION

THE queuing theory in the industry process used simulation models to study behavior of the industry process to improve work station's productivity by reducing waiting time. Engineers have applied results of queuing theory to show how cycle time is related to utilization of machine and statistic of inter-arrival time and service. These analyses provide means and predicting the average cycle time in steady-state condition. Queuing Theory can be applied in the assembly production line by using an analytical model.

Validation of the task time value resulted by comparing it with the task time value based on the company database.

Queuing model is a guide to improve both efficiency and performance in every workstation at the production line.

This work is a case study for application queuing theory in the multi-product multi-stage in the battery production line.

The company produces Canon B. battery lid (CD3-3836-00) as a cover to place battery in camera. Because of the increasing of its product demand, the company needs to examine the efficiency and other parameters of a production

Muhammad Marsudi was at the Department of Industrial Engineering, King Abdulaziz University, Rabigh, Saudi Arabia (e-mail: muhmarsudi@gmail.com)

Hani Shafeek is with the Department of Industrial Engineering, King Abdulaziz University, Rabigh, Saudi Arabia and also with Suez University, Industrial College of Education, Egypt (phone: +966 567434021; e-mail: hanishafeek@yahoo.com)

line to produce Canon B.

The aims of this study are:

- Applied queuing Theory in an assembly production line by using an analytical model
- Using a queuing model to examine performance of a production line

To give information about the studied production line in this case study, Fig. 2 shows the process flow to produce Canon B. battery lid.

Battery consists of the battery lid, inner plate, and lig log produce by injection machine.

Battery lid needs more treatments and processes through the first colour spray, the second colour spray, and ultra-violet spray before this battery lid are combined with other parts to become a product named 'Canon B. battery lid'. As shown in Fig. 2, the other parts, those needs to be combined with battery lid are inner plate, lig log, contact A, contact B, lock spring, open spring, and set plate. Canon B. battery lid product is shown in Fig. 1.

A new approach in designing of semiconductor equipment based Queuing Theory to reduce cycle time. They applied Queuing Theory in calculating optimum batch size for their processing equipment. They have showed the batch size required to achieve short time under different production situations [1].



Fig. 1 Canon B. battery lid product

## II. PROBLEM DEFINITION

As a result of the intense competition between companies resort some companies to develop new products in addition to the continuous improvement of existing products. Proposed a new product is expensive and needs many sources in addition to the time. Development current products depend on the optimal use of available sources. The current study is based on the identification of efficient workstations in an assembly line of batteries. A battery production line as the multi-product multi-stage consists of six workstations. This study used queuing theory to measure performance factors in

every workstation. The used queuing model is a guide to improve both efficiency and performance in every workstation at the batteries production line. Improve both efficiency and performance of workstations lead to improving of the batteries' productivity.

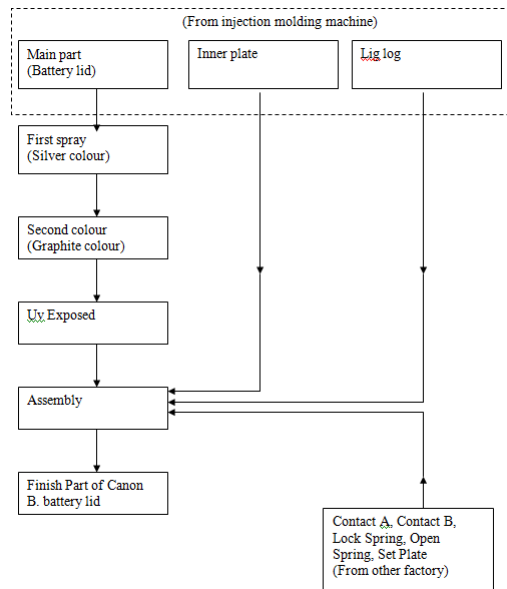


Fig. 2 Process flow to produce Canon B. battery lid

### III. RELATED WORK

Queuing theory in the industry process used simulation models to study behavior of the industry process to improve work station's productivity by reducing waiting time.

Queuing Theory was developed to provide models to predict behavior of systems that attempt to provide service for randomly arising and not unnaturally demand. The earliest problems studied were those of telephone traffic congestion [2]. Queuing analysis is carried out to analyze patient load in outpatient and inpatient services in order to facilitate more realistic resource planning. [3].

Queuing analysis examined medical staff and bed's number as the main resources in the hospital.

The study also applied the results of Queuing Theory to show how the cycle time is related to large and small production. In large production, cycle time is important to determine the amount of work in progress, and it can be determined by using Queuing models. The simulation has showed greater fluctuations in cycle time compared to the value predicated by Queuing Theory [4].

A comparison between Petri net (PN) and queuing network tools to determine the optimum values for flexible manufacturing system (FMS) measures of performance presented [5]. A queuing theory was presented by to calculate the total processing time for the processing time per pizza line at workstation in food production lines [6]. A model proposed and tested, which defines the psychological processes that mediate the relationship between perceived wait duration

(PWD) and satisfaction [7]. The scarce literature existing on modeling material delivery to assembly lines, kitting has received the greater attention. However, most available models utilize queuing theory to analyze dynamic performances of kitting systems and kit-replenished assembly systems [8]. Markov chains and queuing theories are widely used analysis, optimization and decision-making tools in many areas of science and engineering [9].

A new queuing model examined an inventory management system [10]. A model by used computer vision to examine belt speeds and throughput in industry application presented [11]. Many books are studying queuing theory and its industry application as [12]-[17].

A manufacturing system using computer spreadsheet software proposed, in which a static capacity planning model and stochastic queuing models are integrated.

The study finding for this comparison stated that the maximum allowed a relative error was 10% [18]. Number of approaches in the facility design for modeling material flow discussed using queuing networks. In these approaches, Poisson arrival or Markovian job, routing assumptions were used. However, for many manufacturing environments, these assumptions lead to an inaccurate estimation of the material handling system's performance and thus lead to poor facility designs [19]. An analytical model to estimate the performances (the transaction cycle time and waiting times) for product movement presented. The model is based on an open queuing network approach. The model effectiveness in performance estimation was validated through simulation [20].

Many studies focus on the application of simulation to exam cycle time such as [21]-[24].

The use of simulation is a powerful technique that helps decision maker to solve difficult problems in the design, control, or improvement of complex systems to reduce the cost, improve quality or productivity, and shorten time-to-market. However, the technology is still underutilized due to several reasons: (1) simulation modeling is a time-consuming and knowledge-intensive process that requires knowledge not only about simulation but also application and implementation tools; (2) most simulation models developed with current technology are customized "rigid" ones that cannot be reused or easily adapted to other similar problems; and (3) transforming related knowledge and information from an application domain to simulation is an unstructured or ill-defined process dependent on the skill and experience of individual modelers [25]. Based on these facts; the use of analytical approach such as Queuing Theory sometimes is better than simulation to analyze the performance of the manufacturing system.

### IV. METHODOLOGY

To accomplish the objectives of this study, the following steps were used.

1. Select a production line to be studied.
2. Collect data for each workstation. The data are throughput rate, number of operator, and number of parts that arrived and leave during part processing. Data for the number of

- parts that arrives, and leaves are collected at least at the amount of ten samples to make the data is possible to be analyzed by Chi-Squared Goodness Test.
- Analyze the arriving and leaving data by Chi-Squared Goodness Test to determine its variable distribution (Exponential or Poisson distribution).
  - Conduct performance measures of each workstation by using equations based on Queuing theory. The performance measures need to be measured are: utilization factor ( $\rho$ ), percentage of workstation idle time, number of parts in system ( $L_s$ ), number of parts in the queue ( $L_q$ ), waiting time spent in the queue ( $W_q$ ), waiting time spent in the system ( $W_s$ ), and task time.
  - Determine the efficiency of each workstation.
  - Make validation of the task time value resulted at step (4) by comparing it with the task time value based on the company database.

The equations based on Queuing theory that is used in this study can be described as follows:

$$\rho = \frac{\lambda}{\mu} \tag{1}$$

where,  $\rho$  is utilization factor,  $\lambda$  is average number of parts arriving in one unit of time, and  $\mu$  is service rate to parts in one unit of time.

$$\text{Percentage of idle workstation} = (1 - \rho) 100\% \tag{2}$$

$$L_s = \frac{\lambda}{\mu - \lambda} \tag{3}$$

where,  $L_s$  = number of parts in system

$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} \tag{4}$$

where,  $L_q$  = number of parts in queue

$$W_q = \frac{L_q}{\lambda} = \frac{L_s}{\mu} \tag{5}$$

where,  $W_q$  = waiting time spent in

$$W_s = W_q + \frac{1}{\mu} \tag{6}$$

where,  $W_s$  = waiting time spent in system

$$\text{Task time} = \frac{1}{\mu} \tag{7}$$

$$\text{Total task time} = \text{setup time} + \text{inspection time} + \text{task time} + \text{waiting time in queue} \tag{8}$$

$$\text{Cycle time} = \frac{\text{production time available per day}}{\text{demand per day or production per day}} \tag{9}$$

$$\text{Efficiency, } e = \frac{\sum \text{task time}}{(\text{number of operator}) \times (\text{assigned cycle time})} \times 100\% \tag{10}$$

$$\text{Maximum number of operator} = \frac{\sum_{i=1}^m \text{time for task } i}{\text{cycle time}} \tag{11}$$

### V. RESULT AND DISCUSSION

Some performance parameters for a certain process resulted in this study are shown in Table I. These results are based on the queuing theory analysis.

TABLE I  
PERFORMANCE PARAMETERS FOR A CERTAIN PROCESS

Process	performance parameters			
	Arrival distribution (unit/hour)	Leaving Distribution (unit/ hour)	Task time (sec)	Number of Operators (workers)
Injection molding:				
a) Main part	-	2500	6	4
b) Inner plate	-	1000	15	4
1 <sup>st</sup> colour spray	2547	2476	4.38	8
2 <sup>nd</sup> colour spray	2430	2338	3.42	8
UV spray	2172	1810	4.76	10
Assembly	1787	1826	19.53	16
Packing	1876	1828	5.2	5

To make validation for queuing theory model used in this study, all the task time values found by using queuing theory, as described In Table I, were compared to the existing standard task time values in database of the company. This comparison is shown in Table II. Accuracy between data using queuing theory and standard data in the company is 93.80%. Based on this data, it can be stated that queuing theory is valid to be used in this case study.

TABLE II  
TASK TIME VALUES BASED ON QUEUING THEORY AND STANDARD DATA

No.	Process	Task time by queuing theory analysis	Standard data available in IPC
1	Injection Molding		
	a) Main Part	6s	7s
	b) Inner plate	15s	16s
2	1 <sup>st</sup> colour spray	4.38s	3s
3	2 <sup>nd</sup> colour spray	3.42s	4.9s
4	UV spray	4.76s	4.50s
5	Assembly	19.53s	35.7s
6	Packing	5.2m	5.3m
	Total	365.09s ~ 6.085 minutes	389.1s ~ 6.485 minutes

Total production time to produce each part of Canon B. battery lid can be determined by adding the total task time in each workstation. As shown in Table III, the total production time is 42.79 minutes. The efficiency in each workstation is also shown in Table III.

TABLE III  
TOTAL TASK TIME AND EFFICIENCY IN EACH WORKSTATION

No.	Process	Total Task time	Efficiency
1	Injection Molding		
	a)Main Part	6s	30.25%
	b)Inner plate	15s	
2	1 <sup>st</sup> colour spray	15.43 s	54.85%
3	2 <sup>nd</sup> colour spray	22.42 s	39.11%
4	UV spray	13.10 s	67.97%
5	Assembly	35.30 s	80.98%
6	Packing	41.00 m	37.38%
	Total	2567.25 s ~ 42.79 minutes	

Packing process is the last process to produce Canon B. battery lid and it suffered the most in queue time. The process production is performed by batch process, once in ten days. This happened because the time to pack single part is less than the time in previous workstation (assembly workstation). In a single day, average part leaving assembly workstation is only 13500 units a day, but in packing workstation the total part can be packed is 14625 units for an hour.

Process in this packing workstation is handled manually by five operators and the average value of queuing time is 76%. This value is higher than the values in other workstations which have range between 20–42%. Value of 76% of queuing time is equal to 36 minute per carton. In each carton, there is 1000 pieces of parts. This estimated queue time is very high and will increase the total task time. As time goes on, the queue size will grow unlimited and it is difficult to control and monitor. It will also create a problem in storing the parts.

To reduce the queuing time and increase the efficiency, any possible improvement should be done. One possible action is by increasing the number of operators that manually operate this workstation. Based on the analysis, we found out that the efficiency of this workstation was only 37.78% when it was operated by five operators and it increased to 99.94% when it was operated by ten operators. Therefore, it is better for the company to hire at least ten operators to get a maximum efficiency. As informed before, currently there are 5 workers. To get a maximum efficiency, the company should hire:

$$\text{Maximum number of operators} = \frac{\sum_{i=1}^m \text{time for task } i}{\text{cycle time}} = \frac{41 \text{ min} \times 60}{246.15} = 9.99 \sim 10$$

From this data we can derive the efficiency of packing workstation:

$$\text{Efficiency, } e = \frac{41 \text{ min} \times 60}{(10) \times (246.15)} \times 100\% = 99.94\%$$

## VI. CONCLUSION AND RECOMMENDATION

This work used queuing system to an examined battery production line consistence of six work stations.

The study has shown that queuing theory can analyze production system such as utilization ( $\rho$ ), percentage of idle workstation, number of batch in the system ( $L_s$ ), number of batch in the queue ( $L_q$ ), expected time spends in the queue ( $W_q$ ), and expected time spent in the system ( $W_s$ ).

The workstations were injection molding, first colour spray, second colour spray, Ultra-violet, assembly and packing workstation. Arrival and leaving distribution were determined by using analysis of Chi-Squared Goodness of fit test to determine whether it was in Poisson or exponential distribution. The resulted data by queuing model used in this study was compared to the standard data in the company to check the validity of the queuing model. Accuracy between data using queuing theory and standard data in the company were 93.80%.

This study is very valuable for the company, because by knowing all information related to the performance of its production line, it is more effective and easier for the management of company to plan their production in the future. This study is also important because it examines multi-stage production line in a manufacturing industry by queuing theory approach, as such it is one of a few such studies to improve the performance of multi-product multi-stage production lines.

## REFERENCES

- [1] Hideaki Takagi, "Queuing Analysis," 1st Edition, Vol. 2, Elsevier Science Publisher, Netherland, 1993.
- [2] Syski, R. "Introduction to Congestion Theory in Telephone Systems," 2nd Edition, Elsevier Science Publisher, Amsterdam, 1986.
- [3] Mital, K.M., "Queuing analysis for outpatient and inpatient services: a case study," Management Decision, Vol. 48 No. 3, pp. 419-439, 2010.
- [4] Cooper, R.B., "Queuing Theory: Chapter 10 in Stochastic Models," Heyman and M.J. Sobel Edition, North Holland, 1990.
- [5] Ullah, H., "Petri net versus queuing theory for evaluation of FMS," Assembly Automation, Vol.31, No. 1, pp. 29-37, 2011.
- [6] Tsarouhas, P.A., "A comparative study of performance evaluation based on field failure data for food production lines," Journal of Quality in Maintenance Engineering, Vol. 17 No. 1, pp. 26-39, 2011.
- [7] McGuire, A.M., "A framework for evaluating the customer wait experience," Journal of Service Management, Vol. 21 No. 3, pp. 269-290, 2010.
- [8] Caputo, A.C. and Pelagagge, P.M. "A methodology for selecting assembly systems feeding policy," Industrial Management & Data Systems, Vol. 111 No. 1, pp. 84-112, 2011.
- [9] Mehmood, R. and Lu, J.A. "Computational Markovian analysis of large systems," Journal of Manufacturing Technology Management, Vol. 22 No. 6, pp. 804-817, 2011.
- [10] Diaz, D.Z. "New ways of thinking about nurse scheduling," Journal of Advances in Management Research, Vol. 7 No. 1, pp. 76-93, 2010.
- [11] Gudmundsson, D. and Goldberg, K., "Optimizing robotic part feeder throughput with queuing theory," Assembly Automation, Vol. 27 No. 2, pp. 134-140, 2007.
- [12] Papadopoulos, H.T., Heavy, C. and Browne, J., "Queueing Theory in Manufacturing System Analysis and Design," Springer Verlag GmbH, 2013.
- [13] Smith, J.M. and Tan, B., "Handbook of Stochastic Models and Analysis of Manufacturing System Operations," Springer, 2013.
- [14] Guy L.C. and , Feldman, R.M., Manufacturing Systems Modeling and Analysis, Second Edition, Springer, 2010.
- [15] Gershwin, S.B., "Manufacturing Systems Engineering, Englewood Cliffs, NJ: Prentice-Hall, 1994.
- [16] Yao, D.D., "Stochastic Modeling and Analysis of Manufacturing Systems," Berlin: Springer-Verlag, 1994.
- [17] Askin, R.G. and Standridge, C.R., "Modeling and Analysis of Manufacturing Systems," New York: Wiley, 1993.
- [18] Koo, P.H., Moodie, C.L., and Talavage, J.J., "A spreadsheet model approach for integrating static capacity planning and stochastic queuing model," International Journal of Production Research, vol. 33, no. 5, pp. 1369-1385, 1995.
- [19] Sukhotua, V. and Peters, B.A., "Modelling of material handling systems for facility design in manufacturing environments with job-specific routing," International Journal of Production Research, vol. 50, no. 24, pp. 7285-7302, 2005.

- [20] Marcheta, G., Melacina, M., Perottia, S. and Tappiaa, E., "Analytical model to estimate performances of autonomous vehicle storage and retrieval systems for product totes," *International Journal of Production Research*, Vol. 50, No. 24, pp. 7134-7148, 2012.
- [21] Sivakumar, A.I. and Chong, C.S., "A simulation based analysis of cycle time distribution, and throughput in semiconductor backend manufacturing," *Computer in Industry*, Vol. 59, pp. 78-45, 2001.
- [22] Domaschke, J. and Brown, S., "Effective implementation of cycle time reduction," *Proceeding of the 1998 Winter Simulation Conference*, USA, 1998.
- [23] Wang, M., Sun, G. and Wang D., "Manufacturing simulation – An effective tool for productivity improvement productivity and reducing manufacturing cycle time through simulation modeling," *Proceeding of 3rd International Microelectronic & Systems Conference*, Malaysia, 1993.
- [24] Toh, G.K., Teck, U.W., Lie, A., Sun, G., Ming, W., and Kok, K. "Reducing manufacturing cycle time of wafer fab with simulation," *World Scientific*, Vol. July 1995, pp. 889-896, 1995.
- [25] Zhou, M., Chen, Z., He, W. and Chen, X., "Representing and matching simulation cases: a case-based reasoning approach," *Computers & Industrial Engineering*, Vol. 59, pp. 115-125. 2010.