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Loudspeaker Parameters Inverse Problem for Improving Sound Frequency Response Simulation

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***Abstract*—**The sound pressure level (SPL) of the moving-coil loudspeaker (MCL) is often simulated and analyzed using the lumped parameter model. However, the SPL of a MCL cannot be simulated precisely in the high frequency region, because the value of cone effective area is changed due to the geometry variation in different mode shapes, it is also related to affect the acoustic radiation mass and resistance. Herein, the paper presents the inverse method which has a high ability to measure the value of cone effective area in various frequency points, also can estimate the MCL electroacoustic parameters simultaneously. The proposed inverse method comprises the direct problem, adjoint problem, and sensitivity problem in collaboration with nonlinear conjugate gradient method. Estimated values from the inverse method are validated experimentally which compared with the measured SPL curve result. Results presented in this paper not only improve the accuracy of lumped parameter model but also provide the valuable information on loudspeaker cone design.

***Keywords*—**Inverse problem, cone effective area, loudspeaker, nonlinear conjugate gradient method.

I. Introduction

An important acoustical characteristic of moving-coil loudspeakers (MCL) operated in a free-field or an half-space free-field condition is the sound pressure level (SPL) transfer response between electrical input voltage and the sound pressure output generated at an on-axis reference point in the far field [1], [2]. The frequency response and the distortion characteristics of the moving-coil loudspeaker are often analyzed and simulated using the lumped parameter model (LPM) [3]. The LPM contains an important set of electroacoustic parameters, including parameters, namely, voice-coil inductance L_e, electrical resistance R_e, mechanical mass M_m, mechanical resistance R_m, mechanical stiffness K_m, and force factor Bl, that are used to represent the electrical fields and mechanical domains of the moving-coil loudspeaker. In addition, the cone effective area S_d is the effective projected area of the loudspeaker cone for transferring the effective cone displacement into the sound pressure. The cone effective area S_d depends largely on the shape and properties of the surround. It generally estimated as the cone diameter plus 1/3 to 1/2 width of the loudspeaker surround. The S_d is difficult to measure, because the cone effective area would be varied which depended on the geometry change of the cone. Klippel et al. [4] addressed the idea of using the laser vibrometer to scan the geometrical characteristics of cone mode shapes during the loudspeaker unit vibration. The experimental study of non-linear vibrations in a loudspeaker cone was made by Zhang et al. [5]; the experimental results show that non-linear vibrations in a thin shell are related to bending resonance. The amplitude and phase measurements have been made of the mechanical motion of different points on the cone diaphragm for various critical frequencies [6]. The behavior of a loudspeaker diaphragm beyond the piston range of operation has previously only been investigated using analytic techniques such as the Finite Element Method [7]. The study of sound reproduction quality of loudspeaker [8] is highly dependable on the vibration and radiation properties of the loudspeaker cone. However, accurate estimations of the cone effective area, without the need for expensive equipment and complex operational processes, are still being designed, developed, and improved. This work helps to fill certain gaps in these efforts.

Having elicited the interest of many scientists in recent years due to their wide range of applications, the inverse problems of differential equations now constitute a discipline between mathematics and engineering. Inverse problems have been successfully applied to identify the surface temperature [9]-[11], Heat Conduction [12] and geometric shape [13], which are either not measurable or too difficult to measure. The proposed method involves the following four calculation procedures: (i) direct problem, for solving both the voice-coil displacement and current simultaneously from the transduction equations of an MCL; (ii) conjugate gradient method (CGM) [14]-[17], for identifying the search direction to reach optimal values of parameters; (iii) adjoint problem, wherein the Lagrange equations derived from the transduction equations are applied to obtain the gradient of the objective function; and (iv) sensitivity problem, for obtaining the appropriate step length of the CGM. All unknown parameters can be obtained using these procedures.

This study is organized as follows. Section II presents the mathematical framework of MCL. Section III contains the description of the construction of the presented method and its calculation steps. The experiment is presented in Section IV. Finally, related work is summarized in Section V.

II. The Mathematical Framework of Moving-coil Loudspeaker

The typical MCL is shown in Fig. 1, consists of a magnetic system (magnet under yoke and polar piece) and a vibration system (diaphragm and voice coil). The magnetic system of MCL transfers electrical-to-magnetic force to drive the voice coil, and that a diaphragm suspension system is used to

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generate vibration, causing the voice-coil to suspend and begins to vibrate. Parameters in the electrical domain include input voltage signal $e(t)$, electrical inductance L_e, and resistance R_e. Meanwhile, electroacoustic parameters in the mechanical domain include mechanical stiffness K_m, mechanical mass M_m, and mechanical resistance R_m. In addition, the force factor Bl converts magnetic force to mechanical force between the electrical and mechanical domains, $M_{a\text{-}rad}$ and $R_{a\text{-}rad}$ represent the acoustic radiation mass and acoustic radiation resistance of air loading. These parameters are essential in the operation of the MCL lumped parameter model.

Fig. 1 The graph of moving-coil loudspeaker

The governing equations of the MCL can be expressed as

$$\begin{aligned} Bli(t) &= M_m\ddot{x}(t) + R_m\dot{x}(t) + K_m x(t) \\ &\quad + M_{a-rad}S_d\dot{u}(t) + R_{a-rad}S_d u(t) \end{aligned} \tag{1}$$

$$L_e\dot{i}(t) + R_e i(t) + Bl\dot{x}(t) = e(t) \tag{2}$$

where $u(t) = S_d dx(t)/dt$ is the volume velocity. If the far-field pressure radiated by a baffled piston, the simplified model of $M_{a\text{-}rad}$ and $R_{a\text{-}rad}$ are given by,

$$M_{a-rad} = 0.85\rho_0 a / S_d \tag{3}$$

$$R_{a-rad} = 0.5\rho_0 c / S_d \tag{4}$$

where a is the radius of cone, ρ_0 (=1.29kg/m^3) is the sound pressure density in air, c (=343m/s) is the sound speed in air. Subscribing (3) and (4) into (1) and (2), then (1) can be rewritten as,

$$\begin{aligned} Bli(t) &= \left(M_m + 1.0965aS_d\right)\ddot{x}(t) \\ &\quad + \left(R_m + 171.5S_d\right)\dot{x}(t) + K_m x(t) \end{aligned} \tag{5}$$

If the given parameters (M_m, R_m, K_m, Bl, R_e, L_e, a, and Sd) and the initial conditions of the displacement $x(t)$, current $i(t)$, and volume velocity $u(t)$ are known, the solutions for voice-coil displacement and current for (2) and (5) are the direct problems and the solutions are direct solutions. In contrast, within any given time interval $t \in (0, t_f)$, if input voltage $e(t)$, voice-coil displacement $x(t)$, and current $i(t)$ are known, the solutions for electroacoustic parameters for (2) and (5) are the inverse problems and the solutions are inverse solutions. Inverse solutions are the focus of this study, and the calculation steps are discussed in the next section.

III. Inverse Method

A. Nonlinear Conjugate Gradient Method

Through the measured value $i_{mea}(t)$ and estimated value $i(t)$, the objective function J can be defined as

$$J(\mathbf{w}) = \int_0^{t_f} \left[i(t;\mathbf{w}) - i_{mea}(t)\right]^2 dt \tag{6}$$

where $\mathbf{w}$ is a unknown vector to be determined. The above equation reveals that when the objective function J is at minimum value, the estimated value $i(t)$ will approach the measured value $i_{mea}(t)$. Solving the unknown parameters in direct problem as it gradually moves toward the minimum value will obtain the solution for an optimal set through iterations. Therefore, the nonlinear conjugate gradient method is designed to optimize by repeated iteration and which leads to objective function minimization. The iterative equation is

$$\mathbf{w}^{(k+1)} = \mathbf{w}^{(k)} - \beta^{(k)}\mathbf{P}^{(k+1)} \tag{7}$$

Here the superscript k represents the k^{th} iteration, $\beta^{(k)}$ denotes the k^{th} search step length, and $\mathbf{P}^{(k+1)}$ is the $(k+1)^{th}$ search direction with decreased value:

$$\mathbf{P}^{(k+1)} = \nabla J^{(k)} + \gamma^{(k)}\mathbf{P}^{(k)} \tag{8}$$

where $\nabla J^{(k)}$ represents the gradient of the objective function at the k^{th} iteration. The conjugate gradients update parameter $\gamma^{(k)}$ is given by

$$\gamma^{(k)} = \left\|\nabla J^{(k)}\right\|^2 / \left\|\nabla J^{(k-1)}\right\|^2 = \sum_{j=0}^{N_k}\left(\frac{\partial J}{\partial r_j}^{(k)}\right)^2 / \sum_{j=0}^{N_k}\left(\frac{\partial J}{\partial r_j}^{(k-1)}\right)^2 \tag{9}$$

Note that, when descending direction does not take into account $\gamma^{(k)}\mathbf{P}^{(k)}$, then $\mathbf{P}^{(k+1)} = \nabla J^{(k)}$ in (8). At this point, CGM will degenerate into steepest decent method.

During the convergence process of CGM, the voice-coil displacement $x(t)$, the gradient of objective function ∇J, and the step length β must be solved. They are respectively the solutions of the direct problem, adjoint problem, and sensitivity problem, which will be explained in following subsections.

B. Solving Adjoint Problem for ∇J

Using Lagrange multiplier method to multiply the direct problem by a Lagrange multipliers $\boldsymbol{\lambda}(t)$, and substituting it into the objective function in (6) to obtain a new objective function. The adjoint problem is established to solve the Lagrange multipliers $\boldsymbol{\lambda}(t)$ as

$$\begin{aligned}&\hat{M}_m\ddot{\lambda}_1(t)-\hat{R}_m\dot{\lambda}_1(t)+K_m\lambda_1(t)=Bl\dot{\lambda}_2(t)\\&-L_e\dot{\lambda}_2(t)+R_e\lambda_2(t)-Bl\lambda_1(t)=-2\left(i(t)-i_{mea}(t)\right)\\&,\ t\in(t_f,0)\end{aligned}\tag{10}$$

with the final conditions

$$\lambda_{1,2}(t_f)=0 \text{ and } d\lambda_{1,2}(t_f)/dt=0\tag{11}$$

where $\hat{M}_m=M_m+1.0965aS_d$ and $\hat{R}_m=R_m+171.5S_d$.

Therefore, the gradient of the objective function can be obtained as [17],

$$\nabla J=\left[\frac{\partial J}{\partial\hat{M}_m},\ \frac{\partial J}{\partial\hat{R}_m},\ \frac{\partial J}{\partial K_m},\ \frac{\partial J}{\partial Bl},\ \frac{\partial J}{\partial R_e},\ \frac{\partial J}{\partial L_e}\right]^T\tag{12}$$

where

$$\begin{aligned}&\frac{\partial J}{\partial\hat{M}_m}=\int_0^{t_f}\lambda_1(t)\ddot{x}(t)dt,\ \frac{\partial J}{\partial\hat{R}_m}=\int_0^{t_f}\lambda_1(t)\dot{x}(t)dt,\\&\frac{\partial J}{\partial K_m}=\int_0^{t_f}\lambda_1(t)x(t)dt,\ \frac{\partial J}{\partial Bl}=\int_0^{t_f}\left(\lambda_2(t)\dot{x}(t)-\lambda_1(t)i(t)\right)dt,\\&\frac{\partial J}{\partial R_e}=\int_0^{t_f}\lambda_2(t)i(t)dt,\ \frac{\partial J}{\partial L_e}=\int_0^{t_f}\lambda_2(t)\dot{i}(t)dt\end{aligned}\tag{13}$$

Once ∇J is solved, the search direction $\mathbf{P}^{(k+1)}$ is readily solved.

C. Sensitivity Problem

After confirming the search direction $\mathbf{P}$, the step length β of (7) must be determined. The step length β can be obtained as [17],

$$\beta=\frac{\int_0^{t_f}\left(i(t)-i_{mea}(t)\right)\delta i(t)dt}{\int_0^{t_f}\delta i(t)^2\,dt}\tag{14}$$

where $\delta i(t)$ can be solved by following equations,

$$\begin{aligned}&\hat{M}_m\delta\ddot{x}(t)+\hat{R}_m\delta\dot{x}(t)+K_m\delta x(t)-Bl\delta i(t)\\&=-\delta M_m\ddot{x}(t)-\dot{x}(t)\delta R_m-x(t)\delta K_m+i(t)\delta Bl\end{aligned}\tag{15}$$

$$\begin{aligned}&L_e\delta\dot{i}(t)+R_e\delta i(t)+Bl\delta\dot{x}(t)\\&=-\delta L_e\dot{i}(t)-\delta R_e i(t)-\delta Bl\dot{x}(t)\end{aligned}\tag{16}$$

with the initial conditions:

$$\begin{aligned}&\delta x(0)=0,\qquad \delta i(0)=0,\\&\left.\frac{d\delta x(t)}{dt}\right|_{t=0}=0,\text{ and }\left.\frac{d\delta i(t)}{dt}\right|_{t=0}=0\end{aligned}\tag{17}$$

In (15) and (16), $\delta\mathbf{w}$ is the search direction $\mathbf{P}$ of its iteration.

IV. Results and Discussion

The loudspeaker is excited by a sweep-tone test signal with 2 Vrms generated from the KLIPPEL analyzer system [1]. The loudspeaker is place on the fixture of the Klippel analyzer system as shown in Fig. 2. The voltage $e(t)$ and current $i_{mea}(t)$ are also measured simultaneously from KLIPPEL analyzer system. From the measured RMS sound pressure P, the RMS displacement x_{mea} can be obtained by [18],

$$x_{rms,mea}=\frac{P_{rms}}{\rho_0 S_d f}\tag{18}$$

where f is the frequency. Using the numerical solver proposed by [17] to solve the differential equations for obtaining the estimated $i(t)$ and $x(t)$, the unknown parameters $(\hat{M}_m,\hat{R}_m,K_m,Bl,L_e,R_e)$ can be solved by proposed inverse method.

Fig. 2 A two-inch MCL placed on the fixture of the Klippel analyzer system.

First, the laser vibrometer was used for measuring the center displacement $x_{mea}(t)$ in cone piston motion at resonance frequency (210Hz). The values of estimated parameters at resonance frequency are listed in Table I. Then for determining the values of parameters $(\hat{M}_m,\hat{R}_m)$ versus the frequencies, the estimated displacement x must be checked by $|x_{rms}|=|x_{rms,mea}|$. As a result, Figs. 3 to 5 shows the estimated results for parameters $(\hat{M}_m,\hat{R}_m)$ and cone effective area S_d versus frequencies, it is observed that the values of $(\hat{M}_m,\hat{R}_m)$ and S_d are not constant in each frequency. The trend of results curves in Figs. 3 to 5 goes dip at high frequency below 2300Hz. Fig. 6 shows the comparison of the simulated and measured SPL curves, it is seen that the proposed simulated curve SPL is fitted by the measured SPL curve. The original curve which simulated by the estimation values from Table I is not matched by the measured SPL curve. It indicates that the simulation result from proposed method is in high agreement with considering the acoustic radiation mass, resistance and the cone effective area versus frequencies.

TABLE I
THE PARAMETERS OF MOVING-COIL LOUDSPEAKER

M_m (kg)	R_m (kg/s)	K_m (N/m)	Bl (N/A)	L_e (H)	R_e (Ohm)	S_d (πm^2)
0.710E-3	0.343	1.240E3	1.265	3.602E-5	3.921	12.56E-4

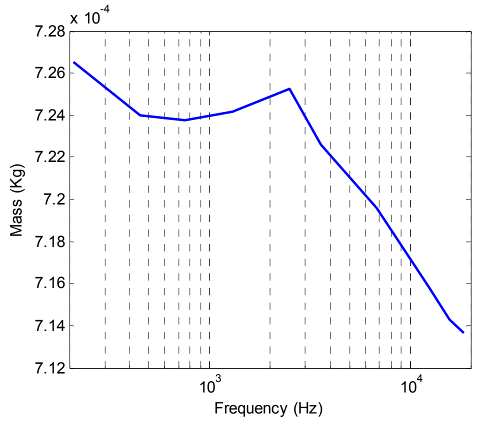

Fig. 3 The mechanical mass with acoustic radiation mass $\hat{M}_m$ versus frequencies

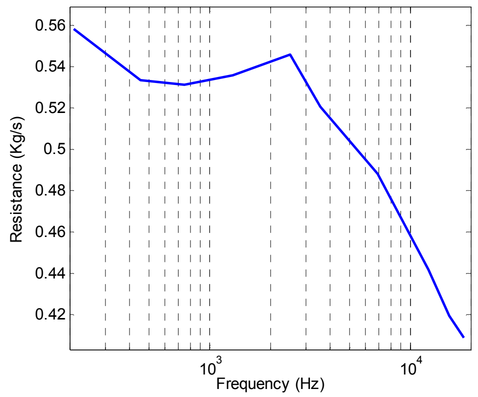

Fig. 4 The mechanical resistance with acoustic radiation resistance $\hat{R}_m$ versus frequencies

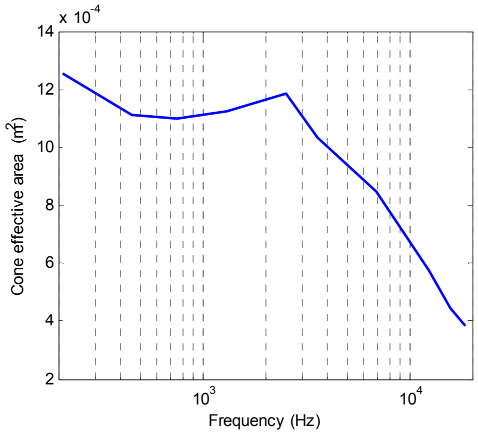

Fig. 5 The cone effective area S_d versus frequencies

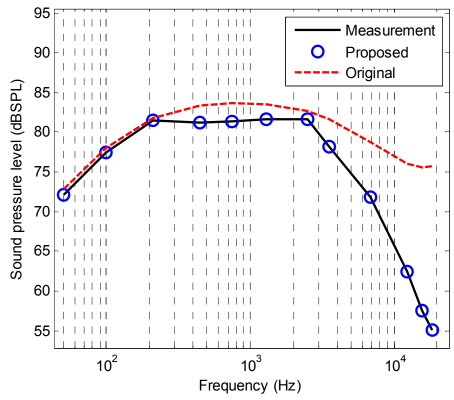

Fig. 6 The comparison curves of simulated (Proposed and original) and measured SPL curves. The original simulation curve is estimated by the parameters from Table I

V. Conclusion

The inverse method for estimating the electroacoustic parameters with considering the acoustic radiation mass, resistance and the cone effective area versus frequencies are presented in this study. The calculation steps for inverse method address the direct problem for solving the equations, adjoint equations for solving the gradient, sensitivity problem for solving the step length, CGM for solving the search direction. The accuracy of the proposed model was ascertained by comparing the measured and estimated SPL result. Through the measurement, the results show that inverse solutions can be estimated in high agreement. The proposed method is useful not only in parameter evaluation but also in the diagnosis of loudspeaker cone vibration, which is of great importance in quality control.

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