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An EWMA p Chart Based On Improved Square Root Transformation

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***Abstract*—**Generally, the traditional Shewhart p chart has been developed by for charting the binomial data. This chart has been developed using the normal approximation with condition as low defect level and the small to moderate sample size. In real applications, however, are away from these assumptions due to skewness in the exact distribution. In this paper, a modified Exponentially Weighted Moving Average (EWMA) control chat for detecting a change in binomial data by improving square root transformations, namely ISRT p EWMA control chart. The numerical results show that ISRT p EWMA chart is superior to ISRT p chart for small to moderate shifts, otherwise, the latter is better for large shifts.

***Keywords*—**Number of defects, Exponentially Weighted Moving Average, Average Run Length, Square root transformations.

I. Introduction

NOWADAYS, the manufacturing processes often produce a low defective level in process due to process improvement and modern technology. The classical control charts for number/proportion of defective products as np/p charts are not good enough for this situation. Because the np and p charts are developed by using the normal approximation which is often used to deal with the conditions $np \geq 5$ and $n(1-p) \geq 5$, where n denotes the number of items produced and inspected and p denotes the defect level of process.

In the literatures, there are several method to deal with this situations, for example, Schader-Schmid (1989) [1] presented that the normal approximation had poorly performance when $np \geq 5$ and the accuracy of the approximation based on the values of p. Next, Ryan and Schwertman (1997) [2] proposed the Arcsine p chart which the lower tail probability is usually too small and the upper tail probability of np chart is usually too large. Later, the Q chart proposed by Quesenberry [3]-[5] which concluded that the Arcsine p-chart gives a better approximation to the nominal lower tail area and the Q-chart provides a better approximation to the nominal upper tail area. Winterbottom [5] and Chen [6] used the Cornish-Fisher expansion of quantiles to construct a modified p-chart. Recently, Tsai, T-R et al. (2006) [7] have been proposed the Improved Square Root Transformation (ISRT) for attribute control charts- ISPRT p, np and c charts for binomial and Poisson data, respectively. This method can be applied to three attribute control charts- ISRT np, p and c charts. In general, the traditional attribute control chart like np chart is often used for moderate to large shifts [8]; however, an alternative Exponentially Weighted Moving Average (EWMA) control chart performs better than traditional chart

Consequently, this paper proposed the modified EWMA control charts based on ISRT method for detecting a change in binomial observation.

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II. Control Charts

A. ISRT p Chart

In 2006, Tsai, T-R et al. [7] proposed the Improved Square Root Transformation (ISRT) method for binomial data and implemented to attribute control charts, namely, ISRT p chart.

Suppose the process observations are taken from a binomial distribution. Let X be a binomial random variable with parameter n and p, where n is number of trial and p, is the proportion of defects in process. Then, $\hat{p} = \frac{X}{n}$ is the sample defect level. The normal approximation would be poor, if p is small and n is small to moderate sample sizes. Then, the ISRT can be used to overcome to this situation which is applied in control chart for binomial observation, namely, ISRT p chart. The upper and lower control limits are shown by [8] as following:

$$UCL_p = \sqrt{p} + \frac{3}{2}\sqrt{\frac{1-p}{n}} - \frac{1}{2}\left(\frac{1-p}{n\sqrt{p}}\right) \tag{1}$$

and

$$LCL_p = \sqrt{p} - \frac{3}{2}\sqrt{\frac{1-p}{n}} - \frac{9}{8}\left(\frac{1-p}{n\sqrt{p}}\right). \tag{2}$$

This control limits is adequate for the low defect level of $p \leq 0.1$. If the parameter p is unknown, it can be estimated by $\bar{p} = \frac{\sum_{i=1}^{m} D_i}{mn} = \frac{\sum_{i=1}^{m} \hat{p}_i}{m}$, where D_i is defect items in the sample i, m is pre-samples each of size n.

B. Binomial EWMA Chart

In 1959, Roberts [9] who first proposed an Exponentially Weighted Moving Average (EWMA) control chart which is an effective alternative to the traditional Shewhart control chart in term of sensitive to the detection of small shifts. The EWMA statistics can be written as follows:

$$Z_i = \lambda \bar{X}_i + (1-\lambda) Z_{i-1} \quad , i = 1,2,...$$

where λ is weight of past information, $0 < \lambda < 1$. The control limits of binomial EWMA control chart are

$$UCL_{EWMA} = p + k\sqrt{\frac{pq}{n}\frac{\lambda}{2-\lambda}\left(1-(1-\lambda)^{2i}\right)} \quad (3)$$

and

$$LCL_{EWMA} = p - k\sqrt{\frac{pq}{n}\frac{\lambda}{2-\lambda}\left(1-(1-\lambda)^{2i}\right)} \quad (4)$$

where k is coefficient of EWMA control limit and $i \to \infty$ the control limits of EWMA chart can be rewritten as the following:

$$UCL_{EWMA} = p + k\sqrt{\frac{pq}{n}\frac{\lambda}{2-\lambda}}$$

and

$$LCL_{EWMA} = p - k\sqrt{\frac{pq}{n}\frac{\lambda}{2-\lambda}}.$$

III. ISRT p EWMA Control Chart

According to ISRT p and EWMA control charts, the modified ISRT to binomial EWMA control chart is proposed in this paper. From (1), it is the upper control limit of ISRT p chart which the expectation and variance are

$$E(X) = \sqrt{p}$$

and

$$V(X) = \left[\frac{1}{2}\sqrt{\frac{1-p}{n}} - \frac{1}{6}\left(\frac{1-p}{n\sqrt{p}}\right)\right]^2.$$

Besides, (2) is lower control limits which the expectation and variance are

$$E(X) = \sqrt{p}$$

and

$$V(X) = \left[\frac{1}{2}\sqrt{\frac{1-p}{n}} - \frac{3}{8}\left(\frac{1-p}{n\sqrt{p}}\right)\right]^2.$$

Therefore, the modified ISRT p EWMA control chart can be presented the control limits as the following:

$$UCL_{ISRTEWMA} = \sqrt{p} + L\sqrt{\frac{\lambda}{2-\lambda}\left[\frac{1}{2}\sqrt{\frac{1-p}{n}} - \frac{1}{6}\left(\frac{1-p}{n\sqrt{p}}\right)\right]} \quad (5)$$

and

$$LCL_{ISRTEWMA} = \sqrt{p} - L\sqrt{\frac{\lambda}{2-\lambda}\left[\frac{1}{2}\sqrt{\frac{1-p}{n}} - \frac{3}{8}\left(\frac{1-p}{n\sqrt{p}}\right)\right]}. \quad (6)$$

where L is coefficient of ISRT p EWMA control limit.

IV. The Properties of the Average Run Length

One of the common characteristics of control charts is in-control Average Run Length (ARL_0) or a mean of false alarm. Ideally, an acceptable ARL_0 of in-control process should be enough large. Otherwise it should be small when the process is out-of-control, so-called out-of-control Average Run Length (ARL_1) or a mean of true alarm. In general, the ARL of Shewhart control chart can be calculated as follows:

$$ARL_0 = \frac{1}{P_I} \text{ and } ARL_0 = \frac{1}{P_{II}}$$

where P_I and P_{II} are probability of type I and type II errors.

V. Numerical Results

In this section, the comparison of the Average Run Length for ISRT p and ISRT p EWMA chart are presented when the process observation are from binomial distribution. On Tables I and II, the comparison of the performance of ISRT p and ISRT p EWMA charts are shown when given ARL_0=370, $n = 50$, $p = 0.05$ and $p = 0.1$, respectively. The ISRT p EWMA control chart performs better than ISRT p chat when the magnitudes of shifts $\delta \le 0.5$, otherwise, the ISRT p chart is superior to ISRT p EWMA chart for $\delta > 0.5$.

TABLE I
Values of the ARL of ISRT p and ISRT p EWMA Control Charts When $p = 0.05,\ n = 50, \lambda = 0.05$ and ARL_0=370

δ	**ISRT p UCL=0.399985**	**ISRT p EWMA UCL=3.03631**
0	370.435	370.683
0.01	346.723	291.491*
0.03	283.874	211.793*
0.05	216.925	201.043*
0.10	152.681	108.164*
0.30	58.463	42.219*
0.50	29.431	25.527*
1.00	7.681*	13.913
1.50	1.912*	9.362
2.00	1.205*	6.994

*Minimum ARL_1

TABLE II
VALUES OF THE ARL OF ISRT P AND ISRT P EWMA CONTROL CHARTS
WHEN $p = 0.1,\ n = 50, \lambda = 0.05$ AND ARL_0=370

δ	ISRT p UCL=0.488999	ISRT p EWMA UCL=5.71498
0	370.417	370.683
0.01	319.458	294.422*
0.03	240.269	206.494*
0.05	205.831	151.309*
0.10	141.448	89.629*
0.30	39.460	37.194*
0.50	27.855	24.919*
1.00	2.637*	14.069
1.50	0.612*	9.845
2.00	0.179*	7.545

*Minimum ARL_1

TABLE III
VALUES OF THE ARL OF ISRT P AND ISRT P EWMA CONTROL CHARTS
WHEN $p = 0.05,\ n = 70, \lambda = 0.05$ AND ARL_0=370

δ	ISRT p UCL=0.368669	ISRT p EWMA UCL=4.12781
0	370.891	370.986
0.01	358.772	306.369*
0.03	305.223	231.101*
0.05	271.998	173.509*
0.10	206.002	102.269*
0.30	65.471	39.564*
0.50	22.387*	25.572
1.00	5.384*	14.089
1.50	1.764*	9.706
2.00	0.612*	7.367

*Minimum ARL_1

TABLE IV
VALUES OF THE ARL OF ISRT P AND ISRT P EWMA CONTROL CHARTS
WHEN $p = 0.1,\ n = 70, \lambda = 0.05$ AND ARL_0=370

δ	ISRT p UCL=0.368669	ISRT p EWMA UCL=4.12781
0	370.891	370.085
0.01	319.985	298.933*
0.03	216.962	193.174*
0.05	171.777	139.179*
0.10	118.861	83.189*
0.30	29.538	35.942*
0.50	28.897	24.537*
1.00	1.146*	14.152
1.50	0.280*	9.995
2.00	0.039*	7.715

*Minimum ARL_1

Furthermore, the numerical results are verified the performance of ISRT p EWMA chart is superior to ISRT p chart for small shifts $\delta \le 0.5$, for the case of ARL_0=370, $n = 70$, $p = 0.05$ and $p = 0.1$.

VI. CONCLUSION

In this paper, the modified Exponentially Weighted Moving Average (EWMA) control chart based on Improved Square Root Transformation (ISRT) with binomial data is proposed namely, ISRT p EWMA control chart. The control limits of this chart are presented. The numerical results show that the performance of ISRT p EWMA chart is better than ISPRT p chart for small shifts $\delta \le 0.5$ otherwise, the latter performs better than the former.

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