Reliability-Based Life-Cycle Cost Model for Engineering Systems

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Abstract—The effect of reliability on life-cycle cost, including initial and maintenance cost of a system is studied. The failure probability of a component is used to calculate the average maintenance cost during the operation cycle of the component. The standard deviation of the life-cycle cost is also calculated as an error measure for the average life-cycle cost. As a numerical example, the model is used to study the average life-cycle cost of an electric motor.

Keywords—Initial Cost, Life-cycle cost, Maintenance Cost, Reliability.

I. INTRODUCTION

THE automotive industry is being influenced by the move toward sustainable development. To find a sustainable alternative to current internal combustion engine vehicles (ICEV), a candidate option must meet many criteria, including little environmental pollution, overall effciency, performance, and cost-effectiveness. Hybric electric vehicles (HEV) are one proposed solution for technology replacement [1]-[2].

The initial cost of a HEV is typically higher than that of an ICEV-with similar performance-mainly because the former contains some additional components when compared to the latter; this includes the electric motor (EM), controllers and the battery pack. On the other hand, an HEV benefits from a much better fuel economy. Consequently, HEVs become economically competetive with ICEVs once their life-cycle cost (LCC) is considered [1]. This implies a need for a comprehensive LCC model for HEVs when designing such a vehicle.

Lipmen and Delucchi developed a detailed model to estimate the LCC of battery powered electric vehicles (BPEV) [3] and HEVs [4]. The model considers the vehicle as a system composed of subsystems including the body, the powertrain, the electric motor, the battery pack, etc. The overal characteristics of the vehicle are expressed in terms of the characteristics of the subsystems. Also, the gasoline price is assumed to be constant. In case of HEVs [4], the results show a moderate hybridization is economically competetive with the corresponding baseline ICEV.

A mathematical model is developed to study fuel cell hybrid vehicles in [5]. The model is based on a linear relation between the cost and the performance of each component. A minimum payback period (PP) of nine years is reported for a mass production of five hundred thousand units.

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The LCC of HEVs in chinese market is studied in [6]-[7]. The energy cost is obtained from investigating 80 vehicles working for 10 days [6]. The maintenace cost is calculated based on the maintenance cost of ICEVs with some corrections to consider the effect of EM and battery set [6]. Similar to [5], a linear cost-performance function is assumed for the components in both papers. Results show that if the gasoline price climbs or the government increases carbon taxes, HEVs become economically competetive with conventional vehicles.

To calculate the LCC of plug-in hybrid vehicles, Al-Alawi and Bradley [8] obtain the long-term trend of electricity and gasoline prices from the Annual Energy Outlook report [9]. The maintenance cost is also calculated based on online published heuristic data presented in [10]-[11]. Their sensetivity analysis shows that driving distance and energy price are the mose influencial parameters on the LCC of HEVs. They also point out that a more detailed analysis of maintenance cost and fuel economy is required to achieve a more precise cost model.

A large range of LCC is reported in literature for HEVs mainly because of the wide range of parameters considered in calculations. Further, though the studies try to compare cost of vehicles with similar characteristics, they neglect the effect of reliability on cost. However, the cost of a component may be sensitive to its reliability. A list of cost-reliability functions proposed by different researchers may be found in [12].

Mettas [13] and Twum et al. [14] use an exponential cost-reliability function to solve a multi-objective optimization problem of minimizing the cost and maximizing the total reliability for a system of series-parallel components. They deal with the initial or purchase cost and no maintenance is considered.

Study of the literature indicates that a cost model which takes into account both the performance and the reliability is required to improve the understanding of the LCC of a system of components. This article focuses on the effect of reliability on cost of a system. First an argument is provided to develop an initial cost-reliability function. Then, a statistical tool is established to calculate the average and the standard deviation of maintenance cost as a function of the system reliability. Finally through some numerical examples, the accuracy of the proposed model is studied; the model is used to minimize the LCC of an electric motor by finding a trade-off between its initial and maintenance cost.

II. MODEL FORMULATION

A. Initial Cost and Reliability

There are generally three approaches to increasing the reliability of a system. In the first approach the reliability of

the system components are increased by using higher quality materials and manufacturing processes. Higher reliability in the component level leads to higher reliability of the whole system. The second approach is changing the components' size to change their reliability. This approach is mainly appropriate wherever increasing the size enhances the fatigue resistance of the components. The idea of the third approach is changing the system configuration by using components preferentially in parallel than in series. The system reliability is then increased without altering the reliability of individual components. The first two approaches are discussed here and the last approach is left for future work.

1) Size and reliability: The effect of size on reliability is evident wherever resizing drastically affects failure growth and fatigue. In this section the effect of resizing on reliability of spur gears is discussed.

The allowable bending stress of a spur gear for 10 millions operation cycle is given by [15]

$$\sigma_{all.} = \frac{A_1}{Y_Z}. \tag{1}$$

Parameter A_1 contains information on the material properties, safety factor, and temperature factor. Y_Z is reliability factor given by

$$Y_Z = \alpha_1 - \alpha_2 Ln(1 - R) \tag{2}$$

where R is the end of cycle reliability and constants α_1 and α_2 are given in [15]. The reliability factor for $R = 0.99$ equals unity.

The maximum bending stress of a spur gear is calculated from [15]

$$\sigma = A_2 \frac{W^t}{b}, \tag{3}$$

where W^t and b are transverse load and gear face width, respectively, and A_2 summarizes the information about load distribution on the gear teeth, overload factor, dynamic factor, geometry factor and the gear module.

One may set left-hand sides of (3) and (1) equal to calculate Y_Z of a specific design, and then use (2) to calculate reliability of the designed gear. This two-step procedure is followed here to study how varying the size of gear affects its reliability keeping the working condition unchanged.

Consider a set of gears with similar face widths and modules but different pitch diameters. Any of the gears may be chosen as a reference gear and all other gears of the set may be geometrically mapped into the reference gear by a scaling γ

$$d_g^* = \gamma d_g \ , \ b_g^* = b_g, \tag{4}$$

where d_g and b_g are the pitch diameter and face width of the reference gear g, respectively, and the parameters with superscipt asterisks are the corresponding quantities for the scaled gear g^*. Furthermore, consider that each of the gears in the set may be used to transmit a specific torque with a specific angular velocity. If the gears have shaped or ground profiles, the sensitivity of the dynamic factor to the linear velocity of

the pitch circle can be assumed negligible [15]. This way, A_2 in (3) remains unchanged under the scaling (4). On the other hand, the transverse load on the gear will be scaled as

$$W_t^* = \frac{1}{\gamma} W_t, \tag{5}$$

which along with (3) leads to

$$\sigma^* = \frac{1}{\gamma} \sigma. \tag{6}$$

Comparison of (6) with (1) implies that the reliability of the scaled gear is related to the reliability of the original gear through

$$Y_{Z_g}^* = \gamma Y_{Z_g}, \tag{7}$$

Substitution of (7) into (2) and some mathimatical manipulation yields

$$1 - R_g^* = (1 - R_g)^\gamma exp[-\frac{\alpha_1}{\alpha_2}(\gamma - 1)], \tag{8}$$

which relates the reliability of scaled gear R_g^* to the reliability of the reference gear R_g.

2) Size and cost: The initial cost C_0 of a component may be written as a function of its mass m in the form

$$C_0 = \alpha_3 + \alpha_4 m. \tag{9}$$

In (9), α_3 is the fixed cost of manufacturing the component whereas α_4 is the variable cost which takes into account material and processes cost per unit mass of the component [16]. For the scaling (4), the mass of the reference and scaled gears are related through

$$m_g^* = \gamma^2 m_g, \tag{10}$$

which along with (9) implies

$$C_{0_g}^* - \alpha_3 = \gamma^2 (C_{0_g} - \alpha_3). \tag{11}$$

3) Cost-reliability correlation: In order to understand how initial cost is correlated to reliability, one approach is varying γ and using (8) and (11) to plot initial cost versus reliability. However, one may be interested in omiting γ and deriving an equation for initial cost-reliability. To this end, calculating γ from (11) and substituting the result in (7) along with (2) gives

$$C_{0_g}^* = \alpha_3 + (C_{0_g} - \alpha_3)\left(\frac{\frac{\alpha_1}{\alpha_2} - Ln(1 - R_g^*)}{\frac{\alpha_1}{\alpha_2} - Ln(1 - R_g)}\right)^2. \tag{12}$$

Equation (12) is quadratic in terms of $Ln(1 - R_g^*)$. Hence, for the set of gears described in this section, the initial cost-reliability function may be written in the general form

$$C_0 = a_1 + a_2 Ln(1 - R) + a_3 \big(Ln(1 - R)\big)^2, \tag{13}$$

which contains three unknown constants. By knowing the cost and reliability of three gears in that set, the initial cost-reliability function in (13) can be determined.

It should be noted that (12) is a consequence of mapping (4). For another set of gears with smilar module and pitch diameter but different face widths, instead of (4) the scaling has the form

$$d_g^* = d_g \ , \ b_g^* = \gamma b_g. \tag{14}$$

In this case the scalings (6), (7) and (8) still hold. However, relation (10) will be replaced by

$$m_g^* = \gamma m_g, \tag{15}$$

which along with (7) and (2) implies

$$C_0 = a_1 + a_2 Ln(1 - R). \tag{16}$$

In this case the initial cost is linearly correlated to $Ln(1-R)$. Therefore by knowing the initial cost and reliability of two gears in the set, the initial cost-reliability correlation can be determined.

4) Materials and processes: A thorough review on the reliability of EMs is presented in [17]. The discussion of reliability of EM's winding is briefly presented here as an example of the effect of materials on reliability of a component.

The time decay of winding's reliability is modeled as [17]

$$R_w(t) = exp(-\lambda_w t), \tag{17}$$

where the failure intensity is given by

$$\lambda_w = \lambda_{wT} exp[\alpha_t(\theta - \theta_{max})] \tag{18}$$

in which θ is the working temperature of the winding and λ_{wT} is the failure intensity of the winding at temperature θ_{max}. Coefficients θ_{max} and α_t depend on the insulation class of the winding. The value of these quantities for three insulation classes are presented in Table I [17].

TABLE I
INSULATION CLASSES OF MOTORS

Class	B	F	H
θ_{max}, C	130	155	180
$\alpha_t, 1/C$	0.073	0.078	0.085

The WEG motor Catalogue indicates that price of motors with F and H insulation classes are, respectively, 5% and 10% higher than that of the B insulation class [18].

B. Maintenance Cost and Reliability

As the reiability of a component decreases by time, the component may fail before its desired life cycle. In this case an unplanned maintenance must be performed to replace the component with a new one. Here, a mathematical model is developed to calculate the average and the standard deviation of maintenance cost of a component.

1) Expected value of maintenance cost: The probability of having a failure between instants t_1 and $t_1 + dt_1$ equals [12]

$$P(t_1)dt_1, \tag{19}$$

where

$$P = -\frac{\partial R}{\partial t}. \tag{20}$$

Generalization of (19) implies that the probability of having n failures at instant $t_1, t_2, ...t_n$ equals

$$f(t_1, t_2, ..., t_n)dt_n...dt_2 dt_1, \tag{21}$$

where

$$f(t_1, t_2, ..., t_n) = P(t_1)P(t_2 - t_1)...P(t_n - t_{n-1}). \tag{22}$$

The function f may be used as a weighting function in calculating the expected value of n-th maintenace cost $\langle C_n \rangle$ through

$$\langle C_n \rangle = \int_0^T \int_{t_1}^T ... \int_{t_{n-1}}^T \tilde{C}_{t_n} f(t_1, t_2, ..., t_n)dt_n...dt_2 dt_1. \tag{23}$$

in which $\tilde{C}_{t_n}$ denotes the equivalent present value (EPV) of n-th maintenance cost at instant t_n. By summing the average of maintenance costs for all possible number of failures n, the total average maintenance cost $\langle C_{main.} \rangle$ is obtained

$$\langle C_{main.} \rangle = \sum_{n=1}^{\infty} \langle C_n \rangle, \tag{24}$$

and, therefore, the average LCC $\langle C \rangle$ of the component is

$$\langle C \rangle = C_0 + \langle C_{main.} \rangle. \tag{25}$$

2) Standard deviation of maintenance cost: The failure probability function (21) may be used to calculate the average of the maintenance cost squared

$$\langle C_n^2 \rangle = \int_0^T \int_{t_1}^T ... \int_{t_{n-1}}^T \tilde{C}_{t_n}^2 f(t_1, t_2, ..., t_n)dt_n...dt_2 dt_1, \tag{26}$$

and the standard deviation of the maintenance cost is calculated using

$$\Delta C_{main.} = \sum_{n=1}^{\infty} \left[\langle C_n^2 \rangle - \langle C_n \rangle^2\right]^{\frac{1}{2}}. \tag{27}$$

C. Numerical Examples

1) Initial Cost-Reliability of Spur Gears: The Stock Production Catalogue (volume 8) of Toronto Gear is used to obtain Price of two sets of gears, one with diametral pitch 24 and another with diametral pitch 4 [19]. Pressure angle of gears of both sets is 14.5^o. In each set, one gear is chosen as the reference gear whose reliability after $T=10$ million cycles equals 0.99. The reliability of the rest of gears of each set is calculated using (8) where constants α_1 and α_2 are obtained from [15].

The initial cost-reliability curves predicted by the present model, the Aggarwal and Misra models [12] are illustrated in Fig. 1 and 2. The results of the present model show an excellent agreement with the catalogue prices.
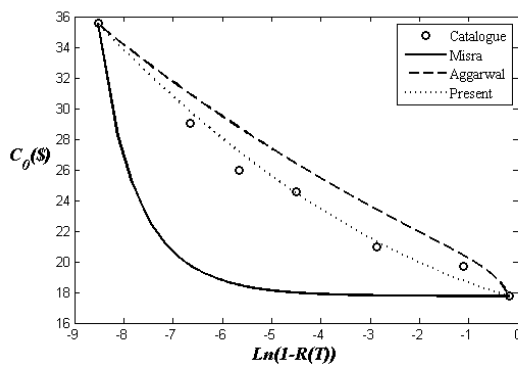

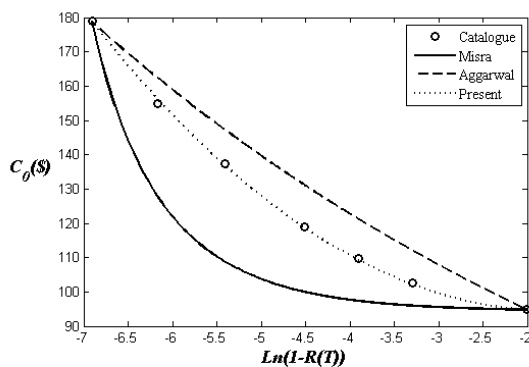Fig. 1. Initial cost-reliability of spur gears, diametral pitch=24

Fig. 2. Initial cost-reliability of spur gears, diametral pitch=4

2) Initial cost and reliability of roller bearings: Equations (13) and (16) are derived for spur gears. However, if two components undergo similar loading regimes and have similar failure mechanisms, they may have similar initial cost-reliability curves. If this conjecture is correct, one may expect that cost-reliability curve of roller bearings follow either (13) or (16). The reason is that, similar to gears, the rollers of a bearing undergo a cyclic loading and one of the main failure mechanisms of bearings is the pitting of the rollers.

To verify the validity of this model, the price of a set of 62xx and 160xx series FAG deep groove ball bearings are obtained from the supplier. The part number, dynamic load rating C_r, applied radial load P_r, reliability $R(T)$ (at $T=1$ million cycles) and initial cost C_0 of the 62xx and 160xx bearings are listed in Tables II and III, respectively. Reliability of each bearing is calculated using the formula presented in [20]. The cost-reliability data of these tables are plotted in Fig. 3 and 4, along with predictions from the Misra and the Aggarwal models [12], as well as the prediction of the present model (13). Again, a good agreement is observed between the present model and the catalogue prices. A study on initial cost-reliability correlation of bearings will be addressed in a future work to establish a solid basis for this consistency.

TABLE II
RELIABILITY AND INITIAL COST OF 62XX FAG BEARINGS

Part no.	C_r (kN)	P_r (kN)	$R(T)$	C_0 (€)
6205	14.9	29.9	0.0751	13.37
6206	20.7	29.9	0.5597	17.93
6207	27.5	29.9	0.8554	23.55
6208	31.0	29.9	0.9151	31.30
6209	33.0	29.9	0.9364	38.25
6210	39.0	29.9	0.9716	45.25
6211	46.0	29.9	0.9880	53.40
6212	56.0	29.9	0.9963	66.70
6213	63.0	29.9	0.9985	97.50
6214	66.0	29.9	0.9990	108.50

TABLE III
RELIABILITY AND INITIAL COST OF 160XX FAG BEARINGS

Part no.	C_r (kN)	P_r (kN)	$R(T)$	C_0 (€)
16012	21.2	29.5	0.6132	63.20
16013	22.5	29.5	0.6893	73.50
16014	29.5	29.5	0.9000	89.90
16015	30.5	29.5	0.9140	100.00
16016	34.0	29.5	0.9481	117.00
16017	36.0	29.5	0.9606	130.50
16018	44.0	29.5	0.9858	146.00
16020	46.5	29.5	0.9895	181.50
16022	61.0	29.5	0.9982	254.00
16024	65.0	29.5	0.9990	296.50

3) Cost-reliability of electric motors: The main failure mechanisms of EMs are failure in winding, bearings, commutator, soldered connections (SC) or gears (if applicable) [17]. Failure of other components of EMs such as magnets and shaft is rare and, therefore, their reliability can be considered to be unity. The present model is used in cost-reliability analysis of an EM. For simplicity, the effect of winding and bearings is studied here and it is assumed there is no failure in SCs,
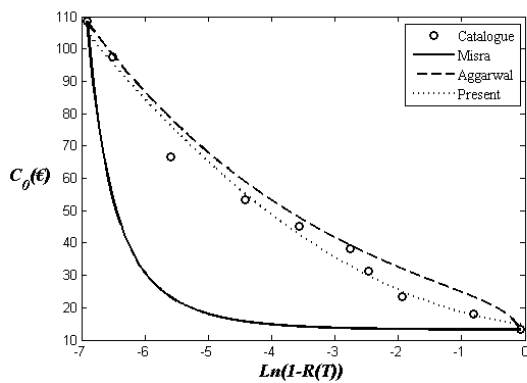
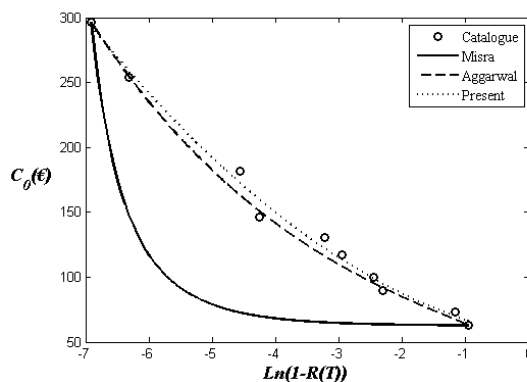Fig. 3.   Initial cost-reliability of 62xx FAG bearings

Fig. 4.   Initial cost-reliability of 160xx FAG bearings

commutator or gears. Comprehensive study of effect of SCs, gears and commutator on cost-reliability curves of EMs is left for a future work.

The electric motor studied here is 01012EG3E256T-W22 WEG motor. This motor has two 6209 bearings. Consider that a designer is allowed to replace 6209 bearings with any of the 62xx FAG bearings listed in Table II and the radial load on each of the EM's bearings is 29.9 kN. The bearings' prices are converted from Euro to US Dollars using a converstion factor of 1.36.

The time evolution of bearings' reliability R_b is often modelled by a Weibull distribution [20]

$$R_b(t) = exp\big(- (\lambda t)^k \big).   \quad (28)$$

Since the Weibull parameter k is not known for the aforementioned bearings, for simplicity it is assumed to be unity and (28) reduces to an exponential decay.

For a constant interest rate η compounded continuously, the EPV of a maintenance at instant t_n equals [16]

$$\tilde{C}_{t_n} = C_0 exp(-\eta t_n).   \quad (29)$$

Equation (28) along with (20)–(25) gives the LCC-reliability curve for the 62xx bearings. For an annual interest rate of

$\eta = 5\%$, the initial, maintenance and LCC curves are plotted in Fig. 5. It is evident that by decreasing the reliability of the component, its initial cost goes down whereas the maintenance cost increases because of higher chance of failure. Further, higher chance of failure leads to a higher uncertainty in the LCC of such components. The standard deviation of LCC calculated from (26) and (27) may be used to obtain higher and lower limits of the average LCC. Fig. 6 depicts the average LCC and also its higher and lower bounds.
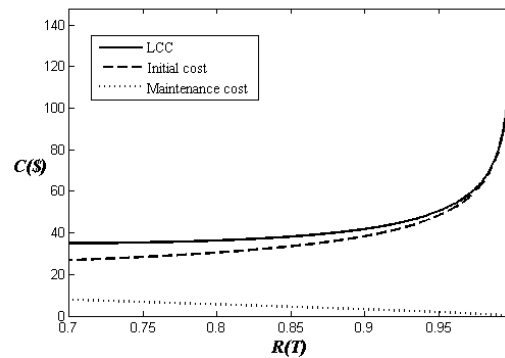

Fig. 5.   LCC, initial, and maintenance cost of EM's bearings
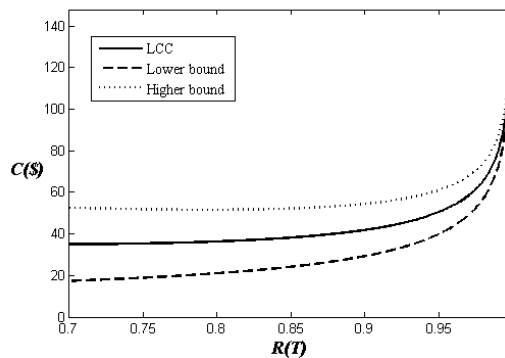

Fig. 6.   LCC and its higher and lower bounds for EM's bearings

For the 01012EG3E256T-W22 WEG motor with 6209 bearings, the catalogue price is US\$ 2636.00 for F insulation class [18]. Using this price, the price of the motor with the other two insulation classes are calculated and presented in Table IV. The price of 6209 bearing is obtained from Table II and is converted to US Dollars by a factor 1.36.

TABLE IV
INITIAL COST OF WEG MOTOR WITH DIFFERENT INSULATION CLASSES

Insulation class	Motor price with bearings	6209 bearings price	Motor price without bearings
B	2510.48	104.04	2406.44
F	2636.00	104.04	2531.96
H	2761.52	104.04	2657.48

The nominal speed of the motor is 1200 rpm [18]. Hence, the motor will reach 1 million revolutions after $T = 13.89$ h. If the failure intensity of the winding is $\lambda_{wT} = 10^{-3} 1/h$ and the motor operates at $\theta = 150^oC$ [17], the winding's reliability after 1 million revolution for different insulation classes can be calculated using (17) and (18) as well as the data in Table I. These results are presented in Table V.

TABLE V
RELIABILITY OF WINDINGS WITH DIFFERENT INSULATION CLASSES

Insulation class	$R_w(T)$
B	0.9419
F	0.9906
H	0.9989

Having the reliability and initial cost of the bearings and the motor (bearings excluded), one may deal with system cost and reliability after 1 million revolutions. Since the bearings and the winding are in series, the system reliability is [12]

$$R(T) = R_b^2(T) R_w(T). \qquad (30)$$

The average motor cost equals sum of average bearings cost $\langle C_b \rangle$ and average motor (bearings excluded) cost $\langle C_w \rangle$ through

$$\langle C \rangle = 2\langle C_b \rangle + \langle C_w \rangle. \qquad (31)$$

One may use (20)-(25), (28), (29) along with (30) and (31) to calculate the reliability and average LCC of the EM. The result is plotted in Fig. 7. This figure shows the regions of cost-reliability space achievable by varying the reliability of system components. A designer may select a design point on this plot according to his performance-economic requirements. It should be noted that there are intersections between the curves at reliabilities $R(T) = 0.8997$ and $R(T) = 0.9886$. This implies that to optimize the system cost and reliability, if the desired system reliability is below 0.8997, insulation class B must be used. For system reliabilities between 0.8997 and 0.9886, insulation class F must be selected. If reliabilities higher than 0.9886 is required, the minimum cost is achieved by using insulation class H.
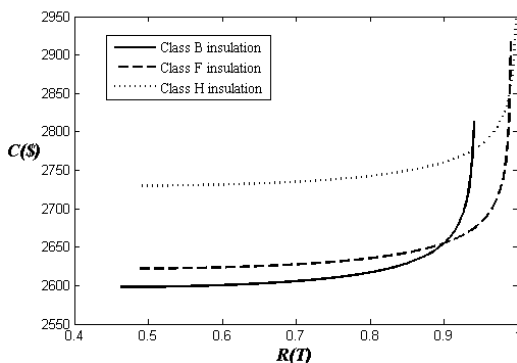

Fig. 7.   System cost versus system reliability for WEG motor

III. CONCLUSION

In this article, an argument is provided to develop a correlation between the inital cost and the reliability of a component. The proposed formulation is simple and accurately models the cost-reliability curves of gears and bearings. Further, a mathematical model is developed to calculate the average and standard deviation of the life-cycle cost of a system of components. This model gives the cost-reliability space achievable for a system of components which may be a valuable tool in selecting a design point with desirable reliability and cost. Moreover, a manufacturer may be interested in setting specific limits on the uncertainty of the life-cycle cost of its product. The proposed model can be used to manage the risk of future warranty costs for manufacturers.

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