Optimal Assessment of Faulted Area around an Industrial Customer for Critical Sag Magnitudes

Marios N. Moschakis

Abstract—This paper deals with the assessment of faulted area around an industrial customer connected to a particular electric grid that will cause a certain sag magnitude on this customer. The faulted (critical or exposed) area's length is calculated by adding all line lengths in the neighborhood of the critical node (customer). The applied method is the so-called Method of Critical Distances. By using advanced short-circuit analysis, the Critical Area can be accurately calculated for radial and meshed power networks due to all symmetrical and asymmetrical faults. For the demonstration of the effectiveness of the proposed methodology, a study case is used.

Keywords—Critical area, fault-induced voltage sags, industrial customers, power quality.

I. INTRODUCTION

VOLTAGE sags are sudden drops in the rms voltage and are usually characterized by the remaining (retained) voltage. The most severe sags are caused by short-circuits in the transmission or distribution system. Their effect on sensitive equipment, such as computers, adjustable speed drives or control devices can be as important as voltage interruptions, which are more severe but less frequent. Industrial customers that use widely the aforementioned type of equipment may face enormous financial losses due to the frequency of sags and sensitivity of equipment [1], [2].

Several mitigation methods for the consequences of sags have been proposed [1], [2], [10]. The first step for the selection of the appropriate mitigation method is the assessment of the expected number of sags per year. Three methods have been proposed for the assessment of voltage sags due to faults: the method of Critical Distances [1]-[3], the method of Fault Positions [4] and the Monte Carlo method [5], [6]. All methods combine the response of the system to faults with stochastic data.

Apart from the assessment of the expected number of sags per year, the assessment of Critical Area around a sensitive industrial customer is also of high significance. This area can be an index of the sensitivity of a particular industrial customer but also reveals the location of faults. Thus, the examined area to apply methods for the elimination of short-circuits (faults) can be limited to a smaller area.

All the three aforementioned methods can be used for the calculation of the Critical Area. In this paper, the Method of Critical Distances is developed and thoroughly analyzed. The pros and cons of the particular method compared with the

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other two methods for the estimation of Critical Area are fully examined. The main points for a proper application of the examined method are listed along with suggestions for the elimination of calculation effort. For the demonstration of its effectiveness and the advantages against the method of Fault Positions, a generic meshed power network is used.

II. ADVANCED SHORT-CIRCUIT ANALYSIS

Voltage sag magnitude, which is the minimum retained (or during-fault) voltage among the three phases, can be calculated using analytical expressions derived from short-circuit theory [7] or performing detailed simulations [5], [6]. In [2], [3] simplified expressions based on the voltage divider model have been proposed for fast assessment of the critical distances and the number of sags. This approach is suitable for radial networks but presents many limitations for meshed systems [11], [12].

For the optimal assessment of voltage sag magnitude, it is important to use analytical expressions for the sag magnitude due to symmetrical and asymmetrical faults that are applicable to meshed and radial power networks. The methodology and the analytical expressions for the sag magnitude of the observation (sagged) node in relation with the fault distance due to faults at every point of a power line are given in [8].

A. Faults and Observation node at the Same Voltage Level

When a fault occurs at the same voltage level with the observation node (i.e. the examined industrial customer), the faulty phases coincide with the most sagged phases. For three-phase (3ph) faults and single-phase-to-ground faults (1ph) on phase A, the during-fault voltage of phase A gives the sag magnitude. For two-phase (2ph) and two-phase-to-ground (2ph-g) faults e.g. between phases B and C, sag magnitude is the minimum retained voltage among the sagged phases. Specifically, when ohmic resistances of lines are not neglected, sag magnitude of phases B and C are not always equal, even for common assumptions used in fault analysis [8], [9], i.e.:

$$Z_{bus}^{(1)} = Z_{bus}^{(2)} \ , \ \tilde{V}^{pref} = 1 \ , \ Z_{flt} = 0 \qquad (1)$$

In Table I, all the symbols, quantities and properties used in the current short-circuit analysis are presented.

B. Faults and Observation Node at Different Voltage Levels

When a fault occurs at different voltage level from the observation node due to the presence of a power transformer,

the effect of transformer's winding connectivity and phase shift should be introduced to the analytical expressions. The effect of the winding's connectivity is well known how to be handled in the formulation of Z_{bus}. The effect of power transformer's phase shift in case that an unbalanced fault occurs at the other side of the transformer with relation to the observation node is explained in [8], [9]. E.g. for a Dyn1 power transformer, in order to incorporate into the analytical expressions the phase shift introduced when faults occur at the High Voltage (HV) side and observed by the Low Voltage (LV) side, a phase shift of $+60^0$ (or $-a^2$) in the negative-sequence voltage can be applied.

Consequently, except for 3ph faults, new expressions are derived for the sag magnitude observed at the LV side due to unbalanced faults at the HV side. Those expressions for unbalanced faults are given in [8]. The accuracy of the derived expressions which incorporate the transformer's effect can be easily verified using a simulation software e.g. PSCAD [13].

TABLE I
NOMENCLATURE

Symbol	Quantity
(1), (2), (0)	Superscripts defining positive, negative and zero sequence respectively
A, B, C	Superscripts defining phases A, B and C
$\widetilde{V}_k^{pref}$, $\widetilde{V}_f^{pref}$	Pre-fault voltage (phase A) of nodes k, f
$\widetilde{V}_{kf}^A$, $\widetilde{V}_{kf}^B$, $\widetilde{V}_{kf}^C$	Phase A, B, C voltages at node k during a fault at position f
α	Fortescue operator ($a = e^{j2\pi/3}$)
Z_{flt}	Fault impedance
Z_{bus}	Bus impedance matrix
Z_{ff}	Driving-point impedance of bus f
Z_{kf}	Transfer impedance of buses k and f
z	Physical impedance of a branch
ℓ ($0 \leq \ell \leq 1$)	Distance to the fault and critical length

III. ASSESSMENT OF CRITICAL AREA

For the assessment of Critical area, the methods of Critical Distances, Fault Positions and Monte Carlo have been proposed. The first method can be applied only using analytical methods for the calculation of voltage sag magnitude. The other two methods can be applied by using either analytical methods or simulations e.g. using PSCAD/EMTDC software package [13].

In the method of Fault Positions, the sag magnitude is calculated for various fault positions (lengths) spread at equal distances for each line. To obtain a good accuracy on the results, a large number of fault positions for each fault type must be used. The usual procedure to find the minimum number of fault positions is to increase the number of fault positions until the results remain unaltered.

The Monte Carlo technique is widely used for solving stochastic problems. It is based on an iterative procedure that uses in every step a new set of values of the random variables involved in the process. These values are generated according to the probability density function associated to each variable. The response of the system studied converges to a solution theoretically after an infinite number of iterations.

IV. METHOD OF CRITICAL DISTANCES

The method of Critical Distances is based on the determination of the critical (exposed) area around the sensitive customer (observation node) for a given critical voltage sag magnitude V_{crit}, which takes values within the range 0.1-0.9 pu [14], [15]. Adding the line lengths within the exposed area the length of Critical Area is determined [1]-[3]. More specifically, it involves the solution of the following expression for each line in the neighborhood of the examined critical node in order to find the critical length ℓ per line for which the voltage will become lower than the examined critical voltage:

$$|\widetilde{V}_{kf}(\ell)| \leq V_{crit} \qquad (2)$$

where $\widetilde{V}_{kf}(\ell)$ is the minimum voltage among the three phases, k the observation node of a sag due to a fault at position f. The expressions for $\widetilde{v}_{kf}(\ell)$ are given in [8].

V. STUDY CASE

The assessment of Critical Area is mainly important for industrial customers and is usually performed in distribution networks. A suitable power network for the application of the Method of Critical Distances method is as the one shown in Fig. 1. Six industrial customers are connected at six nodes of the same 20 kV distribution line through a solidly grounded Dyn1 transformer, widely used in Greece. The equivalent transmission system consists of three 150 kV lines and is relatively of large size to take into account the fact that faults even at hundred kilometers away from the critical customers will cause them severe sags [1].
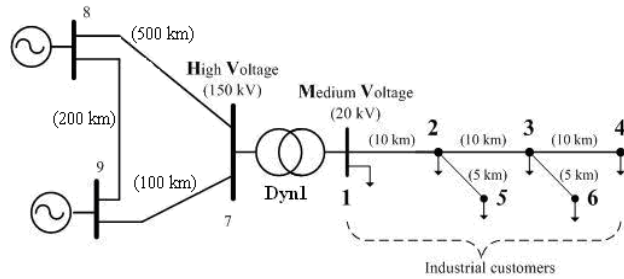

Fig. 1 Single-line diagram of the studied power network

The method of Critical Distances will be applied for the assessment of Critical Area of node 1. The sag magnitude for each fault type is given by the minimum retained voltage among the three phases. By graphical analysis [9], it can be shown that the sag magnitude for 2ph and 2ph-g faults at MV side is given by the during-fault voltage of phase C. Using the common assumptions given in (1), the analytical expressions

for the calculation of sag magnitude for 3ph, 1ph, 2ph and 2ph-g faults at MV side, are given from the following expressions (respectively):

$$\tilde{V}_{kf}(\ell) = 1 - \frac{Z_{kf}^{(1)}(\ell)}{Z_{ff}^{(1)}(\ell)} \tag{3}$$

$$\tilde{V}_{kf}^{A}(\ell) = 1 - \frac{2 \cdot Z_{kf}^{(1)}(\ell) + Z_{kf}^{(0)}(\ell)}{2 \cdot Z_{ff}^{(1)}(\ell) + Z_{ff}^{(0)}(\ell)} \tag{4}$$

$$\tilde{V}_{kf}^{C}(\ell) = a - \frac{(a - a^2) \cdot Z_{kf}^{(1)}(\ell)}{Z_{ff}^{(1)}(\ell)} \tag{5}$$

$$\tilde{V}_{kf}^{C}(\ell) = a - \frac{a \cdot Z_{kf}^{(1)}(\ell) \cdot [Z_{ff}^{(1)}(\ell) + Z_{ff}^{(0)}(\ell)]}{Z_{ff}^{(1)}(\ell) \cdot [Z_{ff}^{(1)}(\ell) + 2 \cdot Z_{ff}^{(0)}(\ell)]} - $$
$$- \frac{a^2 \cdot Z_{kf}^{(1)}(\ell) \cdot Z_{ff}^{(0)}(\ell) - Z_{kf}^{(0)}(\ell) \cdot Z_{ff}^{(1)}(\ell)}{Z_{ff}^{(1)}(\ell) \cdot [Z_{ff}^{(1)}(\ell) + 2 \cdot Z_{ff}^{(0)}(\ell)]} \tag{6}$$

For faults at HV side and calculation Critical Area of node 1, the transformer's phase shift effect must be incorporated into the analytical expressions of during-fault voltages. The minimum retained voltage can be used for the extraction of the sag magnitude. By an extensive examination of those expressions it can be proved that, in case of a Dyn1 HV/MV transformer, the sag magnitude of node 1 is equal for 3ph, 2ph and 2ph-g faults [9]. For 1ph faults, the minimum among phase voltages A and B of node 1 will give the sag magnitude [9]. Thus, the following expressions for 1ph faults and (3) can be used for the calculation of sag magnitude for each fault type at HV side:

$$\tilde{V}_{kf}^{A}(\ell) = 1 - \frac{(a - a^2) \cdot Z_{kf}^{(1)}(\ell) + Z_{kf}^{(0)}(\ell)}{2 \cdot Z_{ff}^{(1)}(\ell) + Z_{ff}^{(0)}(\ell)} \tag{7}$$

$$\tilde{V}_{kf}^{B}(\ell) = a^2 - \frac{(a^2 - a^3) \cdot Z_{kf}^{(1)}(\ell) + Z_{kf}^{(0)}(\ell)}{2 \cdot Z_{ff}^{(1)}(\ell) + Z_{ff}^{(0)}(\ell)} \tag{8}$$

Combining (2) with (3)-(8) will give the critical length of each power line in which a fault will give a sag magnitude below the critical voltage V_{crit}. For 3ph, 1ph and 2ph faults, (2) is further processed and the final expression to be solved is extracted as follows:

$$|\tilde{V}_{kf}(\ell)| = \left| \frac{n_2 \cdot \ell^2 + n_1 \cdot \ell + n_0}{d_2 \cdot \ell^2 + d_1 \cdot \ell + d_0} \right| \leq V_{crit} \Rightarrow$$
$$\Rightarrow s_4 \cdot \ell^4 + s_3 \cdot \ell^3 + s_2 \cdot \ell^2 + s_1 \cdot \ell + s_0 \leq 0 \tag{9}$$

In case of 2ph-g faults, the final expression for the

calculation of the critical length is extracted as follows:

$$|\tilde{V}_{kf}(\ell)| = \left| \frac{n_4 \cdot \ell^4 + n_3 \cdot \ell^3 + n_2 \cdot \ell^2 + n_1 \cdot \ell + n_0}{d_4 \cdot \ell^4 + d_3 \cdot \ell^3 + d_2 \cdot \ell^2 + d_1 \cdot \ell + d_0} \right| \leq V_{crit} \Rightarrow$$
$$\Rightarrow s_8 \cdot \ell^8 + s_7 \cdot \ell^7 + s_6 \cdot \ell^6 + s_5 \cdot \ell^5 +$$
$$s_4 \cdot \ell^4 + s_3 \cdot \ell^3 + s_2 \cdot \ell^2 + s_1 \cdot \ell + s_0 \leq 0 \tag{10}$$

The coefficients n, d are complex numbers and the coefficients s are real numbers. The coefficients n, d and s are given in the Appendix.

The Critical Area (CA) in km for faults at MV and HV side for each node, fault type and critical voltage can be expressed as follows:

$$CA_k^{flt}(|\tilde{V}_{kf}| \leq V_{crit}) = \sum_{i=1}^{m} (\ell_i \cdot L_i) + \sum_{j=1}^{h} (\ell_j \cdot L_j) \tag{11}$$

where m, h are the total number of power lines inside the critical area, L the length of each line and ℓ_i, ℓ_j the pu critical lengths per line given by (9) and (10).

A. Depiction of Critical Areas on the Electric Grid

The Critical Areas for all fault types and three critical voltages are graphically presented in Fig. 2 on the electric grid. It should be noted that in cases where two phases are sagged, two different critical lengths are found and the maximum among them is kept. In case of faults on HV side, the sag magnitude on the MV side is the same for 3ph, 2ph and 2ph-g faults. Thus, the critical lengths are equal for lines 7-8, 8-9, 7-9 and observation node 1-6 for any critical voltage.
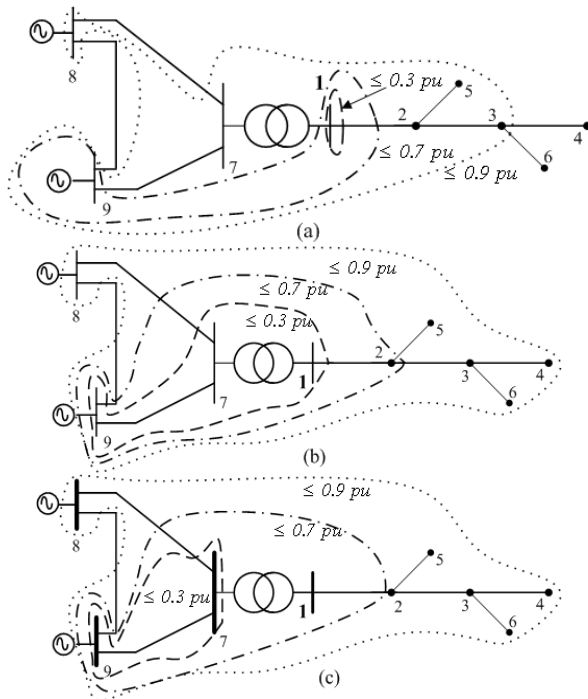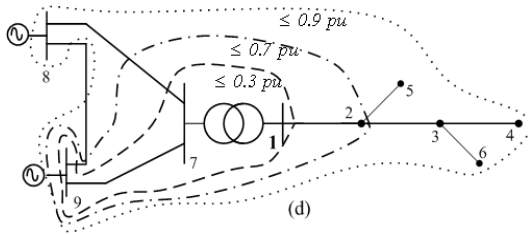
Fig. 2 Critical areas of node 1 for a critical voltage of 0.5 pu, 0.7 pu and 0.9 pu. (a) 1ph, (b) 3ph, (c) 2ph, (d) 2ph-g faults

Moreover, there may be one or two acceptable solutions for analytical expressions (9), (10) and the part of the line that these critical lengths correspond to should be determined. In case of 1ph faults, for critical voltage equal to 0.9 pu there are two acceptable solutions on line 7-8 with a total length of 0.204 pu or 102 km. There are also two solutions on line 8-9 for the same critical voltage that give a total critical length of 0.088 pu or 17.6 km. Those findings by solving (9) are graphically depicted on Fig. 2 (a).

In Figs. 2 (b)-(d) the Critical Areas on HV and MV side is graphically presented for 3ph, 2ph and 2ph-g faults for three different critical voltages (0.3, 0.7, 0.9 pu). The Critical Areas on HV side for every critical voltage is exactly the same for the three fault types. In case of line 8-9 and a 0.9 pu critical voltage, there are two acceptable solutions that give a total critical length of 0.619 pu or 123.8 km.

B. Total Length of Critical Areas per Fault Type and V_{crit}

In Fig. 3, the Critical Area of Nodes 1 and 4 is presented for every fault type on MV side vs. critical voltage. These nodes have the best and worst performance respectively after a fault on the MV side. It can be observed that the maximum critical length equals to 40 km for a sag magnitude below 0.9 pu for any fault type except for 1ph faults and Node 1. This means that any fault within the 40 km length of the MV grid will cause a sag magnitude below 0.9 pu. Moreover, for a sag magnitude equal to or below 0.8 pu, 2ph faults give larger critical length than 3ph faults because of the behavior of Node in faults on the lateral feeders. In case of Node 4, the 1ph faults dominate on deep voltage sags even compared to 3ph faults. Furthermore, 2ph faults contribute to critical areas for sag magnitudes equal to or over 0.5 pu.

In Fig. 4, the Critical Area of Nodes 1 to 6 is presented for faults on HV side vs. critical voltage. It can be observed that the length of critical area for 1ph faults is much shorter than for the other fault types. It should also be noted that 1 ph faults cause 2ph sags and 2ph faults 1ph sags on MV side.

In Fig. 5, the Critical Area of Nodes 1 and 4 is presented for faults on MV and HV side vs. critical voltage. The Critical Area that corresponds to 1ph faults has a much shorter length than the Critical Areas of the other fault types, which are almost equal. The effect on Critical Areas for 1ph faults is due to the power transformer's phase shift, which mitigates their severity despite the fact that they are the most frequent ones.
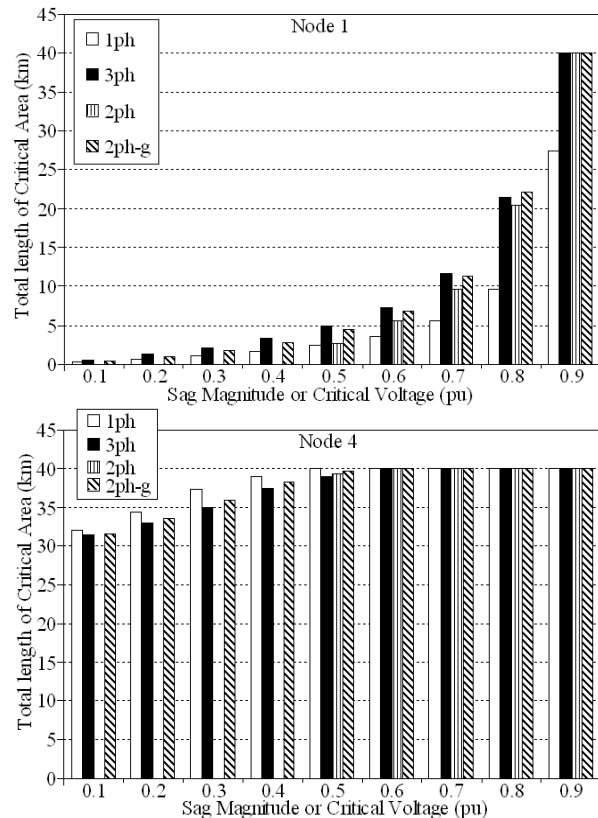

Fig. 3 Total length of Critical Area for faults on MV side vs. critical voltage (sag magnitude) for Nodes 1 and 4
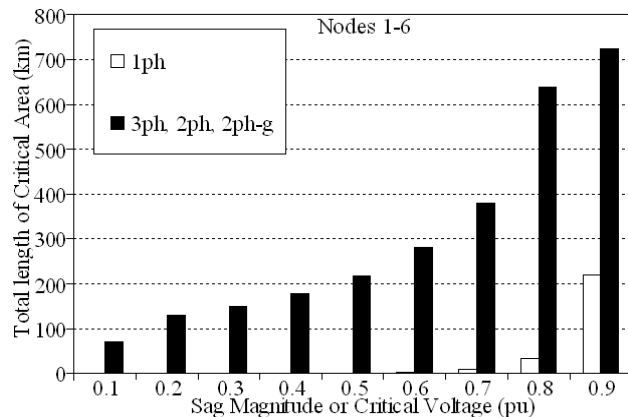

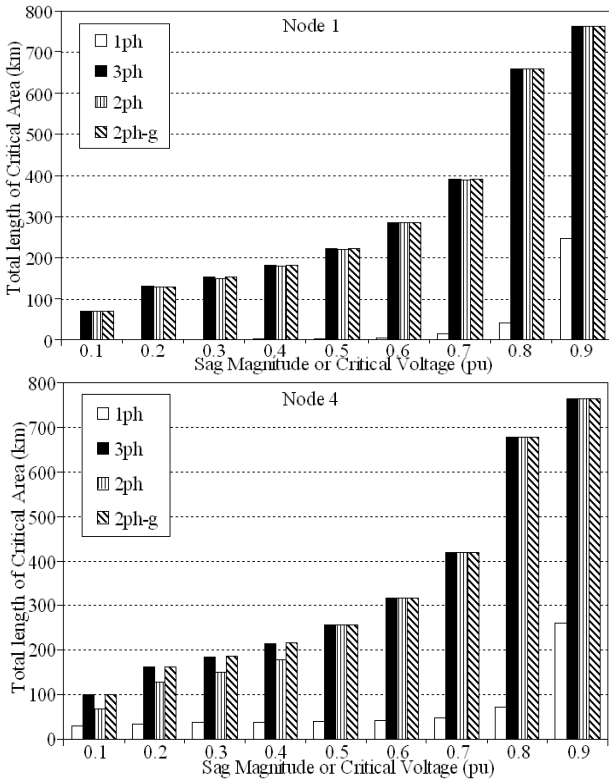Fig. 4 Total length of Critical Area for faults on HV side vs. critical voltage (sag magnitude) for Nodes 1 to 6

Fig. 5 Total length of Critical Area for faults on HV and MV side vs. critical voltage (sag magnitude) for Nodes 1 and 4

VI. CONCLUSION

In this paper, the method of Critical Distances is used to a test power system in order to assess the Critical Area in which a fault will give a certain sag magnitude on a node where an industrial customer may be connected. This method gives high accuracy on the results. The main drawbacks of this method in relation with the Methods of Fault Positions and Monte Carlo include the high calculation effort and the selection of the appropriate root or roots obtained by solving the inversed expression. However, with a proper short-circuit analysis and the analytical expressions given in the current paper the calculation effort can be significantly reduced.

APPENDIX

A. Coefficients n and d

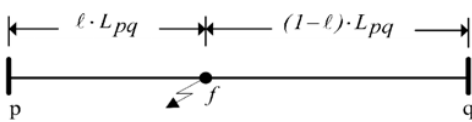The coefficients n and d for all faults at position f of a power line p-q (Fig. 6), are given below:

Fig. 6 Fault position f on line p-q

For 3ph faults:

$$\mathbf{d}_2 = \mathbf{n}_2 = Z_{pp}^{(1)} + Z_{qq}^{(1)} - 2 \cdot Z_{pq}^{(1)} - z_{pq}^{(1)}$$

$$\mathbf{d}_1 = 2 \cdot Z_{pq}^{(1)} - 2 \cdot Z_{pp}^{(1)} + z_{pq}^{(1)} \quad , \quad \mathbf{n}_1 = \mathbf{d}_1 + Z_{pk}^{(1)} - Z_{qk}^{(1)}$$

$$\mathbf{d}_0 = Z_{pp}^{(1)} \quad , \quad \mathbf{n}_0 = \mathbf{d}_0 - Z_{pk}^{(1)}$$

For 1ph faults:

$$\mathbf{d}_2 = 2 \cdot Z_{pp}^{(1)} + 2 \cdot Z_{qq}^{(1)} + Z_{pp}^{(0)} + Z_{qq}^{(0)} - 4 \cdot Z_{pq}^{(1)} - 2 \cdot Z_{pq}^{(0)} - 2 \cdot z_{pq}^{(1)} - z_{pq}^{(0)}$$

$$\mathbf{n}_2^A = \mathbf{d}_2 - (Z_{pk}^{(0)} + Z_{pk}^{(1)} + Z_{pk}^{(2)^*})$$

$$\mathbf{n}_2^B = a^2 \cdot \mathbf{d}_2 - (Z_{pk}^{(0)} + a^2 \cdot Z_{pk}^{(1)} + a \cdot Z_{pk}^{(2)^*})$$

$$\mathbf{d}_1 = 4 \cdot Z_{pq}^{(1)} + 2 \cdot Z_{pq}^{(0)} - 4 \cdot Z_{pp}^{(1)} - 2 \cdot Z_{pp}^{(0)} + 2 \cdot z_{pq}^{(1)} + z_{pq}^{(0)}$$

$$\mathbf{n}_1^A = \mathbf{d}_1 + (Z_{pk}^{(0)} + Z_{pk}^{(1)} + Z_{pk}^{(2)^*}) - (Z_{qk}^{(0)} + Z_{qk}^{(1)} + Z_{qk}^{(2)^*})$$

$$\mathbf{n}_1^B = a^2 \cdot \mathbf{d}_1 + (Z_{pk}^{(0)} + a^2 \cdot Z_{pk}^{(1)} + a \cdot Z_{pk}^{(2)^*}) - (Z_{qk}^{(0)} + a^2 \cdot Z_{qk}^{(1)} + a \cdot Z_{qk}^{(2)^*})$$

$$\mathbf{d}_0 = 2 \cdot Z_{pp}^{(1)} + Z_{pp}^{(0)}$$

$$\mathbf{n}_0^A = \mathbf{d}_0 - (Z_{pk}^{(0)} + Z_{pk}^{(1)} + Z_{pk}^{(2)^*})$$

$$\mathbf{n}_0^B = a^2 \cdot \mathbf{d}_0 - (Z_{pk}^{(0)} + a^2 \cdot Z_{pk}^{(1)} + a \cdot Z_{pk}^{(2)^*})$$

For 2ph faults:

$$\mathbf{d}_2 = 2 \cdot Z_{pp}^{(1)} + 2 \cdot Z_{qq}^{(1)} - 4 \cdot Z_{pq}^{(1)} - 2 \cdot z_{pq}^{(1)}$$

$$\mathbf{n}_2^B = a^2 \cdot \mathbf{d}_2 - a^2 \cdot Z_{pk}^{(1)} + a \cdot Z_{pk}^{(2)^*}$$

$$\mathbf{n}_2^C = a \cdot \mathbf{d}_2 - a \cdot Z_{pk}^{(1)} + a^2 \cdot Z_{pk}^{(2)^*}$$

$$\mathbf{d}_1 = 4 \cdot Z_{pq}^{(1)} - 2 \cdot Z_{pp}^{(1)} + 2 \cdot z_{pq}^{(1)}$$

$$\mathbf{n}_1^B = a^2 \cdot \mathbf{d}_1 + a^2 \cdot Z_{pk}^{(1)} - a \cdot Z_{pk}^{(2)^*} - a^2 \cdot Z_{qk}^{(1)} + a \cdot Z_{qk}^{(2)^*}$$

$$\mathbf{n}_1^C = a \cdot \mathbf{d}_1 + a \cdot Z_{pk}^{(1)} - a^2 \cdot Z_{pk}^{(2)^*} - a \cdot Z_{qk}^{(1)} + a^2 \cdot Z_{qk}^{(2)^*}$$

$$\mathbf{d}_0 = 2 \cdot Z_{pp}^{(1)}$$

$$\mathbf{n}_0^B = a^2 \cdot \mathbf{d}_0 - a^2 \cdot Z_{pk}^{(1)} + a \cdot Z_{pk}^{(2)^*}$$

$$\mathbf{n}_0^C = a \cdot \mathbf{d}_0 - a \cdot Z_{pk}^{(1)} + a^2 \cdot Z_{pk}^{(2)^*}$$

For 2ph –g faults:

$$d_4 = (Z_{pp}^{(1)} + Z_{qq}^{(1)} - Z_{pq}^{(1)} - z_{pq}^{(1)})^2$$

$$+ 2 \cdot (Z_{pp}^{(1)} + Z_{qq}^{(1)} - Z_{pq}^{(1)} - z_{pq}^{(1)}) \cdot (Z_{pp}^{(0)} + Z_{qq}^{(0)} - Z_{pq}^{(0)} - z_{pq}^{(0)})$$

$$n_4^{B} = a^2 \cdot d_4 \quad , \quad n_4^{C} = a \cdot d_4$$

$$d_3 = [2 \cdot (Z_{pp}^{(1)} + Z_{qq}^{(1)} - 2 \cdot Z_{pq}^{(1)} - z_{pq}^{(1)}) + Z_{pp}^{(0)} + Z_{qq}^{(0)} - 2 \cdot Z_{pq}^{(0)}$$

$$- z_{pq}^{(0)}] \cdot (2 \cdot Z_{pq}^{(1)} - 2 \cdot Z_{pp}^{(1)} + z_{pq}^{(1)} + 2 \cdot Z_{pq}^{(0)} - 2 \cdot Z_{pp}^{(0)} + z_{pq}^{(0)})$$

$$n_3^{B} = a^2 \cdot [d_3 - (Z_{qk}^{(1)} - Z_{pk}^{(1)}) \cdot (Z_{pp}^{(1)} + Z_{qq}^{(1)} - 2 \cdot Z_{pq}^{(1)} - 2 \cdot z_{pq}^{(1)}$$

$$+ Z_{pp}^{(0)} + Z_{qq}^{(0)} - 2 \cdot Z_{pq}^{(0)} - z_{pq}^{(0)}) + (Z_{pp}^{(1)} + Z_{qq}^{(1)} - 2 \cdot Z_{pq}^{(1)} - z_{pq}^{(1)}) \cdot (Z_{qk}^{(0)} - Z_{pk}^{(0)})$$

$$+ a \cdot (Z_{pp}^{(0)} + Z_{qq}^{(0)} - 2 \cdot Z_{pq}^{(0)} - z_{pq}^{(0)}) \cdot (Z_{qk}^{(2)*} - Z_{pk}^{(2)*})]$$

$$n_3^{C} = a \cdot [d_3 - (Z_{qk}^{(1)} - Z_{pk}^{(1)}) \cdot (Z_{pp}^{(1)} + Z_{qq}^{(1)} - 2 \cdot Z_{pq}^{(1)} - 2 \cdot z_{pq}^{(1)}$$

$$+ Z_{pp}^{(0)} + Z_{qq}^{(0)} - 2 \cdot Z_{pq}^{(0)} - z_{pq}^{(0)}) + (Z_{pp}^{(1)} + Z_{qq}^{(1)} - 2 \cdot Z_{pq}^{(1)} - z_{pq}^{(1)}) \cdot (Z_{qk}^{(0)} - Z_{pk}^{(0)})$$

$$+ a^2 \cdot (Z_{pp}^{(0)} + Z_{qq}^{(0)} - 2 \cdot Z_{pq}^{(0)} - z_{pq}^{(0)}) \cdot (Z_{qk}^{(2)*} - Z_{pk}^{(2)*})]$$

$$d_2 = 2 \cdot (Z_{pp}^{(1)} + Z_{qq}^{(1)} - 2 \cdot Z_{pq}^{(1)} - z_{pq}^{(1)}) \cdot (Z_{pp}^{(1)} + Z_{pp}^{(0)})$$

$$+ 2 \cdot Z_{pp}^{(1)} \cdot (Z_{pp}^{(0)} + Z_{qq}^{(0)} - 2 \cdot Z_{pq}^{(0)} - z_{pq}^{(0)}) + (2 \cdot Z_{p1}^{(1)} - 2 \cdot Z_{pp}^{(1)}$$

$$+ z_{pq}^{(1)}) \cdot (2 \cdot Z_{pq}^{(1)} - 2 \cdot Z_{pp}^{(1)} - 4 \cdot Z_{pp}^{(0)} + 4 \cdot Z_{pq}^{(0)} + z_{pq}^{(1)} + 2 \cdot z_{pq}^{(0)})$$

$$n_2^{B} = a^2 \{d_2 - [Z_{pk}^{(1)} \cdot (Z_{pp}^{(1)} + Z_{qq}^{(1)} - 2 \cdot Z_{pq}^{(1)} - z_{pq}^{(1)} + Z_{pp}^{(0)} + Z_{qq}^{(0)} - 2 \cdot Z_{pq}^{(0)} - z_{pq}^{(0)})$$

$$+ (Z_{qk}^{(1)} - Z_{pk}^{(1)}) \cdot (2 \cdot Z_{pq}^{(1)} - 2 \cdot Z_{pp}^{(1)} + z_{pq}^{(1)} + 2 \cdot Z_{pq}^{(0)} - 2 \cdot Z_{pp}^{(0)} + z_{pq}^{(0)})\} +$$

$$Z_{pk}^{(0)} \cdot (Z_{pp}^{(1)} + Z_{qq}^{(1)} - 2 \cdot Z_{pq}^{(1)} - z_{pq}^{(1)}) + (Z_{qk}^{(0)} - Z_{pk}^{(0)}) \cdot (2 \cdot Z_{pq}^{(1)} - 2 \cdot Z_{pp}^{(1)} + z_{pq}^{(1)})$$

$$+ a[Z_{pk}^{(2)*} \cdot (Z_{pp}^{(0)} + Z_{qq}^{(0)} - 2 \cdot Z_{pq}^{(0)} - z_{pq}^{(0)}) + (Z_{qk}^{(2)*} - Z_{pk}^{(2)*}) \cdot (2 \cdot Z_{pq}^{(0)} - 2 \cdot Z_{pp}^{(0)} + z_{pq}^{(0)})]$$

$$n_2^{C} = a\{d_2 - [Z_{pk}^{(1)} \cdot (Z_{pp}^{(1)} + Z_{qq}^{(1)} - 2 \cdot Z_{pq}^{(1)} - z_{pq}^{(1)} + Z_{pp}^{(0)} + Z_{qq}^{(0)} - 2 \cdot Z_{pq}^{(0)} - z_{pq}^{(0)})$$

$$+ (Z_{qk}^{(1)} - Z_{pk}^{(1)}) \cdot (2 \cdot Z_{pq}^{(1)} - 2 \cdot Z_{pp}^{(1)} + z_{pq}^{(1)} + 2 \cdot Z_{pq}^{(0)} - 2 \cdot Z_{pp}^{(0)} + z_{pq}^{(0)})\} +$$

$$Z_{pk}^{(0)} \cdot (Z_{pp}^{(1)} + Z_{qq}^{(1)} - 2 \cdot Z_{pq}^{(1)} - z_{pq}^{(1)}) + (Z_{qk}^{(0)} - Z_{pk}^{(0)}) \cdot (2 \cdot Z_{pq}^{(1)} - 2 \cdot Z_{pp}^{(1)} + z_{pq}^{(1)})$$

$$+ a^2 [Z_{pk}^{(2)*} \cdot (Z_{pp}^{(0)} + Z_{qq}^{(0)} - 2 \cdot Z_{pq}^{(0)} - z_{pq}^{(0)}) + (Z_{qk}^{(2)*} - Z_{pk}^{(2)*}) \cdot (2 \cdot Z_{pq}^{(0)} - 2 \cdot Z_{pp}^{(0)} + z_{pq}^{(0)})]$$

$$d_1 = 2 \cdot (2 \cdot Z_{pq}^{(1)} - 2 \cdot Z_{pp}^{(1)} + z_{pq}^{(1)}) \cdot (Z_{pp}^{(1)} + Z_{pp}^{(0)})$$

$$+ 2 \cdot Z_{pp}^{(1)} \cdot (2 \cdot Z_{pq}^{(0)} - 2 \cdot Z_{pp}^{(0)} + z_{pq}^{(0)})$$

$$n_1^{B} = a^2 \cdot \{d_1 - [(Z_{qk}^{(1)} - Z_{pk}^{(1)}) \cdot (Z_{pp}^{(1)} + Z_{pp}^{(0)}) + Z_{pk}^{(1)} \cdot (2 \cdot Z_{pq}^{(1)} - 2 \cdot Z_{pp}^{(1)} + z_{pq}^{(1)}$$

$$+ 2 \cdot Z_{pq}^{(0)} - 2 \cdot Z_{pp}^{(0)} + z_{pq}^{(0)})]\} + \{Z_{pp}^{(1)} \cdot (Z_{qk}^{(0)} - Z_{pk}^{(0)}) + Z_{pk}^{(0)} \cdot (2 \cdot Z_{pq}^{(0)} - 2 \cdot Z_{pp}^{(0)} + z_{pq}^{(0)})$$

$$+ a \cdot [Z_{pp}^{(0)} \cdot (Z_{qk}^{(2)*} - Z_{pk}^{(2)*}) + Z_{pk}^{(2)*} \cdot (2 \cdot Z_{pq}^{(0)} - 2 \cdot Z_{pp}^{(0)} + z_{pq}^{(0)})]\}$$

$$n_1^{C} = a \cdot \{d_1 - [(Z_{qk}^{(1)} - Z_{pk}^{(1)}) \cdot (Z_{pp}^{(1)} + Z_{pp}^{(0)}) + Z_{pk}^{(1)} \cdot (2 \cdot Z_{pq}^{(1)} - 2 \cdot Z_{pp}^{(1)} + z_{pq}^{(1)}$$

$$+ 2 \cdot Z_{pq}^{(0)} - 2 \cdot Z_{pp}^{(0)} + z_{pq}^{(0)})]\} + \{Z_{pp}^{(1)} \cdot (Z_{qk}^{(0)} - Z_{pk}^{(0)}) + Z_{pk}^{(0)} \cdot (2 \cdot Z_{pq}^{(0)} - 2 \cdot Z_{pp}^{(0)} + z_{pq}^{(0)})$$

$$+ a^2 \cdot [Z_{pp}^{(0)} \cdot (Z_{qk}^{(2)*} - Z_{pk}^{(2)*}) + Z_{pk}^{(2)*} \cdot (2 \cdot Z_{pq}^{(0)} - 2 \cdot Z_{pp}^{(0)} + z_{pq}^{(0)})]\}$$

$$d_0 = Z_{pp}^{(1)} \cdot (Z_{pp}^{(1)} + 2 \cdot Z_{pp}^{(0)})$$

$$n_0^{B} = a^2 \cdot [d_1 - (Z_{pp}^{(1)} + Z_{pp}^{(0)}) \cdot Z_{pk}^{(1)}] + Z_{pp}^{(1)} \cdot Z_{pk}^{(0)} + a \cdot Z_{pk}^{(2)*} \cdot Z_{pp}^{(0)}$$

$$n_0^{C} = a \cdot [d_1 - (Z_{pp}^{(1)} + Z_{pp}^{(0)}) \cdot Z_{pk}^{(1)}] + Z_{pp}^{(1)} \cdot Z_{pk}^{(0)} + a^2 \cdot Z_{pk}^{(2)*} \cdot Z_{pp}^{(0)}$$

where (*) means that those properties should be multiplied by $(-\alpha^2)$ for faults at HV side and sag is observed at MV side

B. Coefficients s

The coefficients *s* of are real numbers and for 3ph, 1ph and 2ph faults given below:

$$s_4 = V_{crit}^2 \cdot [\text{ Re}\{d_2\}^2 + \text{Im}\{d_2\}^2] - [\text{Re}\{n_2\}^2 + \text{Im}\{n_2\}^2]$$

$$s_3 = V_{crit}^2 \cdot [2 \cdot (Re\{d_2\} \cdot Re\{d_1\} + Im\{d_2\} \cdot Im\{d_1\})]$$

$$- [2 \cdot (Re\{n_2\} \cdot Re\{n_1\} + Im\{n_2\} \cdot Im\{n_1\})]$$

$$s_2 = V_{crit}^2 \cdot [Re\{d_1\}^2 + Im\{d_1\}^2 + 2(Re\{d_2\} \cdot Re\{d_0\} + Im\{d_0\} \cdot Im\{d_0\})]$$

$$- [Re\{n_1\}^2 + Im\{n_1\}^2 + 2(Re\{n_2\} \cdot Re\{n_0\} + Im\{n_0\} \cdot Im\{n_0\})]$$

$$s_1 = V_{crit}^2 \cdot [Re\{d_2\}^2 + Im\{d_2\}^2] - [Re\{n_2\}^2 + Im\{n_2\}^2]$$

$$s_0 = V_{crit}^2 \cdot [Re\{d_0\}^2 + Im\{d_0\}^2] - [Re\{n_0\}^2 + Im\{n_0\}^2]$$

For 2ph-g faults:

$$s_8 = V_{crit}^2 \cdot [Re\{d_4\}^2 + Im\{d_4\}^2] - [Re\{n_4\}^2 + Im\{n_4\}^2]$$

$$s_7 = V_{crit}^2 \cdot [2 \cdot (Re\{d_4\} \cdot Re\{d_3\} + Im\{d_4\} \cdot Im\{d_3\})]$$

$$- [2 \cdot (Re\{n_4\} \cdot Re\{n_3\} + Im\{n_4\} \cdot Im\{n_3\})]$$

$$s_6 = V_{crit}^2 \cdot [Re\{d_3\}^2 + Im\{d_3\}^2 + 2(Re\{d_4\} \cdot Re\{d_2\} + Im\{d_4\} \cdot Im\{d_2\})]$$

$$- [Re\{n_3\}^2 + Im\{n_3\}^2 + 2(Re\{n_4\} \cdot Re\{n_2\} + Im\{n_4\} \cdot Im\{n_2\})]$$

$$s_5 = V_{crit}^2 \cdot [2 \cdot (Re\{d_3\} \cdot Re\{d_2\} + Re\{d_4\} \cdot Re\{d_1\}$$

$$+ Im\{d_3\} \cdot Im\{d_2\} + Im\{d_4\} \cdot Im\{d_1\})] - [2(Re\{n_3\} \cdot Re\{n_2\} + Re\{n_4\} \cdot Re\{n_1\}$$

$$+ Im\{n_3\} \cdot Im\{n_2\} + Im\{n_4\} \cdot Im\{n_1\})]$$

$$s_4 = V_{crit}^2 \cdot [Re\{d_2\}^2 + Im\{d_2\}^2 + 2 \cdot (Re\{d_3\} \cdot Re\{d_1\} + Re\{d_4\} \cdot Re\{d_0\}$$

$$+ Im\{d_3\} \cdot Im\{d_1\} + Im\{d_4\} \cdot Im\{d_0\})] - [Re\{n_2\}^2 + Im\{n_2\}^2$$

$$+ 2 \cdot (Re\{n_3\} \cdot Re\{n_1\} + Re\{n_4\} \cdot Re\{n_0\} + Im\{n_3\} \cdot Im\{n_1\} + Im\{n_4\} \cdot Im\{n_0\}))]$$

$$s_3 = V_{crit}^2 \cdot [2 \cdot (Re\{d_3\} \cdot Re\{d_0\} + Re\{d_2\} \cdot Re\{d_1\}$$

$$+ Im\{d_3\} \cdot Im\{d_0\} + Im\{d_2\} \cdot Im\{d_1\})] - [2(Re\{n_3\} \cdot Re\{n_0\} + Re\{n_2\} \cdot Re\{n_1\}$$

$$+ Im\{n_3\} \cdot Im\{n_0\} + Im\{n_2\} \cdot Im\{n_1\})]$$

$$\mathbf{s}_2 = V_{crit}^2 \cdot [Re\{\mathbf{d}_1\}^2 + Im\{\mathbf{d}_1\}^2 + 2(Re\{\mathbf{d}_2\} \cdot Re\{\mathbf{d}_0\}$$
$$+ Im\{\mathbf{d}_2\} \cdot Im\{\mathbf{d}_0\})] - [Re\{\mathbf{n}_2\}^2 + Im\{\mathbf{n}_2\}^2$$
$$+ 2(Re\{\mathbf{n}_2\} \cdot Re\{\mathbf{n}_0\} + Im\{\mathbf{n}_2\} \cdot Im\{\mathbf{n}_0\})]$$

$$\mathbf{s}_1 = V_{crit}^2 \cdot [2 \cdot (Re\{\mathbf{d}_1\} \cdot Re\{\mathbf{d}_0\} + Im\{\mathbf{d}_1\} \cdot Im\{\mathbf{d}_0\})]$$
$$- [2 \cdot (Re\{\mathbf{n}_1\} \cdot Re\{\mathbf{n}_0\} + Im\{\mathbf{n}_1\} \cdot Im\{\mathbf{n}_0\})]$$

$$\mathbf{s}_0 = V_{crit}^2 \cdot [Re\{\mathbf{d}_0\}^2 + Im\{\mathbf{d}_0\}^2] - [Re\{\mathbf{n}_0\}^2 + Im\{\mathbf{n}_0\}^2]$$

where *Re{}* and *Im{}* are the real and imaginary part of coefficients *n* and *d* .

REFERENCES

[1] M. Moschakis, S. Loutridis, V. Dafopoulos, A. Anastasiadis, T. Tomtsi, E. Karapidakis, A. Tsikalakis, "Prediction of Voltage Sags Applying the Method of Critical Distances to Meshed Power Networks", in *Proc. IEEE PMAPS (Probabilistic Methods Applied to Power Systems) Conference*, Istanbul, Turkey, June 2012, pp. 570-575.

[2] M. Bollen, "Understanding Power Quality Problems: Voltage Sags and Interruptions", IEEE Press Series on Power Engineering, P.M. Anderson Series Editor, 2000.

[3] M. N. Moschakis, I. G. Andritsos, V. V. Dafopoulos, J. M. Prousalidis, E. S. Karapidakis, "An Evaluation of Sag Detection Techniques for Fast Solid-State Electronic Transferring to Alternate Electrical Energy Sources", International Journal of Electrical Science and Engineering, International Science Index 80, Vol:7, No:8, 2013.

[4] L. E. Conrad, "Proposed Chapter 9 for Predicting Voltage Sags (Dips) in revision to IEEE Std 493, the Gold Book", *IEEE Trans. Ind. Applicat.*, vol. 30, no. 3, pp. 805-821, May/June 1994.

[5] M. Moschakis, E. Leonidaki, N. Hatziargyriou, "Considerations for the Application of Thyristor Controlled Series Capacitors to Radial Power Distribution Circuits", IEEE PowerTech Conference Proceedings, Bologna, Italy, 2003.

[6] M. N. Moschakis, V. V. Dafopoulos, I. G. Andritsos, E. S. Karapidakis, J. M. Prousalidis, "The Effect of Transformer's Vector Group on Retained Voltage Magnitude and Sag Frequency at Industrial Sites due to Faults", International Journal of Electrical Science and Engineering, International Science Index 79, Vol:7, No:7, 2013.

[7] J. Grainger, W. Stevenson, Power System Analysis, McGraw-Hill, 1994.

[8] M. N. Moschakis, N. D. Hatziargyriou, "Analytical Calculation and Stochastic Assessment of Voltage Sags", *IEEE Transactions on Power Delivery*, vol. 21, no. 3, pp. 1727-1734, July 2006.

[9] M. N. Moschakis, A. Kladas, N. Hatziargyriou, "A Voltage Source Converter Model for Exchanging Active and Reactive Power with a Distribution Network", ELSEVIER -Journal of Materials Processing Technology, vol. 161, pp. 128-135, 2005.

[10] G. Olguin, M. Bollen, "Stochastic Assessment of Unbalanced Voltage Dips in Large Transmission Systems", *IEEE PowerTech Conference*, June 23-26, Bologna, Italy, 2003.

[11] M. N. Moschakis, E. L. Karfopoulos , E. I. Zountouridou , S. A. Papathanassiou , "On Adaptation of Electric Vehicle and Microgrid Issues to EMC-Power Quality Standards", Electrical and Electronic Engineering Journal, Vol. 2, No. 5, pp. 249-257, 2012.

[12] M. N. Moschakis, E. L. Karfopoulos, E. I. Zountouridou, and S. A. Papathanassiou, "Adapting EV-Microgrid Concepts to European Grid Standards Related to Power Quality", 16th IEEE International Conf. on Intelligent System Applications to Power Systems, Crete, Greece, 2011.

[13] PSCAD/EMTDC Power Systems Simulation Software, Manitoba HVDC Research Center., Canada, 1997.

[14] European Standard EN 50160, "Voltage characteristics of electricity supplied by public electricity distribution networks", November 1994.

[15] IEEE Std 1159, "Recommended Practice for Monitoring Electric Power Quality", 1995.