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Higher Order Statistics for Identification of Minimum Phase Channels

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Abstract—This paper describes a blind algorithm, which is compared with two another algorithms proposed in the literature, for estimating of the minimum phase channel parameters. In order to identify blindly the impulse response of these channels, we have used Higher Order Statistics (HOS) to build our algorithm. The simulation results in noisy environment, demonstrate that the proposed method could estimate the phase and magnitude with high accuracy of these channels blindly and without any information about the input, except that the input excitation is identically and independent distribute $(i.i.d)$ and non-Gaussian.

Keywords—System Identification, Higher Order Statistics, Communication Channels.

I. Introduction

IN the literature, blind identification of linear systems systems has attracted considerable attention [1], [4]−[7], [9]−[10], [12]−[14]. We have important results [2], [3], established that blind identification of finite impulse response (FIR) communication channels is possible only from the output AutoCorrelation Function (second order statistics) of the observed sequences. But, these statistics are sensible to additive Gaussian noise. Thus, their performances degrade when the output is noisy, because the second order cumulants for a Gaussian process are not identically zero. Hence, when the processed signal is non-Gaussian and the additive noise is Gaussian, the noise will vanish in the higher order cumulants (3^{rd} and 4^{th} cumulants) domain, where the autocorrelation function does not allow identifying the system correctly. In this contribution, we propose on blind algorithm based on fourth order cumulants, this approach allows the resolution of the insoluble problems using the second order statistics. In order to test the efficiency of the proposed algorithm we have compared with the Safi et al algorithm [5], and Zhang et al algorithm [3]. For that, we will consider a Minimum Phase (MP) channels excited by non Gaussian distribution input, and is contaminated by a Gaussian noise for different signal to noise ratio (SNR) and for different size data input. The proposed method in this paper is based on fourth order cumulants exploiting only $(q + 1)$ equations to estimate q unknown parameters and compared to the Zhang's and Safi's algorithms exploiting $(2q + 1)$ equations. The validity of the proposed algorithm has been demonstrated by simulation results.

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II. Priliminairies and problem statement

A. Model and Assumptions

We consider the single-input single-output $(SISO)$ model (Fig. 1) of the finite impulse response (FIR) system described by the following relationships:
Noise free case :

$$y(k) = \sum_{i=0}^{q} x(i)h(k-i). \tag{1}$$

With noise :

$$r(k) = y(k) + n(k), \tag{2}$$

where $x(k)$ is the input sequence, $h(k)$ is the impulse response coefficients, q is the order of FIR system, $y(k)$ is the output of system and $n(k)$ is the noise sequence.

The completely blind channel identification problem is to

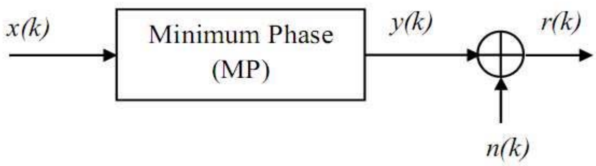

Fig. 1. Channel model

estimate h_q based only on the received signal $r(k)$ and without any information of input channel, nor the energy of noise.
The principal assumptions made on the model can be presented as follows:

- The input sequence, $x(k)$, is independent and identically distributed $(i.i.d)$ zero mean, and non-Gaussian.
- The system is causal and truncated, $i.e.$ $h(k) = 0$ for $k < 0$ and $k > q$, where $h(0) = 1$.
- The system order q is known.
- The measurement noise sequence $n(k)$ is assumed zero mean, $i.i.d$, Gaussian and independent of $x(k)$ with unknown variance.

B. Basic relationships

The general fundamental relations which permit to identify the FIR linear systems using higher order cumulants are presented in this subsection.
Brillinger and Rosenblatt showed that the m^{th} order cumulants

of $y(k)$ can be expressed as a function of impulse response coefficients $h(i)$ as follows [8]:

$$C_{my}(t_1, t_2, ..., t_{m-1}) = \xi_{mx} \sum_{i=0}^{q} h(i)h(i+t_1)...h(i+t_{m-1}), \tag{3}$$

where ξ_{mx} represents the m^{th} order cumulants of the excitation signal $x(i)$ at origin.
Stogioglou−McLaughlin [11] presents the relationship between different n^{th} cumulant slices of the output signal $y(n)$, as follows

$$\begin{aligned} &\sum_{j=0}^{q} h(j)\Big[\prod_{k=1}^{r} h(j+t_k)\Big] C_{ny}(\beta_1, ..., \beta_r, j+\alpha_1, ..., j+\alpha_{n-r-1}) \\ &= \sum_{i=0}^{q} h(i)\Big[\prod_{k=1}^{r} h(i+\beta_k)\Big] C_{ny}(t_1, ..., t_r, i+\alpha_1, ..., i+\alpha_{n-r-1}), \end{aligned} \tag{4}$$

where $1 \le r \le n-2$.
If we take $m = 2$ into (3) we obtain the second order cumulant:

$$C_{2y}(t) = \xi_{2x} \sum_{i=0}^{q} h(i)h(i+t) \tag{5}$$

Analogically, if $m = 4$, (3) reduces:

$$C_{4y}(t_1, t_2, t_3) = \xi_{4x} \sum_{i=0}^{q} h(i)h(i+t_1)h(i+t_2)h(i+t_3). \tag{6}$$

The fourier transforms of the 2^{nd} and 4^{rd} order cumulants are given respectively by the following equations [6], [11]:

$$\begin{aligned} S_{2y}(\omega) &= TF\{C_{2y}(t)\} \\ &= \xi_{2x} \sum_{i=0}^{q} \sum_{t=-\infty}^{+\infty} h(i)h(i+t)\exp(-j\omega t) \\ &= \xi_{2x} H(-\omega)H(\omega) \end{aligned} \tag{7}$$

$$\begin{aligned} S_{4y}(\omega_1, \omega_2, \omega_3) &= TF\{C_{4y}(t_1, t_2, t_3)\} \\ &= \xi_{4x} H(-\omega_1-\omega_2-\omega_3)H(\omega_1)H(\omega_2)H(\omega_3) \end{aligned} \tag{8}$$

Thus, if we take $\omega = \omega_1+\omega_2+\omega_3$, (7) becomes:

$$S_{2y}(\omega_1+\omega_2+\omega_3) = \xi_{2x} H(-\omega_1-\omega_2-\omega_3)H(\omega_1+\omega_2+\omega_3) \tag{9}$$

Then, from the (8) and (9) we construct a relationship between the spectrum, the trispectrum and the parameters of the output system:

$$S_{4y}(\omega_1, \omega_2, \omega_3)H(\omega_1+\omega_2+\omega_3) = \mu H(\omega_1)H(\omega_2)H(\omega_3)S_{2y}(\omega_1+\omega_2+\omega_3), \tag{10}$$

with $\mu = \frac{\xi_{4x}}{\xi_{2x}}$
The inverse fourier transform of (10) is:

$$\begin{aligned} &\sum_{i=0}^{q} C_{4y}(t_1-i, t_2-i, t_3-i)h(i) \\ &= \mu \sum_{i=0}^{q} h(i)h(t_2-t_1+i)h(t_3-t_1+i)C_{2y}(t_1-i). \end{aligned} \tag{11}$$

Based on the relationship (11) we can develop the following algorithm based on the fourth order cumulants.

III. Proposed algorithm: AlgoZ

If we fixe $t_1 = t_2$ into (11) we obtain:

$$\sum_{i=0}^{q} C_{4y}(t_1-i, t_1-i, t_3-i)h(i) = \mu \sum_{i=0}^{q} h^2(i)h(t_3-t_1+i)C_{2y}(t_1-i). \tag{12}$$

Using the AutoCorrelation Function (second order statistics) property of the stationary process such as $C_{2y(t)} \neq 0$ only for $-q \le t \le q$ and vanishes elsewhere.
If we suppose that $t_1 = -q$ (12) reduces:

$$\sum_{i=0}^{q} C_{4y}(-q-i, -q-i, t_3-i)h(i) = \mu h^2(0)h(t_3+q)C_{2y}(-q) \tag{13}$$

The considered system is causal. Thus, the interval of the $t_3 = -q, -q+1, ..., 0$.
Otherwise if we take $t_3 = -q$ into (13), we obtain the following equation:

$$C_{4y}(-q, -q, -q)h(0) = \mu h^3(0)C_{2y}(-q), \tag{14}$$

with $h(0) = 1$ we obtain:

$$C_{4y}(-q, -q, -q) = \mu C_{2y}(-q) \tag{15}$$

Thus, we based on (15) for eliminating $C_{2y}(-q)$ in (13), we obtain the equation constituted of only the fourth order cumulants:

$$\sum_{i=0}^{q} C_{4y}(-q-i, -q-i, t_3-i)h(i) = h(t_3+q)C_{4y}(-q, -q, -q), \tag{16}$$

where $t_3 = -q, -q+1, ..., 0$.
The system of (16) can be written under the matrix form as follows:

$$\begin{pmatrix} C_{4y}(-q-1, -q-1, -q-1) & ... & C_{4y}(-2q, -2q, -2q) \\ C_{4y}(-q-1, -q-1, -q) - \alpha & ... & C_{4y}(-2q, -2q, -2q+1) \\ . & . & . \\ . & . & . \\ . & . & . \\ C_{4y}(-q-1, -q-1, -1) & ... & C_{4y}(-2q, -2q, -q) - \alpha \end{pmatrix}$$

$$\times \begin{pmatrix} h(1) \\ . \\ . \\ . \\ h(i) \\ . \\ . \\ . \\ h(q) \end{pmatrix} = \begin{pmatrix} 0 \\ -C_{4y}(-q, -q, -q+1) \\ . \\ . \\ . \\ . \\ . \\ -C_{4y}(-q, -q, 0) \end{pmatrix} \tag{17}$$

where $\alpha = C_{4y}(-q, -q, -q)$
Or in more compact form, (17) can be written as follows:

$$M h_e = d, \tag{18}$$

with M the matrix of size $(q+1, q)$ elements, h_e a column vector of size $(q, 1)$ and d is a column vector of size $(q+1, 1)$.
The Least Square (LS) solution of the system of equation (18) is given by:

$$\widehat{h}_e = (M^T M)^{-1} M^T d \tag{19}$$

IV. SAFI ET AL ALGORITHM: ALGO−SAFI [5]

If we take $n = 4$ into (4) we obtain the Following equation:

$$\begin{aligned}&\sum_{j=0}^{q} h(j)h(j+t_1)h(j+t_2)C_{4y}(\beta_1,\beta_2,j+\alpha_1)\\&=\sum_{i=0}^{q} h(i)h(i+\beta_1)h(i+\beta_2)C_{4y}(t_1,t_2,i+\alpha_1)\end{aligned} \quad (20)$$

If $t_1 = t_2 = q$ et $\beta_1 = \beta_2 = 0$, (20) take the form :

$$h(0)h^2(q)C_{4y}(0,0,\alpha_1) = \sum_{i=0}^{q} h^3(i)C_{4y}(q,q,i+\alpha_1), \quad (21)$$

with

$$-q \leq \alpha_1 \leq q \quad (22)$$

Then, from (21) and (22) we obtain the following system of equations:

$$\begin{pmatrix} C_{4y}(q,q,-q) & \ldots & C_{4y}(q,q,0) \\ \vdots & \ddots & \vdots \\ C_{4y}(q,q,0) & \ldots & C_{4y}(q,q,q) \\ \vdots & \ddots & \vdots \\ C_{4y}(q,q,q) & \ldots & C_{4y}(q,q,2q) \end{pmatrix}$$

$$\times \begin{pmatrix} h^3(0) \\ \vdots \\ h^3(i) \\ \vdots \\ h^3(q) \end{pmatrix} = h(0)h^2(q) \begin{pmatrix} C_{4y}(0,0,-q) \\ \vdots \\ C_{4y}(0,0,0) \\ \vdots \\ C_{4y}(0,0,q) \end{pmatrix} \quad (23)$$

In more compact form, the system of (23) can be written in the following form:

$$Mb_q = d, \quad (24)$$

where M, b_q and d are defined in the system of (24).
The least squares solution of the system of (24) is given by:

$$\widehat{b_q} = (M^T M)^{-1} M^T d \quad (25)$$

This solution give us an estimation of the quotient of the parameters $h^3(i)$ and $h^2(q)$, i.e., $b_q(i) = \widehat{\frac{h^3(i)}{h^2(q)}}$, $i = 1, ..., q$. Thus, in order to obtain an estimation of the parameters $\widehat{h}(i)$, $i = 1, ..., q$ we proceed as follows:
The parameters $h(i)$ for $i = 1, ..., q-1$ are estimated from the estimated values $\widehat{b}_q(i)$ using the following equation:

$$\widehat{h}(i) = sign\Big[\widehat{b}_q(i) \times (\widehat{b}_q(q))^2\Big]\Big\{abs(\widehat{b}_q(i)) \times (\widehat{b}_q(q))^2\Big\}^{1/3} \quad (26)$$

The $\widehat{h}(q)$ parameters is estimated as follows:

$$\widehat{h}(q) = \frac{1}{2} sign\Big[\widehat{b}_q(q)\Big]\Big\{abs(\widehat{b}_q(q)) + \Big(\frac{1}{\widehat{b}_q(1)}\Big)^{1/2}\Big\} \quad (27)$$

V. ZHANG ET AL ALGORITHM: ALGO−ZHANG [3]

Zhang et al. [3] demonstrates that the coefficients $h(i)$ for an FIR system can be obtained by the following equation:

$$\sum_{i=0}^{q} h(i)C_{ny}^{n-1}(i-t,q,...,0) = C_{ny}(t,0,...,0)C_{ny}^{n-3}(q,...,0)C_{ny}(q,q,...,0). \quad (28)$$

For $n = 4$, from (28), we obtain the following equation:

$$\sum_{i=0}^{q} h(i)C_{4y}^3(i-t,q,0) = C_{4y}(t,0,0)C_{4y}(q,0,0)C_{4y}(q,q,0). \quad (29)$$

for $-q \leq t \leq q$
Then, (29) can be rewritten as follows:

$$Mh_z = d, \quad (30)$$

where M is the matrix of size $(2q+1) \times (q)$ elements, h_z is a column vector constituted by the unknown impulse response parameters $h(k) = k = 1, ..., q$ and d is a column vector of size $(2q+1)$.
The Least Squares (LS) solution of the system of Eq. (30), permits blindly identification of the parameters $h(k)$ and without any information of the input selective channel. Thus, the solution will be written under the following form

$$\widehat{h}_z = (M^T M)^{-1} M^T d. \quad (31)$$

VI. SIMULATION RESULTS

To verify the performance of the proposed algorithm, we have applied a two linear models.
To measure the strength of noise, we define the signal-to-noise ratio (SNR) as:

$$SNR = 10log\Big[\frac{\sigma_y^2(k)}{\sigma_n^2(k)}\Big] \quad (32)$$

To measure the accuracy of parameter estimation with respect to the real values, we define the Mean Square Error (MSE) for each run as:

$$MSE = \frac{1}{q}\sum_{i=1}^{q}\Big[\frac{h(i) - \widehat{h}(i)}{h(i)}\Big]^2, \quad (33)$$

where $\widehat{h}(i)$, $i = 1, ..., q$ are the estimated parameters in each run, and $h(i)$, $i = 1, ..., q$ are the real parameters in the model.

A. first channel

The first channel is defined by the following equation:

$$y(k) = x(k) - 0.860x(k-1) + 0.740x(k-2) \quad (34)$$

Fig. 2 shows that the zeros is intside of the unit circle (i.e. minimum phase channel).
The simulation results are shown in the Tables I and II for different values of sample sizes and different values of signal to noise ratio (SNR). The true parameters are $h(1) = -0.860$ and $h(2) = 0.740$.
From the simulation results, presented in Tables I and II, we can deduce:

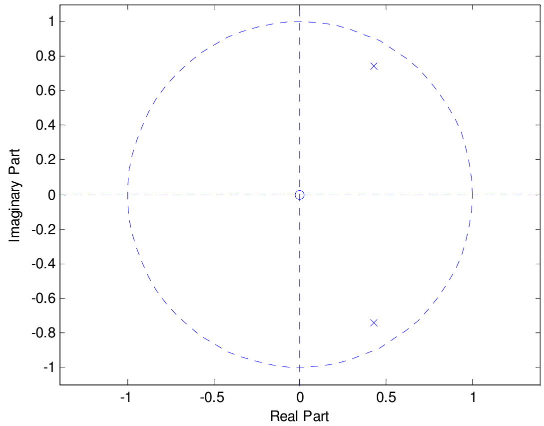

Fig. 2. Zeros of the first channel

TABLE I
ESTIMATED PARAMETERS OF FIRST CHANNEL IN NOISE FREE CASE FOR 50 MONTE CARLO RUNS.

N	$Algo$	$\widehat{h}(1)\pm\sigma$	$\widehat{h}(2)\pm\sigma$	MSE
500	$AlgoZ$	-0.7715 ± 0.1341	0.5662 ± 0.3289	0.0219
	$Algo\text{-}Safi$	-0.6879 ± 0.5040	0.4499 ± 0.7337	0.0646
	$Algo\text{-}Zhang$	-0.6461 ± 0.2046	0.6729 ± 0.2402	0.0234
1500	$AlgoZ$	-0.9007 ± 0.1264	0.6403 ± 0.2625	0.0068
	$Algo\text{-}Safi$	-0.9106 ± 0.2005	0.8008 ± 0.2344	0.0034
	$Algo\text{-}Zhang$	-0.6913 ± 0.1241	0.6638 ± 0.0921	0.0164
2000	$AlgoZ$	-0.8411 ± 0.0850	0.6939 ± 0.1420	0.0015
	$Algo\text{-}Safi$	-0.8922 ± 0.0794	0.8009 ± 0.1349	0.0027
	$Algo\text{-}Zhang$	-0.7460 ± 0.0811	0.6570 ± 0.0696	0.0101

For all sample sizes and all SNR, the values of Mean Square Error (MSE) obtained using the proposed algorithm are small than those obtained by the Safi et al (Algo-Safi) and Zhang et al (Algo-Zhang) algorithms, this implies the true parameters are near the estimates parameters, principally if we used the proposed method (AlgoZ). If we increase the data length, i.e. $N \geq 1500$, we well obtain a very good estimation of the parameters channel impulse response using the developed algorithm, compared with the results obtained by the algorithms proposed in the literature, same in case when the variance of Gaussian noise is high ($SNR = 0\ dB$). In the case $N = 2000$ and $SNR = 0\ dB$ we observe that the noise Gaussian have not the influence to the developed algorithm, but, had an influence on the (Algo-Safi) and (Algo-Zhang) algorithms. This is due to the complexity of the systems of equations for each algorithm, non linear of the cumulants in (Algo-Zhang), and non linear of the parameters in the (Algo-Safi) algorithm, or the fact that the higher order cumulants for a Gaussian noise are not identically zero, but they have values close to zero for higher data length.

Fig. 3 gives us a good idea about the precision of the proposed algorithm.

In the Fig. 4 we have presented the estimation of the magnitude and the phase of the impulse response using the proposed algorithm (AlgoZ), compared with the (Algo-Safi) and (Algo-Zhang) algorithms, for data length $N = 2000$ and an $SNR = 15\ dB$. From the Fig. 4 we remark that the magnitude and phase estimation have the same appearance using proposed method (AlgoZ), but using (Algo-Safi) and (Algo-Zhang) algorithms we have a minor difference between the estimated and true ones.

TABLE II
ESTIMATED PARAMETERS OF FIRST CHANNEL IN NOISE CASE FOR DIFFERENT SNR, $N = 2000$, AND FOR 50 MONTE CARLO RUNS

SNR	$Algo$	$\widehat{h}(1)\pm\sigma$	$\widehat{h}(2)\pm\sigma$	MSE
0 dB	$AlgoZ$	-0.8570 ± 0.1990	0.8298 ± 0.3080	0.0049
	$Algo\text{-}Safi$	-0.8174 ± 0.3625	1.0176 ± 0.3745	0.0477
	$Algo\text{-}Zhang$	-0.2903 ± 0.2338	0.6317 ± 0.1018	0.1534
15 dB	$AlgoZ$	-0.8439 ± 0.1032	0.6705 ± 0.1751	0.0031
	$Algo\text{-}Safi$	-0.9429 ± 0.0884	0.8897 ± 0.1270	0.0167
	$Algo\text{-}Zhang$	-0.7272 ± 0.0803	0.6494 ± 0.0531	0.0129
30 dB	$AlgoZ$	-0.8679 ± 0.0690	0.6788 ± 0.1405	0.0023
	$Algo\text{-}Safi$	-0.9406 ± 0.0811	0.8581 ± 0.1276	0.0114
	$Algo\text{-}Zhang$	-0.7423 ± 0.0765	0.6511 ± 0.0444	0.0111
45 dB	$AlgoZ$	-0.8648 ± 0.0844	0.6894 ± 0.1445	0.0016
	$Algo\text{-}Safi$	-0.9131 ± 0.1190	0.8512 ± 0.1792	0.0088
	$Algo\text{-}Zhang$	-0.7372 ± 0.0685	0.6582 ± 0.0502	0.0109

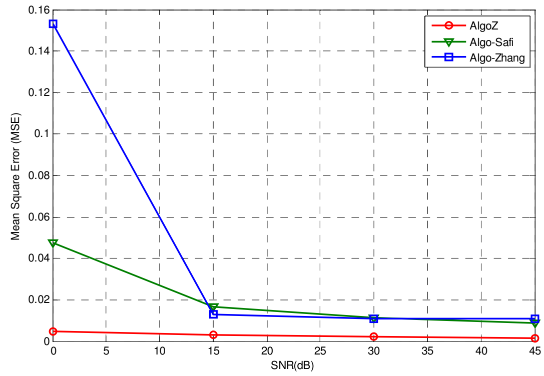

Fig. 3. MSE (first channel) for each algorithm and for different SNR and for a data length $N = 2000$.

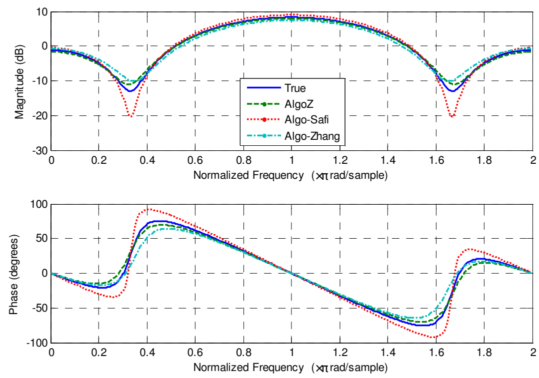

Fig. 4. Estimated magnitude and phase of the first channel impulse response, for $N = 2000$ and $SNR = 15\ dB$

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B. Second channel

The second channel is defined by the following equation:

$$y(k) = x(k) + 0.750x(k-1) - 0.580x(k-2) - 0.750x(k-3) \tag{35}$$

Fig. 5 shows that the zeros is intside of the unit circle (i.e. minimum phase channel).

The simulation results are illustrated in the Tables III and

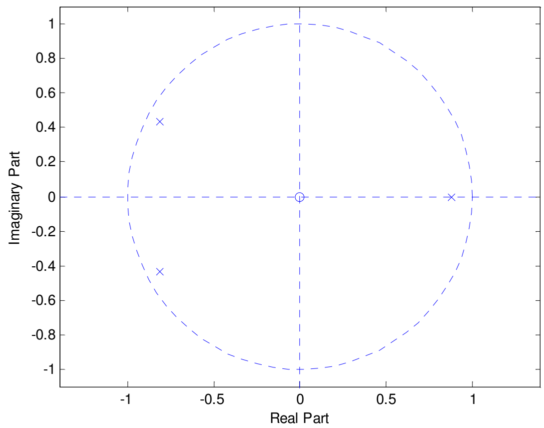

Fig. 5. Zeros of the second channel

IV for different values of sample sizes and different values of signal to noise ratio (SNR). The true parameters are $h(1) = 0.750$, $h(2) = -0.580$ and $h(3) = -0.750$.

The results presented in the Tables III and IV permit to conclude that:

The MSE values obtained using the developed algorithm are lower than those obtained by the (Algo-Safi) and (Algo-Zhang) algorithms. Indeed, the estimates of parameters are approximately closer to real values if we used the developed algorithm, in noisy environment. In addition, the obtained values of the standard deviation (std) diminue if we increase the data length.

In very noise environment $(SNR = 0\ dB)$ and for data length $N = 2000$, we well obtain a good estimation of the parameters channel impulse response using the developed algorithm, than those obtained by the (Algo-Safi) and (Algo-Zhang) algorithms, this is due to the complexity of the systems of equations for each algorithm, the proposed method is optimum , exploiting only $(q + 1)$ equations to estimate q unknown parameters, compared with the (Algo-Zhang) and (Algo-Safi) algorithms exploiting $(2q + 1)$ equations.

Fig. 6 give us a good idea about the precision of the proposed algorithm.

Fig. 7 proof that the proposed algorithm (AlgoZ) gives us a

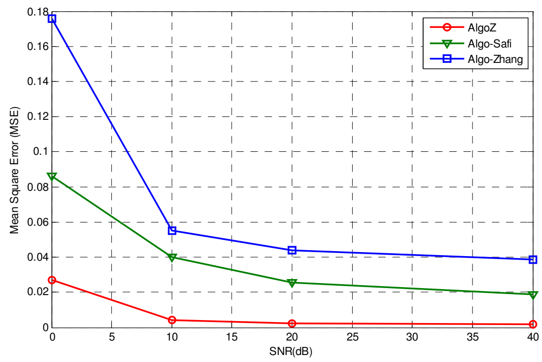

Fig. 6. MSE (second channel) for each algorithm and for different SNR and for a data length $N = 2000$.

very good estimation for magnitude and phase response, the estimated magnitude and phase are closed to the true ones, compared to the (Algo-Safi) and (Algo-Zhang) algorithms we remark more difference between estimated and true ones. To conclude, the proposed algorithm are able to estimate the phase and magnitude of channel impulse response in very noisy environments $(SNR = 0\ dB)$ with very good precision.

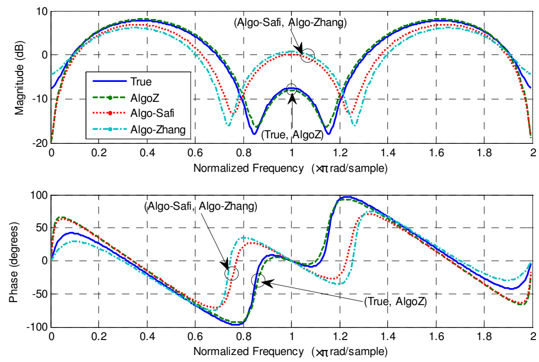

Fig. 7. Estimated magnitude and phase of the second channel impulse response, for $N = 2000$ and $SNR = 0\ dB$

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TABLE III
ESTIMATED PARAMETERS OF SECOND CHANNEL IN NOISE FREE CASE FOR 50 MONTE CARLO RUNS

N	$Algo$	$\widehat{h}(1)\pm\sigma$	$\widehat{h}(2)\pm\sigma$	$\widehat{h}(3)\pm\sigma$	MSE
500	$AlgoZ$	0.6722±0.3750	−0.6143±0.3593	−0.5033±0.4396	0.0306
	$Algo$-$Safi$	0.4345±0.5202	−0.5992±0.3156	−0.7112±0.3219	0.0452
	$Algo$-$Zhang$	0.4476±0.2153	−0.3244±0.2853	−0.6283±0.2141	0.0958
1500	$AlgoZ$	0.6726±0.1706	−0.6399±0.1609	−0.6494±0.2183	0.0098
	$Algo$-$Safi$	0.6977±0.2139	−0.3887±0.3941	−0.8573±0.1425	0.0335
	$Algo$-$Zhang$	0.4683±0.1360	−0.4660±0.2079	−0.6494±0.1122	0.0494
2000	$AlgoZ$	0.7860±0.1583	−0.6196±0.1639	−0.7232±0.1713	0.0021
	$Algo$-$Safi$	0.6110±0.1017	−0.4416±0.1614	−0.7520±0.1098	0.0228
	$Algo$-$Zhang$	0.5138±0.0807	−0.4229±0.1145	−0.6930±0.1000	0.0446

TABLE IV
ESTIMATED PARAMETERS OF SECOND CHANNEL IN NOISE CASE FOR DIFFERENT SNR, $N = 2000$, AND FOR 50 MONTE CARLO RUNS.

N	$Algo$	$\widehat{h}(1)\pm\sigma$	$\widehat{h}(2)\pm\sigma$	$\widehat{h}(3)\pm\sigma$	MSE
0 dB	$AlgoZ$	0.6518±0.6818	−0.7499±0.5255	−0.7976±1.0092	0.0267
	$Algo$-$Safi$	0.3533±0.4946	−0.4383±0.3873	−0.7990±0.2443	0.0859
	$Algo$-$Zhang$	0.4631±0.2827	−0.1489±0.2686	−0.7010±0.1688	0.1758
15 dB	$AlgoZ$	0.7965±0.1733	−0.6320±0.1777	−0.7956±0.1875	0.0039
	$Algo$-$Safi$	0.5409±0.2055	−0.4170±0.2926	−0.7970±0.1142	0.0402
	$Algo$-$Zhang$	0.5566±0.1209	−0.3550±0.1273	−0.7100±0.0848	0.0550
30 dB	$AlgoZ$	0.7642±0.1393	−0.6351±0.1443	−0.7503±0.1313	0.0023
	$Algo$-$Safi$	0.5906±0.1669	−0.4435±0.2336	−0.7752±0.1154	0.0254
	$Algo$-$Zhang$	0.5401±0.0731	−0.4071±0.1137	−0.6785±0.0799	0.0441
45 dB	$AlgoZ$	0.7935±0.1308	−0.5993±0.1086	−0.7172±0.1317	0.0016
	$Algo$-$Safi$	0.6377±0.1075	−0.4497±0.1997	−0.7729±0.1067	0.0185
	$Algo$-$Zhang$	0.5502±0.0879	−0.4169±0.1134	−0.7041±0.0827	0.0384

VII. CONCLUSION

In this paper we have presented an algorithm based on fourth order cumulants, exploiting only $(q + 1)$, compared to the Zhang's and Safi's algorithms, exploiting only $(2q + 1)$, to identify the parameters of the impulse response of the minimum phase channel. The simulation results show the proposed algorithm able to estimate the phase and magnitude blindly, with very good precision, than those obtained by the Zhang's and Safi's algorithms their performances degrade for all target. In the future we will test the efficiency of the proposed algorithm for the identification of the Broadband Radio Access Network channel especially MC-CDMA system.

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