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Sliding Mode Position Control for Permanent Magnet Synchronous Motors Based On Passivity Approach

Jenn-Yih Chen, Bean-Yin Lee, Yuan-Chuan Hsu, Jui-Cheng Lin, Kuang-Chyi Lee

Abstract**—**In this paper, a sliding mode control method based on the passivity approach is proposed to control the position of surface-mounted permanent magnet synchronous motors (PMSMs). Firstly, the dynamics of a PMSM was proved to be strictly passive. The position controller with an adaptive law was used to estimate the load torque to eliminate the chattering effects associated with the conventional sliding mode controller. The stability analysis of the overall position control system was carried out by adopting the passivity theorem instead of Lyapunov-type arguments. Finally, experimental results were provided to show that the good position tracking can be obtained, and exhibit robustness in the variations of the motor parameters and load torque disturbances.

Keywords**—**Adaptive law, passivity theorem, permanent magnet synchronous motor, sliding mode control.

I. Introduction

PERMANENT magnet synchronous motors have low inertia, high efficiency, high power density, and fast response properties, which are widely applied to industrial applications [1]. Employing the field-oriented control technique [2], the dynamical model of a PMSM is rather similar to that of a DC motor, and the control effort is reduced. However, the control performance is still affected by the load torque disturbance or parameter uncertainty.

The passivity theorem is an alternative scheme for the stability analysis of feedback systems [3]. Recently, some sliding mode control approaches for induction machines or PMSMs are presented to show the feasibility by means of simulation or experimental results [4]-[6]. The sliding mode control is one of the effective methods to overcome the variations of system parameters and external disturbances [7]. However, the main drawback of this kind of discontinuous switching control law is the undesirable chattering effects. Some research results have been proposed to remove or alleviate the chattering [8], [9]. The values of the moment of inertia and the viscous friction coefficient of PMSMs are generally small. Thus, the load torque disturbance is a critical term to yield the chattering of the control input. In this paper, we utilized the passivity theorem to design an adaptive load torque estimator to eliminate the undesirable chattering. The position control system is formally verified to be asymptotically stable. Experimental results show the effectiveness and the capability for position tracking of the presented scheme. Moreover, the estimated parameters are bounded and converge to the actual values.

Jenn-Yih Chen, Bean-Yin Lee, and Yuan-Chuan Hsu are with the Department of Mechanical and Computer-Aided Engineering, National Formosa University (e-mail: jychen@nfu.edu.tw, leebyin@nfu.edu.tw, jasonych@nfu.edu.tw).

Jui-Cheng Lin is with the Department of Mechanical Design Engineering, National Formosa University (e-mail: linrc@nfu.edu.tw).

Kuang-Chyi Lee is with the Department of Automation Engineering, National Formosa University (e-mail: kclee@nfu.edu.tw).

II. Mathematic Model and Passivity for Permanent Magnet Synchronous Motors

The dynamic equations of a PMSM in the rotor reference frame can be described as follows [10]:

$$\dot{i}_{ds} = \frac{1}{L_{ds}}\left(V_{ds} - R_s i_{ds} + n_p \omega_m L_{qs} i_{qs}\right) \tag{1}$$

$$\dot{i}_{qs} = \frac{1}{L_{qs}}\left(V_{qs} - R_s i_{qs} - n_p \omega_m L_{ds} i_{ds} - n_p \omega_m \lambda_f\right) \tag{2}$$

$$\dot{\omega}_m = \frac{1}{J}\left(\frac{3}{2} n_p \left(\lambda_f + \left(L_{ds} - L_{qs}\right) i_{ds}\right) i_{qs} - T_L - B\omega_m\right) \tag{3}$$

$$\dot{\theta}_m = \omega_m \tag{4}$$

where

$$\lambda_{ds} = L_{ds} i_{ds} + \lambda_f \tag{5}$$

$$\lambda_{qs} = L_{qs} i_{qs} \tag{6}$$

with i_{ds} and i_{qs} are the *d*- and *q*-axes stator currents; V_{ds} and V_{qs} represent the *d*- and *q*-axes stator voltages; L_{ds} and L_{qs} denote the *d*- and *q*-axes stator inductances; λ_{ds} and λ_{qs} are the *d*- and *q*-axes stator flux linkages; λ_f, R_s, n_p, ω_m, and θ_m represent the rotor permanent magnet flux linkage, stator resistance, number of pole pairs, mechanical rotor angular speed, and mechanical rotor position, respectively; J, B, and T_L denote the moment of inertia, viscous friction coefficient, and load torque, respectively.

In order to prove the property of passivity of a PMSM, the storage function $V(t)$ is defined as

$$V(t) = \frac{1}{2}\left(L_{ds} i_{ds}^2(t) + L_{qs} i_{qs}^2(t)\right) \tag{7}$$

The derivative of $V(t)$ along the trajectories of (1) and (2) is given by

$$\begin{aligned}\dot{V}(t) &= V_{ds}i_{ds} + V_{qs}i_{qs} - R_s\left(i_{ds}^2 + i_{qs}^2\right) \\ &+ n_p\omega_m i_{ds}i_{qs}\left(L_{qs} - L_{ds}\right) - n_p\omega_m\lambda_f i_{qs}\end{aligned} \tag{8}$$

For a surface-mounted PMSM, the *d*- and *q*-axes stator inductances are the same. If V_{ds} , V_{qs} are considered inputs, and i_{ds} , i_{qs} are outputs, and then integrating both sides of (8) and using (6) yields

$$\begin{aligned}\int_0^t \left(V_{ds}i_{ds} + V_{qs}i_{qs}\right)d\tau &= V(t) - V(0) \\ &+ \int_0^t \left(R_s\left(i_{ds}^2 + i_{qs}^2\right) + n_p\omega_m\lambda_f\lambda_{qs}/L_{qs}\right)d\tau\end{aligned} \tag{9}$$

which implies that the PMSM is a state strictly passive system. Then, the origin of (1) and (2) is an asymptotically stable equilibrium point of the unforced system $V_{ds} = V_{qs} = 0$ [11].

The vector control of a PMSM is to force i_{ds} to zero, therefore, (3) can be rewritten as

$$J\ddot{\theta}_m + B\dot{\theta}_m + T_L = k_t u \tag{10}$$

where $k_t = 3n_p\lambda_f/2$ is the torque constant, $u = i_{qs}^*$ is the control input. Thus, the dynamics of the PMSM is simplified to that of the DC motor and the control effort will be reduced.

III. Sliding Mode Position Controller

Generally, motors are adopted as the speed (or positioning) actuator to drive various mechanical loads, such as robotics, elevator, machine tools, etc. These typical load torques are proportional to $\sin(\theta_m)$, ω_m, ω_m^2, and can be described by the torque-speed characteristic curve, so we suppose that the load torque is a function of the mechanical rotor position for the experimental position control system, and can be described as:

$$T_L = K_L \sin(\theta_m) \tag{11}$$

where K_L is an unknown constant. That is, the load torque is characterized by the known type function with unknown magnitude. Applying (10) and (11), the dynamics of a PMSM can be rewritten as

$$J_k\ddot{\theta}_m + B_k\dot{\theta}_m + K_k\sin(\theta_m) = u \tag{12}$$

where $J_k = J/k_t$, $B_k = B/k_t$ and $K_k = K_L/k_t$.

The sliding surface is chosen to be

$$S = \dot{e}_\theta + c_1 e_\theta, \quad c_1 > 0 \tag{13}$$

where e_θ represents the position tracking error, and it is defined as

$$e_\theta = \theta_m - \theta_m^* \tag{14}$$

where θ_m^* represents the position command. Differentiating (13) and utilizing (11) and (12), we obtain

$$J_k\dot{S} + B_k S = u - J_k\dot{z} - B_k z - K_k\sin(\theta_m) \tag{15}$$

where $z = \dot{\theta}_m^* - c_1 e_\theta$. The mechanical parameters of a PMSM are not exactly known, therefore J_k and B_k can be expressed as:

$$J_k = J_{kn} + \Delta J_k \tag{16}$$

$$B_k = B_{kn} + \Delta B_k \tag{17}$$

where J_{kn} and B_{kn} are nominal values of J_k and B_k , respectively. ΔJ_k and ΔB_k are the deviations.

In order to reduce the chattering phenomenon, the control input with an adaptive law is chosen as

$$\begin{aligned}u &= J_{kn}\dot{z} + B_{kn}z + \hat{K}_k\sin(\theta_m) - \sup|\Delta J_k|\mathrm{sgn}(S\dot{z})\dot{z} \\ &- \sup|\Delta B_k|\mathrm{sgn}(Sz)z - c_2 S\end{aligned} \tag{18}$$

where $\hat{K}_k$ represents the estimated value of K_k . $\sup|\cdot|$, $\mathrm{sgn}(\cdot)$, and c_2 are the least upper bound, signum function, and positive constant, respectively. Substituting (18) into (15) gives

$$J_k\dot{S} + (B_k + c_2)S = \tilde{u} \tag{19}$$

where

$$\begin{aligned}\tilde{u} &= \tilde{K}_k\sin(\theta_m) - \Delta J_k\dot{z} - \Delta B_k z - \sup|\Delta J_k|\mathrm{sgn}(S\dot{z})\dot{z} \\ &- \sup|\Delta B_k|\mathrm{sgn}(Sz)z\end{aligned} \tag{20}$$

with $\tilde{K}_k = \hat{K}_k - K_k$. The following adaptive law is adopted to estimate the unknown external load torque disturbances,

$$\dot{\hat{K}}_k = -c_3\sin(\theta_m)S, \quad c_3 > 0 \tag{21}$$

where c_3 denotes the adaptive gain. Hence, the closed-loop equivalent feedback interconnection for the position control of a PMSM is shown in Fig. 1.

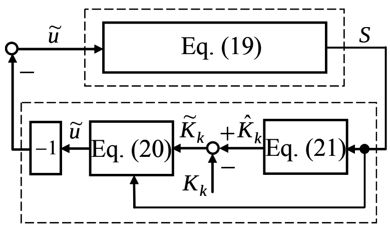


Fig. 1 Equivalent feedback interconnection of a PMSM

In order to carry out the stability analysis via passivity theorem, we employ the verification and lemma [12] and need to prove the passivity property of each block in Fig. 1 before analyzing the stability. Utilizing (19) yields

$$\langle S, \tilde{u}\rangle_t = \int_0^t S^T \tilde{u}\, d\tau = \frac{1}{2}\left(J_k S^2(t) - J_k S^2(0)\right) + (B_k + c_3)\int_0^t S^2 d\tau \tag{22}$$

Thus, according to the definition [13], the mapping $\tilde{u} \mapsto S$ of the feedforward block in Fig. 1 is output strictly passive for all $t \geq 0$. We now consider the passivity property of lower block in Fig. 1. Using (20) and (21) gives

$$\begin{aligned}\langle -\tilde{u}, S\rangle_t &= -\int_0^t \tilde{u}^T S\, d\tau = \frac{1}{2c_3}\left(\tilde{K}_k^{\,2}(t) - \tilde{K}_k^{\,2}(0)\right)\\ &+ \int_0^t \begin{pmatrix}\left(\Delta J_k + \sup|\Delta J_k|\operatorname{sgn}(S\dot{z})\right)S\dot{z}\\ +\left(\Delta B_k + \sup|\Delta B_k|\operatorname{sgn}(Sz)\right)Sz\end{pmatrix} d\tau\end{aligned} \tag{23}$$

Therefore, according to the definition [13], the mapping $S \mapsto -\tilde{u}$ is input strictly passive for all $t \geq 0$.

Combining (18) and (19), we get

$$J_k S^2(t) + \frac{\tilde{K}_k^{\,2}(t)}{c_3} < J_k S^2(0) + \frac{\tilde{K}_k^{\,2}(0)}{c_3} \tag{24}$$

Equation (24) verifies that the sliding mode position control system is asymptotically stable. Fig. 2 shows the schematic diagram of the system.

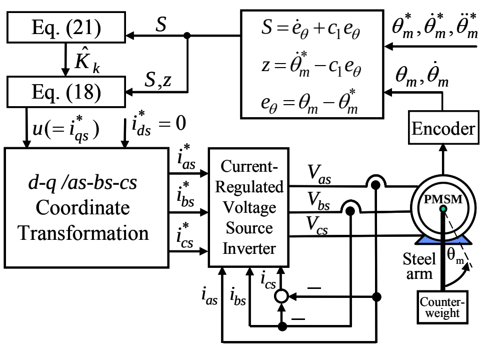


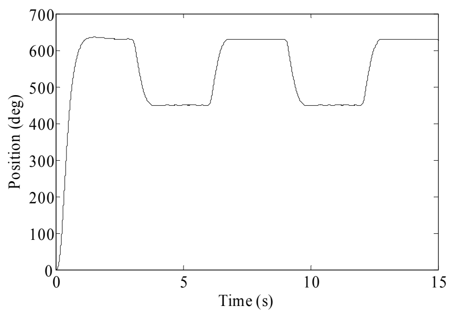
Fig. 2 Schematic diagram of the position control system

IV. Experimental Results

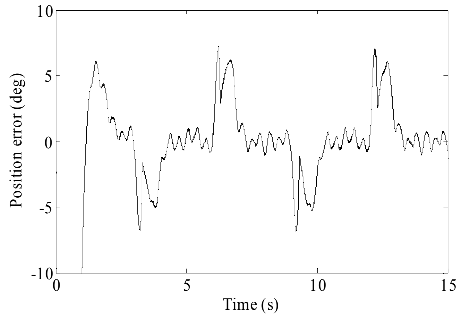
Fig. 3 shows the experimental hardware set-up which consists of a PC, an interface card with 12-bit analog-to-digital and digital-to-analog converters, a ramp comparison current-regulated voltage source inverter with an intelligent power module for the power switching device, and a 750-watt surface-mounted PMSM with a 2000 pulses/rev encoder. The resolution of the encoder is improved by the four times frequency multiplier. A counterweight is affixed to the rotor shaft for simulating the mechanical parameter variations and load torque disturbances. The control algorithm is built by adopting the MATLAB/Simulink blocks, and the sampling time is 0.5 ms.

In order to ensure the convergence of $\tilde{K}_k$ to zero, a square waveform is adopted to evaluate the position tracking capability. Hence, the position command θ_m^* continuously changed from 630° to 450° per three seconds. Fig. 4 shows the position responses and the parameter estimation result when load torque $T_L = \sin(\theta_m)$ Nm is applied to the rotor shaft. As seen in Fig. 4 (b), the position tracking error varies from 1.035° to - 1.08° when the steel arm is approximately horizontal to the ground. From Fig. 4 (e), $\hat{K}_k$ fluctuates between 1.6 and 1.7 due to the torque constant $k_t = 0.61$. This means that $\hat{K}_L = k_t \hat{K}_k$ varies from 0.976 to 1.037. Thus, the relative error is 3.7% for the parameter $\hat{K}_L$ in estimating a load torque. Experimental results show that the percentage of estimation error is acceptable. That is, the proposed scheme is robust to the variations of the motor parameters and external load torques.

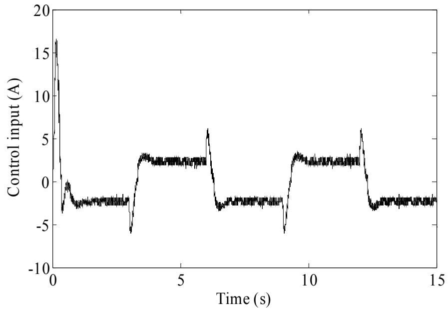
Fig. 3 Experimental set-up



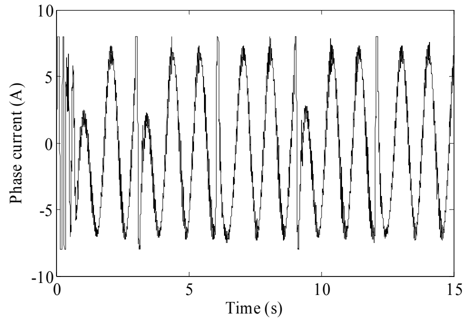
(a)



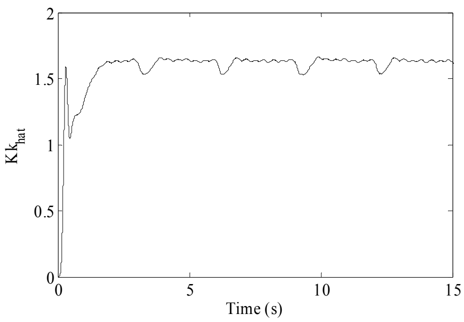
(b)



(c)



(d)



(e)

Fig. 4 Measured position responses and the estimated parameters: (a) position, θ_m; (b) position tracking error, e_θ; (c) control input, u; (d) phase-A current, i_{as}; (e) estimated parameter, $\hat{K}_k$

V. Conclusions

A passivity-based sliding mode position control algorithm with an adaptive load torque estimator is designed to control the position of a PMSM such that the chattering effects associated with the conventional sliding mode position controller can be alleviated. The passivity properties of the PMSM and the overall position control system stability have been proved formally based on the passivity theory. An experimental setup of a PC-based drive system is utilized for demonstrating the characteristics of the proposed scheme. Good position tracking responses can be obtained by the proposed sliding mode position controller. Furthermore, the approach is robust to the variations of motor parameters and load torque disturbances.

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