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Deriving Generic Transformation Matrices for Multi-Axis Milling Machine

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***Abstract*—**This paper proposes a new method to find the equations of transformation matrix for the rotation angles of the two rotational axes and the coordinates of the three linear axes of an orthogonal multi-axis milling machine. This approach provides intuitive physical meanings for rotation angles of multi-axis machines, which can be used to evaluate the accuracy of the conversion from CL data to NC data.

***Keywords*—**CAM, multi-axis milling machining.

I. Introduction

A multi-axis milling machine is equipped with three linear axes and two rotational axes. Generating NC data from multi-axis tool-paths is an essential research topic in the field of post-processing for multi-axis milling machines. Tool-paths stored in CL data describe the locations of the cutting tool (assuming the workpiece is fixed) and NC data is used to drive the motion of the five axes. A major difference between CL data and NC data is that CL data is referenced to the workpiece coordinate system (WCS), whereas NC data uses the machining coordinate system (MCS) as the reference. In addition, CL data for multi-axis machining combines information from both the tool-tip coordinates and the tool-orientation vectors in the WCS. For instance, for the CL data (100, 70, 50, 0.2673, 0.5345, 0.8018), (100, 70, 50) is the tool-tip coordinate and (0.2673, 0.5345, 0.8018) is the tool-orientation vector. In contrast to this, NC data for multi-axis machining uses coordinates of points and rotational angles described in the MCS. For example, in the NC data "X58.138 Y56.176 Z104.232 A–36.699 C–26.561", "X58.138 Y56.176 Z104.232" represents the coordinates of the three linear axes, and "A–36.699 C–26.561" represents the coordinates of the two rotational axes.

Many studies use the concept of "kinematic linkage" to discuss the equations for multi-axis post-processing. For instance, Takeuchi and Watanabe [1] introduced two types of coordinate transformation equations for orthogonal vertical/horizontal multi-axis milling machines. Rüegg and Gygax [2] developed a kinematic model using the linkage method to investigate the coordinate transformation equations for spindle-tilting type multi-axis milling machines with orthogonal rotation axes. Sakamoto and Inasaki [3] developed a kinematic model for multi-axis milling machines based on kinematic chains, and further derived equations to analyze the structure of multi-axis milling machines. Lee and She [4] classified orthogonal multi-axis milling machines into 3 groups, and used homogeneous coordinate transformations and inverse kinematics to convert CL data into NC data. Mahbubur et al. [5] used linkage pairs and the Denavit-Hertenberg method to discuss the equations for post-processing. Jung et al. [6] used the phase-reverse method to avoid interference when a multi-axis milling machine reaches its rotation limits, which also reduced the upward movement of tool motions, resulting in optimal NC tool-paths. She and Chang [7] discussed a post-process transformation with a nutating-head configuration. She and Huang [8] investigated a post-processor coordinate transformation for hybrid multi-axis milling machines where both the spindle and workpiece can rotate. Rahman et al. [9] used a coordinate transformation to convert CL data into NC data. Zheng et al. [10] and Andres et al. [11] applied inverse kinematics to derive equations for coordinate transformation. Others, such as Nagata et al. [12], employed a "robotics" approach to deduce post-processing equations for multi-axis machining. She and Lee [13] and Ashmall [14] proposed coordinate transformation equations of a post-processor for orthogonal multi-axis milling machines. She and Chang [15] proposed a kinematic model to represent the structure of the two rotational axes in both the spindle and the workpiece, so as to derive the post-processor's transformation equations for orthogonal-type multi-axis milling machines.

The aforementioned research works were all carried out for specific multi-axis milling machines, and all used complex methods to derive equations for multi-axis post-processing. In real practice, there are multiple types of multi-axis milling machines, and a generic and simple method which can be applied to any type of multi-axis milling machine is still absent among the above research. This paper thus proposes a spherical two-circle (STC) method to solve this problem, and attempts to develop a generic model for coordinate transformations in multi-axis post-processing. Unlike conventional approaches, the proposed STC method disregards linear movements, instead focusing on the motion of the spindle vector (Vec_s) or the tool-orientation vector (Vec_t) about two rotational axes, and generating two circular moving trajectories, namely the primary circle (C_u) and the secondary circle (C_v), as shown in Fig. 1 (note: the definition of relevant terms and symbols will be covered in subsequent sections.) Vectors Vec_s or Vec_t are set to rotate about these two circles so that the rotation angles of the two axes can be derived easily. The derived rotation angles will lead to a solution for finding the coordinates of points in NC

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data as well as the development of post-processing equations for coordinate transformation.

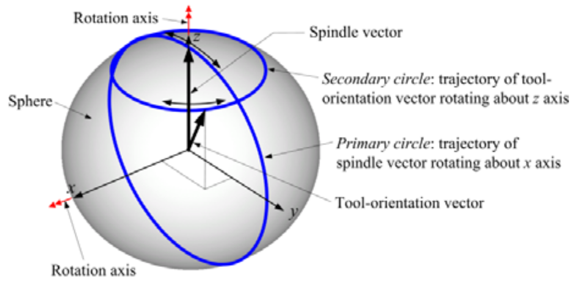


Fig. 1 Illustration of the spherical two-circle method

II. Classification of Multi-Axis Milling Machines

For ease of discussion, multi-axis milling machines are classified into different types based on the following criterion: whether the rotation axis drives the spindle or workpiece. If both rotational axes drive the spindle, the milling machine is considered as 'spindle-tilting type', as shown in Fig. 2. If both rotation axes drive the workpiece, the machine is considered as 'table-tilting type', as shown in Fig. 3. Finally, if one axis drives the spindle and the other drives the workpiece, it is considered as 'hybrid type', as displayed in Fig. 4. In the following sections, the letter *S* represents the rotation axis which drives the spindle, and the letter *T* represents the rotation axis which drives the workpiece. Also, the word 'tilting' is omitted and thus the terms 'spindle-type' and 'table-type' are used instead.

For spindle- or table-type machines, if the movement of one rotation axis brings about a position change of the other rotation axis, this rotation axis is named the primary rotation axis (PRA); otherwise, it is named the secondary rotation axis (SRA). Fig. 5 illustrates the mechanism of PRA and SRA for a spindle-type machine. In the figure, the letter *U* represents a unit vector of PRA, and *V* represents a unit vector of SRA. Both spindle-type and table-type machines include primary and secondary rotation axes. For hybrid-type machines, the axis driving the spindle is called PRA, and the other is called SRA.

For a spindle-type machine, if PRA is parallel to the *z*-axis and SRA is parallel to the *y*-axis, then the machine is of type *SCSB*, as shown in Fig. 2. For a table-type machine, if PRA is parallel to the *x*-axis and SRA is parallel to the *z*-axis, then the machine is of type *TATC*, as shown in Fig. 3. For a hybrid-type machine, if the spindle (or PRA) rotates about the *y*-axis and the workpiece (or SRA) rotates about the *z*-axis, then the machine is of type *SBTC*, as shown in Fig. 4.

III. Generic STC Model

By applying the STC model shown in Fig. 1 to the three types of multi-axis machines, the "generic STC model", as shown in Fig. 6, can be derived. In this generic model, the spindle endpoint P_s rotates about the *x*-axis and *y*-axis separately, and generates two circles C_{xs} and C_{ys}. The tool endpoint P_t rotates about the *x*-axis, *y*-axis, and *z*-axis separately, and generates three circles C_{xt}, C_{yt}, and C_{zt}. The idea of the generic STC model is to translate the two rotation axes, the spindle vector, and the tool-orientation vector to the origin point of the MCS; then, the primary and secondary circles are determined according to the machine type, the transit points are found, and, finally, the motion trajectory is found and the rotation angles are calculated.

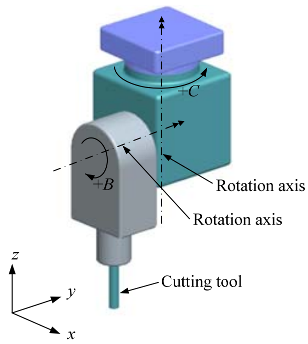


Fig. 2 Rotation mechanism of a spindle-tilting machine

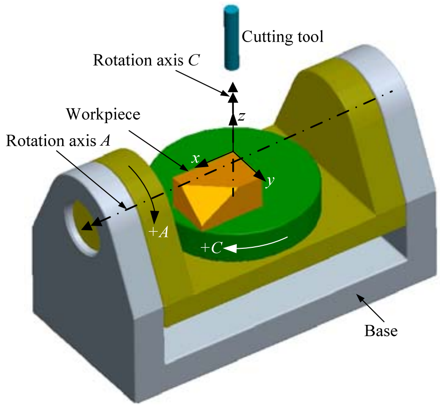


Fig. 3 Rotation mechanism of a table-tilting machine

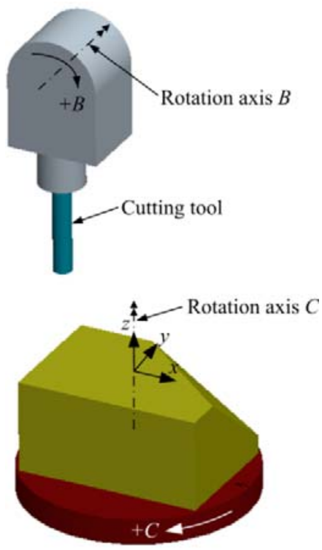
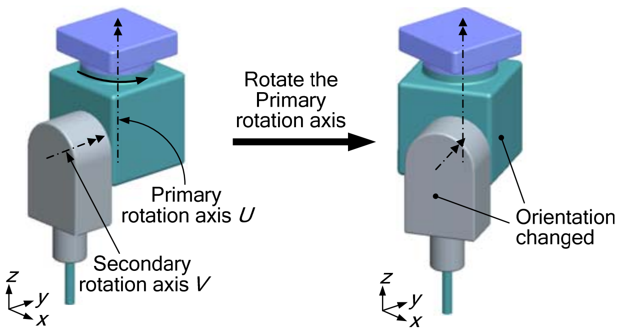
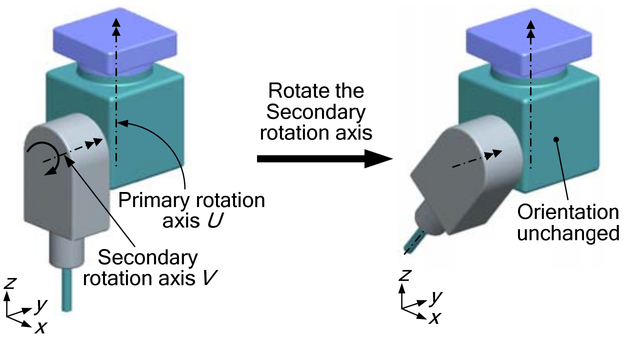


Fig. 4 Rotation mechanism of a hybrid-type machine



(a)



(b)

Fig. 5 Rotation axes of a spindle-type multi-axis milling machine: (*a*) orientation of SRA is changed after PRA is rotated, (*b*) orientation of PRA remains unchanged after SRA is rotated

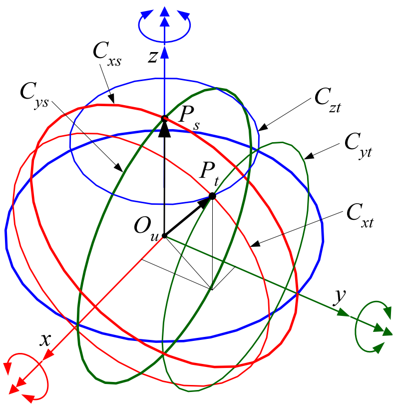


Fig. 6 Generic STC model

By applying the generic STC model to the three types of multi-axis milling machines, equations for rotation angles and point coordinates can be derived as follows:

(1) Spindle-tilting machine, for example *SCSB*

Primary rotation angles: $C = \text{atan2}(\frac{i}{\sin B}, \frac{j}{\sin B})$

Secondary rotation angles: $B = \pm\cos^{-1} k$

$$\begin{bmatrix} X \\ Y \\ Z \\ 0 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 & d_{t\to 2x}+d_{2\to 1x} \\ 0 & 1 & 0 & d_{t\to 2y}+d_{2\to 1y} \\ 0 & 0 & 1 & d_{t\to 2z}+d_{2\to 1z} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos C & -\sin C & 0 & 0 \\ \sin C & \cos C & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
$$\begin{bmatrix} 1 & 0 & 0 & -d_{t\to 2x}-d_{2\to 1x} \\ 0 & 1 & 0 & -d_{t\to 2y}-d_{2\to 1y} \\ 0 & 0 & 1 & -d_{t\to 2z}-d_{2\to 1z} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & d_{t\to 2x} \\ 0 & 1 & 0 & d_{t\to 2y} \\ 0 & 0 & 1 & d_{t\to 2z} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
$$\begin{bmatrix} \cos B & 1 & \sin B & 0 \\ 0 & 1 & 0 & 0 \\ -\sin B & 0 & \cos B & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -d_{t\to 2x} \\ 0 & 1 & 0 & -d_{t\to 2y} \\ 0 & 0 & 1 & -d_{t\to 2z} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{2}$$

(2) Table-tilting machine, for example *TATC*

Primary rotation angles: $A = \pm\cos^{-1} k$

Secondary rotation angles: $C = \text{atan2}(\frac{-j}{\sin A}, \frac{i}{\sin A})$

$$\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & d_{O\to Cx}+d_{C\to Ax} \\ 0 & 1 & 0 & d_{O\to Cy}+d_{C\to Ay} \\ 0 & 0 & 1 & d_{O\to Cz}+d_{C\to Az} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & \cos(-A) & -\sin(-A) & 0 \\ 0 & \sin(-A) & \cos(-A) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
$$\begin{bmatrix} 1 & 0 & 0 & -d_{C\to Ax} \\ 0 & 1 & 0 & -d_{C\to Ay} \\ 0 & 0 & 1 & -d_{C\to Az} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(-C) & -\sin(-C) & 0 & 0 \\ \sin(-C) & \cos(-C) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
$$\begin{bmatrix} 1 & 0 & 0 & -d_{O\to Cx} \\ 0 & 1 & 0 & -d_{O\to Cy} \\ 0 & 0 & 1 & -d_{O\to Cz} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \tag{3}$$

(3) Hybrid-type machine, for example *SATC*

Primary rotation angles: $A = \pm\cos^{-1}(k)$

Secondary rotation angles: $C = \text{atan2}(\frac{-j}{\sin A}, \frac{i}{\sin A})$

$$\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & d_{O\to Cx} \\ 0 & 1 & 0 & d_{O\to Cy} \\ 0 & 0 & 1 & d_{O\to Cz} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(-C) & -\sin(-C) & 0 & 0 \\ \sin(-C) & \cos(-C) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
$$\begin{bmatrix} 1 & 0 & 0 & -d_{O\to Cx} \\ 0 & 1 & 0 & -d_{O\to Cy} \\ 0 & 0 & 1 & -d_{O\to Cz} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 & d_{O\to Ax} \\ 0 & 1 & 0 & d_{O\to Ay} \\ 0 & 0 & 1 & d_{O\to Az} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & \cos A & -\sin A & 0 \\ 0 & \sin A & \cos A & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -d_{O\to Ax} \\ 0 & 1 & 0 & -d_{O\to Ay} \\ 0 & 0 & 1 & -d_{O\to Az} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

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IV. System Implementation

In addition to proposing the STC method, this research developed computer software aimed at implementing post-processing functions for a multi-axis milling machine based on the generic STC model and relevant equations described in the above section. The software converts CL data into NC data for practical multi-axis machining, as illustrated in Fig. 7. Vericut, a well-known software package for NC cutter-path simulation, is used to verify the feasibility of the output NC data before actual machining.

Fig. 8 shows one of the example parts implemented in this research. The letters "ABC" were engraved onto a sculptured surface using NC data generated by the developed software and it required that the cutting tool remain normal to the sculpted surface during machining. A multi-axis machine of type *TATC* was used for milling. Fig. 9 (a) shows the initial setup and Figs. 9 (b)~(d) depict the geometry of the in-process workpiece and the orientations of the cutting tool and workpiece table. The geometry of the final machined part (as displayed in Fig. 10) fulfills the requirement, so the generated NC data is shown to be correct.

Partial CL data

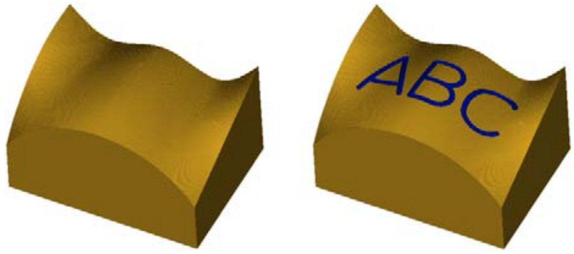
x	*y*	*z*	*i*	*j*	*k*
-25.9774	-8.70539	31.3803	-0.14407	-0.11945	0.982331
-25.257	-8.10815	26.46865	-0.14407	-0.11945	0.982331
-24.6013	-6.30422	26.72031	-0.10365	-0.0668	0.992368
-23.9332	-4.48585	26.85433	-0.07119	-0.01707	0.997317
-23.8192	-4.18156	26.85928	-0.06811	-0.00929	0.997459
-23.7053	-3.87726	26.86423	-0.06504	-0.00152	0.997601
-23.5913	-3.57297	26.86917	-0.06196	0.00626	0.997744
-23.4774	-3.26868	26.87412	-0.05889	0.014035	0.997886
-23.3634	-2.96438	26.87907	-0.05581	0.02181	0.998028
-23.2495	-2.66009	26.88401	-0.05274	0.029585	0.99817
…					

Processed by
TATC equation

Partial NC data

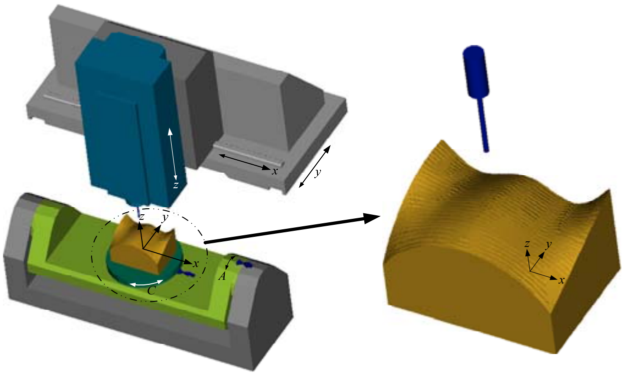
X	Y	Z	A	C
9.878355	19.23007	35.60836	-10.7865	129.6613
9.878355	19.23007	30.60836	-10.7865	129.6613
8.027413	20.61532	29.48744	-7.0832	122.7996
1.21707	22.28844	28.56256	-4.19801	103.4809
-0.92412	22.19097	28.51262	-4.08523	97.76706
-3.32405	21.87253	28.4465	-3.96925	91.33471
-5.92606	21.25657	28.36036	-3.84979	84.23142
-8.62244	20.28663	28.25236	-3.7265	76.59502
-11.2643	18.95409	28.12434	-3.599	68.65665
-13.6946	17.31518	27.98228	-3.46682	60.7094
…				

Fig. 7 Part of CL data and NC data

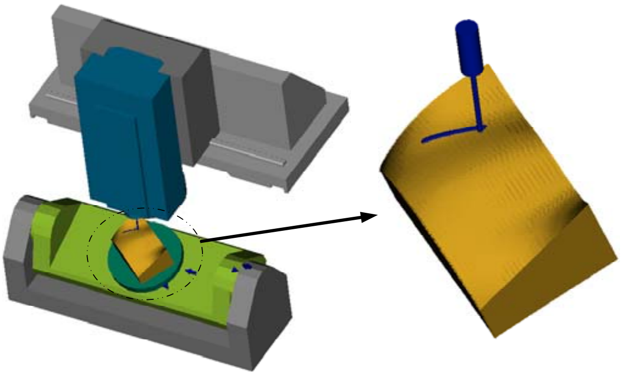


(a) (b)

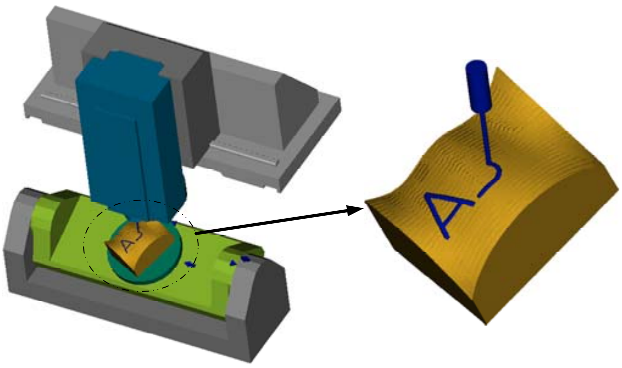
Fig. 8 One of the implemented examples: (a) stock, and (b) finished workpiece



(a)



(b)



(c)

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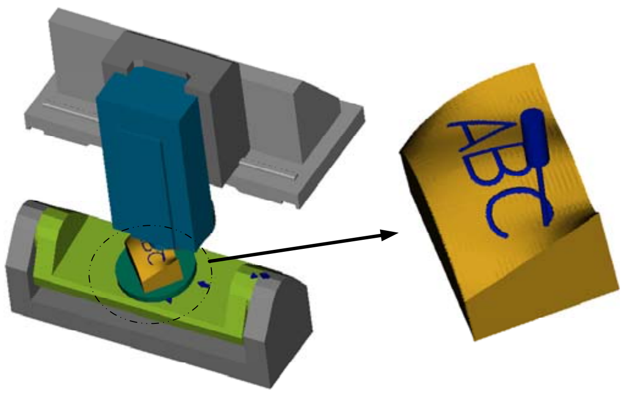

(d)

Fig. 9 (a) Initial setup, (b) & (c) in-process workpiece, (d) finished workpiece

Fig. 10 The final machined part

V. Conclusions

This research proposes an STC method to determine rotation angles of NC data for all types of orthogonal multi-axis milling machines. This method focuses on generating a primary circle and a secondary circle based on the rotation mechanism of multi-axis machines. Transit points are then defined as the intersection points of the two circles. By moving the endpoints of spindle or cutting tool to the transit points along the secondary and primary circles, the two rotation angles can be easily obtained. This STC method allows a quick and intuitive examination of whether or not the NC data is correct. It is practical, generic, easy to use, and can be successfully applied to every type of orthogonal multi-axis milling machine. In this paper, a generic STC model was developed and equations for 3 types of multi-axis machines were derived based on the generic model.

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