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A Comparative Study between Displacement and Strain Based Formulated Finite Elements Applied to the Analysis of Thin Shell Structures

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***Abstract*—**The analysis and design of thin shell structures is a topic of interest in a variety of engineering applications. In structural mechanics problems the analyst seeks to determine the distribution of stresses throughout the structure to be designed. It is also necessary to calculate the displacements of certain points of the structure to ensure that specified allowable values are not exceeded. In this paper a comparative study between displacement and strain based finite elements applied to the analysis of some thin shell structures is presented. The results obtained from some examples show the efficiency and the performance of the strain based approach compared to the well known displacement formulation.

***Keywords*—**Displacement formulation, Finite elements, Strain based approach, Shell structures.

I. INTRODUCTION

CONSIDERABLE attention has been given to applying the finite element method in the analysis of curved structures. Grafton and Strome [1] developed conical segments for the analysis of revolution shells. Later Jones and Strome [2] modified the method and used meridional elements which were found to lead to considerably improved results for the stresses. Curved rectangular and cylindrical shell elements were also developed by Connor and Brebbia [3] Cantin and Clough [4] and Sabir and lock [5]. Several shell elements have been developed to analyze shell structures of different shape, except the last one, all previous elements are developed using the displacement formulation. Meanwhile, at Cardiff University (UK), a different approach has been used to the development of curved and shell elements by Sabir et al. [6]-[13]. This approach is based on the development of displacement functions based on the strain approach and satisfying the compatibility equation. An element for arches deforming in the plane, containing the curvature and out of the plane curvature [6] and [7] as well as for cylindrical shells [8]-[10] were first developed. This approach was further extended to develop a general quadrilateral cylindrical element and was used to investigate the stress concentration problems in cylinders having circular and elliptical holes, and also to obtain a solution to the problem of normally intersecting cylinders [11]. Recently, Sabir and Ramadhani [12] developed an even simpler curved element for general shell analysis. The element is rectangular in plan and has only the essential five external nodal degrees of freedom at each of the four corner nodes. The formulated element was tested by applying it the analysis of cylindrical as well as spherical shells [13] and the results show a high degree of accuracy and can converge to the correct solution with relatively coarse meshes. Some other elements for shells and three- dimensional elasticity have been also developed by Djoudi et al. [14], Sabir et al. [15], Belarbi et al. [16] and Assan [17]; from the validation tests, these elements have been shown to produce results of an acceptable degree of accuracy without the use of large number of elements. Lately, Djoudi and Bahai have developed a new strain based shell element for the linear and nonlinear analysis of cylindrical shells [18] and two other elements for vibration analysis of shell structures [19], [20]. The effectiveness of these elements was demonstrated and good convergence was also observed. Most recently, a new flat shell element was formulated by Hamadi et al. [21]; it is a quadrilateral element obtained by the superposition of the Q4SBE1 (quadrilateral strain based element) membrane strain based element [22] with the ACM standard plate bending element [23], [24]. The performance of the developed shell element is evaluated on standard test problems. Finally, the main objective of this paper is to present the advantageous and efficiency of the shell finite elements based on the strain approach compared to other displacement elements, and this is through some validation tests.

II. PRESENTATION OF DIFFERENT FORMULATIONS

According to the choice of the interpolation field, several models of the finite elements can be generated which are [25]:

A. Displacement Model

This model is the most popular formulation. In this model, the finite elements are based on an interpolation of the displacements field. The displacements are determined in a single and detailed way in the structure, whereas the stresses are not continuous at the boundaries [26].

B. Stress Model

In this model the element is formulated on the base of stress field approximation only.

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C. Mixed Model

This model is based on two independent interpolations of two or more various unknown fields, generally the displacements fields and stresses fields within the element. This model takes the unknown parameters of these fields as degrees of freedom.

D. Hybrid Model

This model takes in consideration an assumed stress distribution within the element and assumed displacements along its edges.

E. Strain Based Approach

This approach is based on the calculation of the exact terms representing all the rigid body modes and the other components of the displacement functions; which are based on assumed independent strain functions insofar as it is allowed by the compatibility equations.

III. ADVANTAGEOUS OF THE STRAIN BASED APPROACH

Direct interpolation based on the strain approach provides a better precision on these values and on constraints and displacements (obtained by integration); compared to the classic formulation where deformations are obtained by derivation of the chosen displacement fields.

The main advantages of this approach are [27], [28]:

1) Easy satisfaction of the main two convergence criteria bound directly to strains (constant strains and rigid body movement).
2) Effortlessly decoupling of the various strain components (a field of uncoupled displacements generates coupled strains).
3) Possibility of enriching the field of displacements by terms of high order without the introduction of intermediate nodes or of supplementary degrees of freedom (allowing so to treat the problem of locking).

IV. PRESENTATION OF SOME DEVELOPED FINITE ELEMENTS BASED ON THE STRAIN APPROACH

A. Hazim Element

A strain-based hyper finite element is developed by Hazim [29]. This element is rectangular in plan with corner nodes only and six degrees of freedom at each node. These degrees of freedom are taken to be the essential external degrees of freedom namely u, y, w, ∂ w/∂ y and ∂ w/∂ x with an additional degree of freedom representing the in plane rotation associated with plane membrane elasticity Ø represented by:

$$\emptyset = 1/2(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}) \tag{1}$$

B. Mousa and El Naggar Element

A spherical shell rectangular strain-based finite element was developed also by A.I. Mousa and M.H. El Naggar [30], using shallow shell formulations. The element has six degrees of freedom at each corner node, the essential five external degrees of freedom as well as an additional sixth degree of freedom representing the in-plane rotation. This element can be used in conjunction with arched beam elements having all the essential six degrees of freedom. This element is applied to the analysis of shell roof with diagonal arched beams, acceptable results are obtained.

C. Djoudi and Bahai Element

A cylindrical strain-based shell element for vibration analysis of shell structures was developed by Djoudi and Bahai [19]. This element have only five degrees of freedom at each of the four corners, u, v, w, ∂ w/∂ x and ∂ w/r∂ θ. Therefore, the shape functions should contain 20 independent constants. First the displacement fields are:

$$u = a_7 x + a_8 r x\theta - a_{10} r^2 \theta^2 = 2 + a_4 r\theta/2 - a_4 r^3 \theta/24 - a_4 r^4 \theta^5/120 - a_4 r^2 \theta^3/12 \tag{2}$$

$$v = - a_4 x2/2 + a_4 r\theta + a_4 r x\theta + a_4 x/2 + a_4 r^2 \theta^3/6 + a_4 r^2 x\theta^3/6 + a_4 r^3 \theta^4/24 + a_4 r^3 x\theta^4/24 + a_4 r x\theta^2/4 \tag{3}$$

$$w = - a_4 x2/2 - a_4 x3/6 - a_{14} r x2\theta/2 - a_{15} r x3\theta/6 - a_{16} r^2 \theta^2/2 - a_{17} r^2 x\theta^2/2 - a_{18} r^3 \theta^3/6 - a_{19} r^3 x\theta^3/6 - a_{20} r x\theta/2 \tag{4}$$

The complete shape function is the sum of corresponding expressions. Consider the cylindrical element shown in Fig. 1, O is the centre of the element and the origin of the curvilinear co-ordinates x, y and z. It is well known that the stiffness matrix for the element can be written in the form:

$$[\mathrm{K}] = [C^{-1}]^T \left\{ \int_{-\alpha}^{\alpha} \int_{-\beta}^{\beta} [B]^T \, [D][B] r dx \, d\theta \right\} [C^{-1}] \tag{5}$$

and the mass matrix can be written as

$$[m] = [C^{-1}]^T \left\{ tp \int_{-\alpha}^{\alpha} \int_{-\beta}^{\beta} [P]^T \, [P] r dx \, d\theta \right\} [C^{-1}] \tag{6}$$

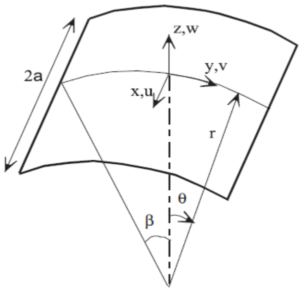

Fig. 1 Shell element

D. Hamadi Et Al Element

Hamadi et al [21] formulated a new shell element baptised ACM_Q4SBE1. It composed by assembling the two elements: Membrane element Q4SBE1 and bending element ACM Fig. 2.

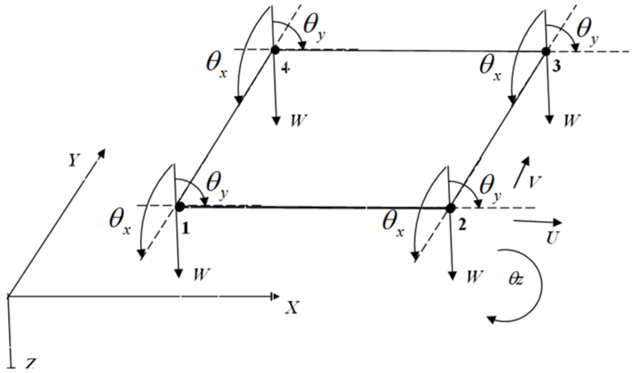

Fig. 2 The shell element ACM_ Q4SBE1

The stiffness matrix of the formulated shell element ACM_ Q4SBE1 is obtained by using the analytical integration of the membrane and bending stiffness matrices. The calculation of the element stiffness matrix is summarized with the following well known expressions equations (7)-(9):

$$[K_e]=[A^{-1}]^T\left[\iint_S [Q]^T[D].[Q].dx.dy\right][A^{-1}] \tag{7}$$

$$[K_e]_{[\mathrm{K0}]} = [A^{-1}]^T [K_e]_{[\mathrm{K0}]}\,[A^{-1}] \tag{8}$$

With:

$$[\mathrm{K}_0] = \iint_S [Q]^{\mathrm{T}}[\mathrm{D}].[Q].\mathrm{d}\,x.\mathrm{d}\,y \tag{9}$$

V. PRESENTATION OF SOME NUMERICAL APPLICATIONS

A. Analysis of Gable Roof

A complex type of hyperbolic paraboloid shell which is usually referred to as the gable (or hipped) roof is analyzed by Hazim [29]. This structure is usually constructed by a combination of four hyper shells together with crown and edge beams, as shown in Fig. 3. The dimensions of the gable roof are shown in Fig. 4.

The analysis is carried out using strain based element of Hazim. Due to symmetry of geometry and the uniform loading, only one quarter of the shell needs to be analyzed. Convergence tests were carried out, using the last strain element, for the maximum deflection at the shell crown and the maximum bending moments in the crown and edge beams. The convergence curves for these three values are given in Figs. 5-(7), which show that an 8x8 mesh of the present elements gives results that are in close agreement to those given by Schnobrich [33].

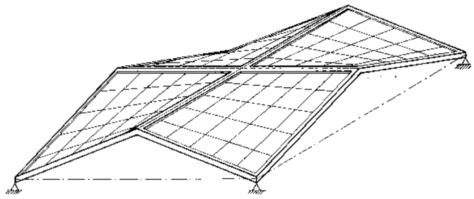

Fig. 3 Gable roof hyperbolic paraboloid shell

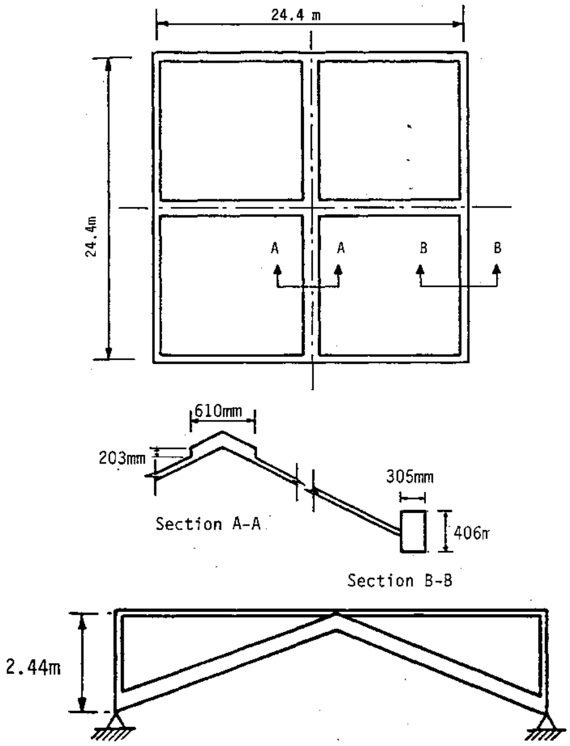

Fig. 4 Gable roof dimensions and configuration

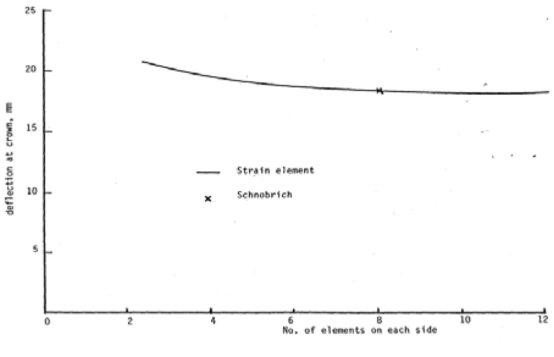

Fig. 5 Gable roof, Convergence of maximum deflection w at crown

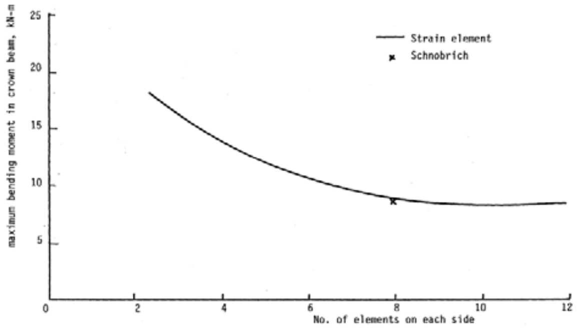

Fig. 6 Gable Roof. Convergence of maximum bending moment in crown beam

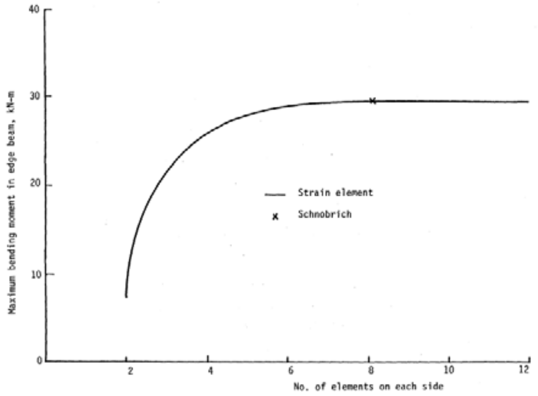

Fig. 7 Gable Roof, convergence of maximum bending moment in edge beam

B. Scordelis-Lo Roof

The next test to be considered which is frequently used to test the performance of shell element is that of Scordelis-Lo roof having the geometry as shown in Fig. 9. The straight edges are free, while the curved edges are supported on rigid diaphragms along their plan. The geometrical and mechanical characteristics are given in Fig. 9. Considering the symmetry of the problem only one quarter of the roof is analysed (part ABCD).

The results obtained by the flat shell elements ACM_Q4SBE1[22], for the vertical displacement at the midpoint B of the free edge and the centre C of the roof are compared to the reference values based on the deep shell theory. Furthermore, the convergence of this element is also compared to other kinds of quadrilateral shell elements Q4$^{\gamma}$ 24, DKQ24 [31] and ACM-SBQ4 [32]. We should mention that the analytical solution based on the shallow shell theory is given by Scordelis and Lo [34], which is slightly different from the deep shell theory.

Convergence curves (Figs. 9 and 10) show the good contribution of the strain based approach.

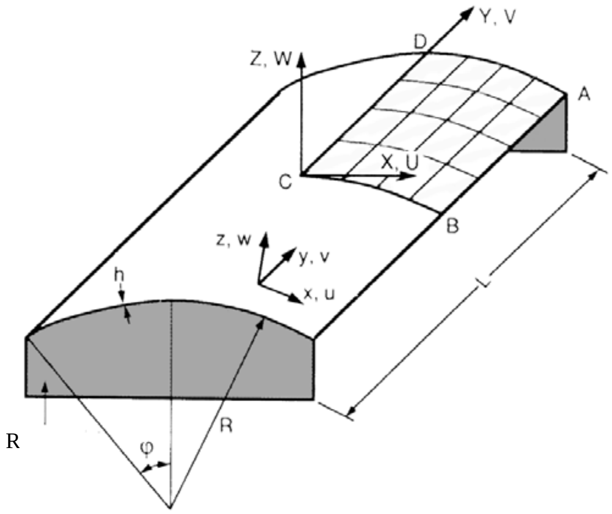

Fig. 8 Scordelis-Lo roof

Data:

L = 6 m; R = 3 m; h = 0.03 m; φ = 40°
E = 3 x 1010 Pa; ν = 0; fz = -0,625 x 104 Pa

Boundary conditions:

U = W = θY = 0 for AD

Symmetry conditions:

U = θY = θZ = 0 for CD
V = θX = θZ = 0 for CB

Reference value (Deep Shell Theory):

WB = -3,61 cm; WC = 0,541 cm

Analytical solution (Shallow Shell theory):

WB = -3,703 cm; WC = 0,525 cm
UB = -1,965 cm; VA = -0,1513 cm

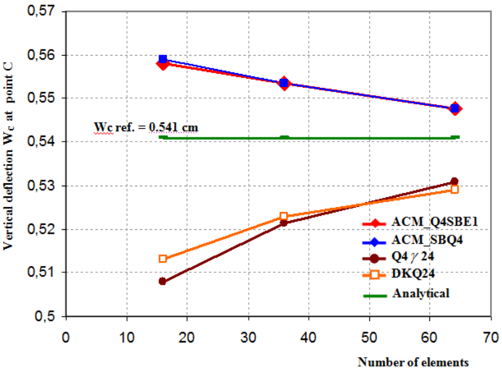

Fig. 9 Convergence curve for the deflection Wc at point C

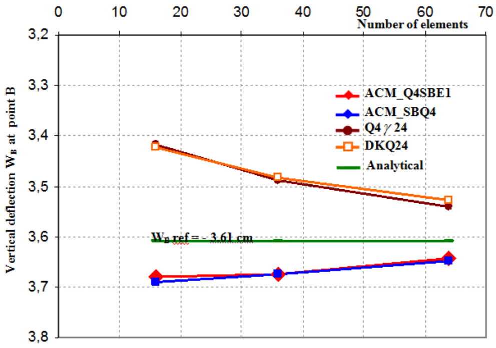

Fig. 10 Convergence curve for the deflection WB at point B

VI. CONCLUSION

From the numerical applications and the results obtained for different finite elements, the following conclusions can be drawn:

Both strain based finite elements ACM_Q4SBE1 and ACM_SBQ4 have similar behavior and they show good converge to the exact solution compared to the corresponding displacements based finite elements Q4$^{\gamma}$24 and DKQ24.

The results obtained by the use of Hazim element are shown to converge more rapidly, for gable roof example.

The deflections and stresses in the hypar are shown to be highly influenced by the stiffness of the edge beams, and a significant reduction in the values of the maximum membrane stresses in the shell are shown to occur when stiffer edge beams are used.

The efficiency of the strain based elements has been demonstrated, and the advantageous of using the strain approach are confirmed.

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