

# The Sustainability of Public Debt in Taiwan

Chiung-Ju Huang

**Abstract**—This study examines whether the Taiwan's public debt is sustainable utilizing an unrestricted two-regime threshold autoregressive (TAR) model with an autoregressive unit root. The empirical results show that Taiwan's public debt appears as a nonlinear series and is stationary in regime 1 but not in regime 2. This result implies that while Taiwan's public debt was mostly sustainable over the 1996 to 2013 period examined in the study, it may no longer be sustainable in the most recent two years as the public debt ratio has increased cumulatively to 3.618%.

**Keywords**—Nonlinearity, public debt, sustainability, threshold autoregressive model.

## I. INTRODUCTION

PUBLIC debt sustainability has been an ongoing debate over decades and a number of empirical studies have attempted to explore the sustainability of country's public debt. However, few studies have assessed the sustainability of Taiwan's public debt. From 1996 to 2013, Taiwan's government had experienced deficits each year with the exception of having a slight government surplus in 1998. Therefore, the general government debt as a percentage of GDP dramatically increased from 24.5% in 1996 to 41.5% in 2013 (in Fig. 1). According to the Global Competitiveness Report 2013-2014 published by the World Economic Forum, although Taiwan ranks 12<sup>th</sup> overall out of 148 economies, Taiwan's government budget deficit is ranked at 91 while its government debt is ranked at 69. Therefore, the aim of this study is to investigate whether the Taiwan's public debt is sustainable.

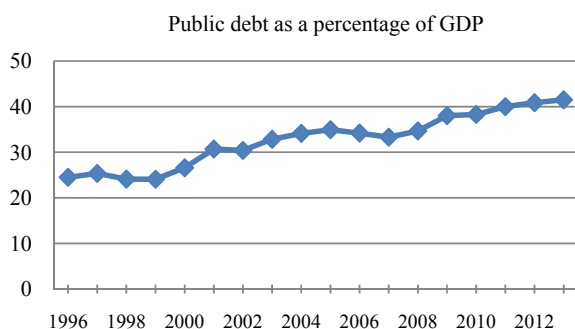


Fig. 1 Taiwan's public debt ratio

Public debt sustainability is measured by whether the intertemporal solvency condition is satisfied. A necessary and

sufficient condition for the intertemporal solvency condition to hold is a stationary discounted stock of public debt. Another necessary condition for the sustainability of public debt is the cointegration between government expenditure and tax revenue. Therefore, one strand of the empirical literature, such as that of Hamilton and Flavin [1], Trehan and Walsh [2], Quintos [3], Lima et al. [4], focuses on the stationarity of government debt or deficit and adopts unit root tests to examine whether government debt or deficit is stationary. Another strand of the empirical literature such as that of Bohn [5], Ahmed and Rogers [6], and Payne et al. [7], has concentrates instead on the long run relationship between government expenditure and tax revenue and employs cointegration tests to examine whether the observed data are consistent with this requirement.

The stationary tests by using traditional unit root tests assume public debt behaves under a continuous and constant speed adjustment process. However, facing the possibility of public debt, adjustments may very well be asymmetric. Recently, some empirical studies have used a nonlinear model to examine the sustainability of public debt or budget deficit (e.g., Sarno [8], Chortareas et al. [9], Arestis et al. [10], Payne et al. [7], and Huang and Ho [11]) given that traditional unit root tests are inadequate when public debt exhibits a threshold behavior. More specifically, the traditional unit root tests frequently fail to reject the unit root null hypothesis and may lead to incorrect interpretation suggesting that a country's public debt is unsustainable.

To address the aforementioned shortcomings, this study adopts a newly developed threshold unit root tests that factors in nonlinearities. The threshold autoregressive (TAR) model allows us to examine whether the data series is a nonlinear process. If public debt is a nonlinear process, then traditional unit root tests that do not take into account possible nonlinearity, such as the augmented Dickey-Fuller (ADF) [12], Phillips-Perron (PP) [13], and the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) [14] tests, may not be practical. This study attempts to add to the existing literature by not only testing for nonlinearities, but also investigating the unit root properties of Taiwan's public debt. Additionally, if public debt is a nonlinear process, then the threshold unit root test proposed by Caner and Hansen [15] can be adopted to identify evidence for the unit root hypothesis.

## II. METHODOLOGY

### A. The Threshold Autoregressive (TAR) Model

Following the work of Caner and Hansen [15], we adopt a two regime TAR (k) model with an autoregressive unit root, and two regimes  $\theta_1$  and  $\theta_2$  as follow:

$$\Delta debt_t = \theta_1' x_{t-1} I\{Z_{t-1} < \lambda\} + \theta_2' x_{t-1} I\{Z_{t-1} \geq \lambda\} + e_t \quad (1)$$

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where  $debt$  is public debt ratio for  $t = 1, 2, \dots, T$ ,  $x_{t-1} = (debt_{t-1}r'_t\Delta debt_{t-1} \dots \Delta debt_{t-k})'$ ,  $I\{\cdot\}$  is the indicator function,  $e_t$  is an i.i.d error,  $Z_t = debt_t - debt_{t-m}$  for some  $m \geq 1$ ,  $Z_{t-1}$  is the threshold variable,  $m$  represents the delay parameter and  $1 \leq m \leq k$ ,  $r_t$  is a vector of deterministic components including an intercept and possible a linear time trend, and  $k \geq 1$  is the autoregressive order. The threshold  $\lambda$  is unknown and takes on values in the interval  $\lambda \in \Lambda = [\lambda_1, \lambda_2]$ , where  $\lambda_1$  and  $\lambda_2$  are selected according to the following:

$$\begin{cases} P(Z_t \leq \lambda_1) = \pi_1 > 0 \\ P(Z_t \leq \lambda_2) = \pi_2 < 1 \end{cases} \quad (2)$$

where  $\pi_1 + \pi_2 = 1$ .

The components of  $\theta_1$  and  $\theta_2$  can be partitioned as follows:

$$\begin{cases} \theta_1 = (\rho_1, \beta_1, \alpha_1) \\ \theta_2 = (\rho_2, \beta_2, \alpha_2) \end{cases} \quad (3)$$

where  $\rho_1$  and  $\rho_2$  are slope coefficients on  $debt_{t-1}$ ,  $\beta_1$  and  $\beta_2$  are scalar intercepts, and  $\alpha_1$  and  $\alpha_2$  are  $1 \times k$  vectors containing the slope coefficients on dynamics regressors ( $\Delta debt_{t-1} \dots \Delta debt_{t-k}$ ) in the two regimes.

#### B. Threshold Test

A particular public debt might have a threshold effect. The threshold effect in (1) is based on the null hypothesis

$$H_0: \theta_1 = \theta_2. \quad (4)$$

To test the null hypothesis of linearity against the alternative of threshold effects, we use the Wald statistic:

$$W_T = W_T(\hat{\lambda}) = \sup_{\lambda \in \Lambda} W_T(\lambda). \quad (5)$$

If the null hypothesis is rejected, then there is a threshold effect.

#### C. Threshold Unit Root Tests

The stationarity of the series  $debt_t$  can be characterized in two ways. First, there is a unit root in both regimes (i.e. complete unit roots). In a complete unit root case, the null hypothesis of unit root is

$$H_0: \rho_1 = \rho_2 = 0. \quad (6)$$

One of an alternative hypothesis to the null hypothesis is

$$H_1: \rho_1 \neq 0 \text{ and } \rho_2 \neq 0. \quad (7)$$

To test the null hypothesis of unit root, we use a two-side Wald test statistic:

$$R_{2T} = t_1^2 + t_2^2 \quad (8)$$

where  $t_1$  and  $t_2$  are the  $t$  ratios for  $\hat{\rho}_1$  and  $\hat{\rho}_2$  from the OLS estimate. When the null hypothesis  $H_0$  holds, the process  $debt_t$  has a unit root in both regimes. The other alternative hypothesis to the null hypothesis is

$$H_2: \rho_1 < 0 \text{ and } \rho_2 < 0 \quad (9)$$

To test the null hypothesis of unit root, we use a one-side Wald test statistic:

$$R_{1T} = t_1^2 I\{\hat{\rho}_1 < 0\} + t_2^2 I\{\hat{\rho}_2 < 0\} \quad (10)$$

when the alternative hypothesis  $H_1$  (or  $H_2$ ) holds, the process  $debt_t$  is stationary in both regimes. Meanwhile, Caner and Hansen [15] claim that the two-side Wald statistic may have less power than a one-side Wald statistic.

Second, there is a unit root in only one regime (i.e. partial unit roots). In a partial unit root case, the null hypothesis of unit root is  $H_0: \rho_1 = \rho_2 = 0$ . An alternative hypothesis of stationarity only in regime 1, that is,

$$H_3: \rho_1 < 0 \text{ and } \rho_2 = 0. \quad (11)$$

The alternative hypothesis of stationarity only in regime 2, that is,

$$H_4: \rho_1 = 0 \text{ and } \rho_2 < 0 \quad (12)$$

### III. EMPIRICAL RESULTS

Annual data on public debt and GDP from Taiwan's National Statistics over the 1996 to 2013 period was used in this analysis. In this study, we focus on public debt ratios as a percentage of GDP which is the conventional method utilized internationally. Series  $debt$  represents public debt as a percentage of GDP.

In the first part of this study, we use the Wald test to examine whether we can reject the liner autoregressive model in favor of a threshold model. The Wald statistics are dependent on the value of the delay parameter,  $m$ . According to the Caner and Hansen [15] study, the preferred value of the delay parameter is one that minimizes the sum of squared errors. Based on this criterion, the preferred model in this study is  $m = 2$ . The result of the Wald test and the bootstrap  $p$ -values for threshold variables of the form  $Z_{t-1} = debt_{t-1} - debt_{t-m-1}$  for the optimal delay parameter  $m = 2$ , are all reported in Table I. The Wald statistic  $W_T$  in Table I is significant at the 1% level of significance. This result implies strong evidence against the null of linearity. Therefore, a simple linear model is inappropriate and the threshold model is preferable. Meanwhile, the threshold estimate is  $\hat{\lambda} = 3.618$  and the percentage of observations in regime 1 is about 67%.

TABLE I  
THRESHOLD TEST

Optimal Delay Parameter $m$	$W_T$	Bootstrap $p$ -value	Threshold Parameter $\hat{\lambda}$	Percentage of Observations in Regime 1 (%)
2	133.54	0.01	3.618	67

Note: The critical Wald-statistics at three significant levels and the  $p$ -value are calculated according to a bootstrap approach with 5000 replications

Next, we explore the threshold unit root properties of  $debt$  based on  $R_{2T}$  and  $R_{1T}$  statistics for the optimal delay

parameter  $m = 2$ , which are reported in Tables II and III, respectively. The test statistics  $R_{2T}$  and  $R_{1T}$  allow us to test whether public debt under this study is stationary. The Wald statistic  $W_T$  obtained from the  $R_{2T}$  and  $R_{1T}$  tests are greater than the bootstrap critical value at the 5% level of significance. As a result, the null hypothesis of a unit root for public debt series is rejected at the 5% level of significance.

We then continued to look for evidence on the partial unit root by examining the individual  $t$  statistics,  $t_1$  and  $t_2$  to confirm the results of complete unit root test, as shown in Tables II and III. The test statistics  $t_1$  and  $t_2$  of partial unit root tests allow us to identify which of the regimes is stationary. These statistics can be used to examine whether the Taiwan's public debt is sustainable in the long run. The results of partial unit root tests are reported in Table IV. Given that the  $t_1$  statistics reported in Table IV is 7.60, which is significant at the 1% level of significance, we are able to reject the null hypothesis, implying that public debt is stationary in regime 1. However, as the  $t_2$  statistics is 1.43, which is nonsignificant at 10% level of significance, we are unable to reject the null hypothesis and do not find evidence that public debt is stationary in regime 2.

TABLE II  
TWO-SIDE UNIT ROOT TEST

$R_{2T}$					
Optimal $m$	$W_T$	Bootstrap Critical Value			$p$ -value
		10%	5%	1%	
2	59.82	30.10	39.73	112.56	0.02

TABLE III  
ONE-SIDE UNIT ROOT TEST

Optimal $m$	$R_{1T}$				$p$ -value
	$W_T$	Bootstrap Critical Value			
		10%	5%	1%	
2	59.82	28.59	39.24	112.56	0.02

TABLE IV  
PARTIAL UNIT ROOT TESTS

Optimal $m$	$t_1$		$t_2$	
	t-statistic	Bootstrap $p$ -value	t-statistic	Bootstrap $p$ -value
2	7.60***	0.006	1.43	0.608

Note: \*\*\* denotes significance at the 1% level.

The least square estimates for the TAR model is reported in Table V where the threshold estimate is  $\hat{\lambda} = 3.618$  and the optimal delay parameter  $m = 2$ . The TAR model split the regression into two regimes depending on whether the variable  $Z_{t-1} = debt_{t-1} - debt_{t-3}$  lies above or below the threshold  $\hat{\lambda} = 3.618$ ; in other words, a two year change in the public debt ratio is above or below 3.618. The first regime is when  $Z_{t-1} < 3.618$ , which occurs when public debt ratio has fallen, remained constant, or has risen by less than 3.618% (e.g. from 38.285% to 40.858%) over a two year period. Approximately 67% of the observations lie in this regime. The second regime is when  $Z_{t-1} \geq 3.618$ , which occurs when public debt ratio has risen by more than 3.618% over a two year period.

Approximately 33% of the observations fall in the regime 2. According to the partial unit root test results, we find that public debt is stationary in regime 1 but we do not find evidence that public debt is stationary in regime 2. Thus, Taiwan's public debt was stationary most of time in the study. This leads us to conclude that Taiwan's public debt was sustainable throughout most of the 1996 – 2013 time period examined in this study.

TABLE V  
ESTIMATION OF TAR

Regressors	Regime 1 $Z_{t-1} < 3.618$	Regime 2 $Z_{t-1} \geq 3.618$
	$t$	$t$
Intercept	22.290 (2.767)	21.153 (14.356)
$debt_{t-1}$	-0.898 (0.118)	-0.665 (0.464)
$\Delta debt_{t-1}$	1.076 (0.110)	-0.580 (0.225)
$\Delta debt_{t-2}$	-0.173 (0.169)	-0.373 (0.599)

Note: Standard errors are in parentheses.

#### IV. CONCLUSIONS

As public debt may be characterized by a nonlinear data generating process, the conventional method of investigating the sustainability of public debt by adopting the traditional unit root tests may not be effective. This study takes into account the possibility of nonlinearity and utilizes the TAR model that allows us to examine whether the data series behaves as a nonlinear process. The objective of this study is to examine whether Taiwan's public debt, during the 1996 – 2013 period, is sustainable by applying threshold unit root tests to public debt.

Two important results emerge from our empirical analysis. First, we find strong evidence of a threshold effect for Taiwan's public debt. In this study, the TAR model is preferred over the linear AR model. Our study results, which suggest that public debt is a nonlinear series, is consistent with that of Chortareas, et al. [9], Arestis, et al. [10], and Jawadi and Sousa [16]. Second, we apply a suite of test statistics for unit roots and find that Taiwan's public debt series is stationary in regime 1 but we do not find evidence suggesting the same for regime 2. Thus, Taiwan's public debt was stationary for most of the 1996 to 2013 period examined in the study. If a two year change in the public debt ratio is above 3.618%, then public debt may not be stationary. To ensure sustainability of public debt, the Taiwanese government should pay attention to the increase in the public debt ratio, and more specifically, be cautious of any increase of public debt ratio that cumulatively reaches more than 3.618% over a two year period. To prevent this from happening, government authorities may actively cut the public debt ratio only when it has reached a certain threshold.

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