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Flexural Strength Design of RC Beams with Consideration of Strain Gradient Effect

Mantai Chen, Johnny Ching Ming Ho

***Abstract*—**The stress-strain relationship of concrete under flexure is one of the essential parameters in assessing ultimate flexural strength capacity of RC beams. Currently, the concrete stress-strain curve in flexure is obtained by incorporating a constant scale-down factor of 0.85 in the uniaxial stress-strain curve. However, it was revealed that strain gradient would improve the maximum concrete stress under flexure and concrete stress-strain curve is strain gradient dependent. Based on the strain-gradient-dependent concrete stress-strain curve, the investigation of the combined effects of strain gradient and concrete strength on flexural strength of RC beams was extended to high strength concrete up to 100 MPa by theoretical analysis. As an extension and application of the authors' previous study, a new flexural strength design method incorporating the combined effects of strain gradient and concrete strength is developed. A set of equivalent rectangular concrete stress block parameters is proposed and applied to produce a series of design charts showing that the flexural strength of RC beams are improved with strain gradient effect considered.

***Keywords*—**Beams, Equivalent concrete stress block, Flexural strength, Strain gradient.

I. Introduction

IN flexural strength design of reinforced concrete (RC) members, it is important to determinate the concrete stress distribution within the compression zone of RC members under flexure. Currently, the stress-strain relationship of concrete in flexure is obtained by incorporating a scale-down factor k_3, which is the ratio of maximum flexural concrete stress to cylinder strength and is taken to be 0.85 as proposed by Hognestad [1] to account for the effects of size, shape and casting position of members, in the uniaxial concrete stress-strain curve. However, experimental investigation by Sturman et al. [2] has revealed that a larger maximum concrete stress was developed in eccentrically loaded RC columns with strain gradient than that in concentrically loaded one without strain gradient due to retardation of the formation of micro-cracking in concrete. Various researchers [3]-[5] have also reported the effect of strain gradient on maximum concrete stress in flexure. By conducting experimental tests on eccentrically loaded RC columns, the authors have reported that the maximum concrete stress is influenced by the strain gradient under flexure [6], [7] and have recommended modeling the variation of maximum concrete stress in terms of factor k_3 against strain gradient in tri-linear manner [8].

M. T. Chen is a MPhil student of the Department of Civil Engineering, The University of Hong Kong, Hong Kong (phone: 852-2241-5390; fax: 852-2559-5337; e-mail: cmt111@hku.hk).

J. C. M. Ho is a senior lecturer of the School of Civil Engineering, The University of Queensland, QLD 4072, Australia (e-mail: johnny.ho@uq.edu.au).

Through imposing a strain-gradient-dependent factor k_3 [8] in concrete stress-strain relationship to consider the effect of strain gradient, the authors have extended the investigation of strain gradient effect on flexural strength to high strength concrete (HSC) up to 100 MPa by nonlinear moment-curvature analysis [9]. From the results of theoretical analysis, it was found that the values of equivalent concrete stress block parameters α and β depend on both strain gradient and concrete strength while a constant value of 0.0032 can be used reasonably well as ultimate concrete strain for flexural design of RC members with various concrete strengths incorporating strain gradient effect.

As a continued study, in this paper, the proposed value of ultimate concrete strain together with equivalent rectangular concrete stress block parameters are applied to develop a new flexural strength design method incorporating the combined effects of strain gradient and concrete strength. A set of design equations and a series of design charts are produced for evaluating the flexural strength of both singly- and doubly-reinforced concrete beams with various concrete strengths incorporating strain gradient effect.

II. Nonlinear Moment-Curvature Analysis

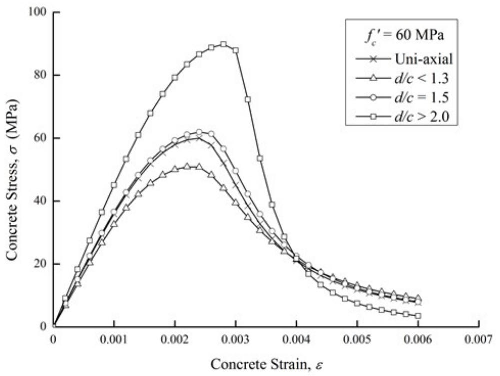
To study strain gradient effect on the flexural behavior of RC beams, nonlinear moment-curvature analysis, which takes into account the constitutive stress-strain curve of concrete and steel, was employed.

It has been revealed in authors' previous study that the ratio of maximum flexural concrete stress to cylinder strength k_3, and that of concrete strain at maximum flexural stress to uni-axial strength k_o, are strain-gradient-dependent [8]. The empirical formulas developed to correlate k_3 and k_o to strain gradient, which adopts a non-dimensional form in the ratio of effective to neutral axis depth d/c to eliminate size effect, are given in (1a) and (1b). The concrete stress-strain curve developed by Attard and Setunge [10], which is applicable to concrete strength from 20 to 130MPa, was modified to include strain gradient effect on k_3 and k_o.

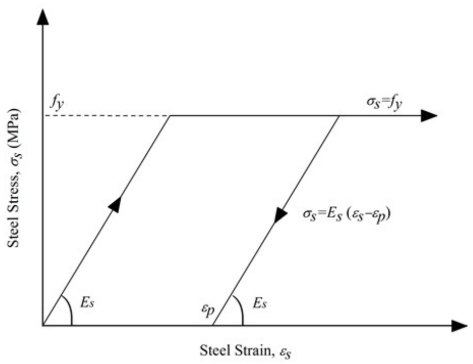
$$k_3 = \begin{cases} 0.85 & \text{for } 0 \le d/c \le 1.3 \\ 0.923(d/c) - 0.35 & \text{for } 1.3 < d/c < 2.0 \\ 1.5 & \text{for } 2.0 \le d/c \end{cases} \quad (1a)$$

$$k_o = \begin{cases} 1.0 & \text{for } 0 \le d/c \le 1.3 \\ 0.143(d/c) + 0.814 & \text{for } 1.3 < d/c < 2.0 \\ 1.1 & \text{for } 2.0 \le d/c \end{cases} \quad (1b)$$

For longitudinal steel reinforcement, an idealized linear elastic-perfectly plastic stress-strain curve with stress-path dependence was adopted. The adopted stress-strain curves of concrete and steel reinforcement are shown in Fig. 1.



(a) Strain-gradient-dependent concrete Stress-strain curve



(b) Stress-strain curve of steel with stress-path dependence considered

Fig. 1 Stress-strain curves of concrete and steel reinforcement

The moment-curvature relation of beam section was analyzed by applying prescribed curvatures to the section incrementally starting from zero. At a prescribed curvature and assumed neutral axis depth, the stresses developed in concrete and steel reinforcement can be determined from strain distribution in the section and their respective stress-strain curves. An iterative procedure of successively adjusting the neutral axis depth was needed until the unbalanced axial force was negligibly small. The resulted neutral axis depth and resisting moment can be therefore evaluated. Such procedure was repeated until the resisting moment increased to the peak and decreased to half of the peak value.

Flexural strength of a beam section is defined as the peak moment in the moment-curvature curve. A parametric study using non-linear moment-curvature analysis is conducted to evaluate effect of strain gradient on flexural strength of RC beams. The sections analyzed and the values of various factors are shown in Fig. 2.

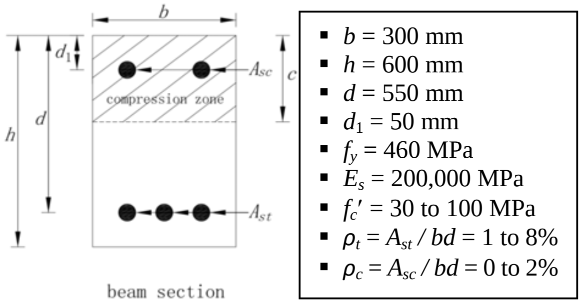


Fig. 2 Beam sections analyzed

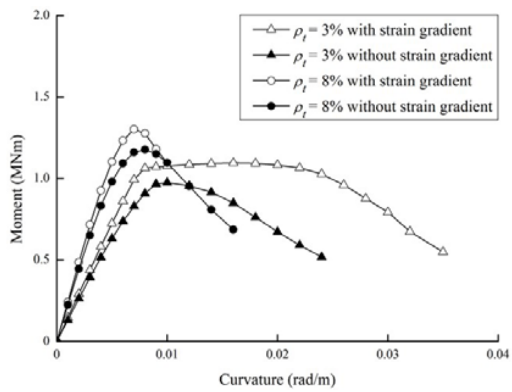
III. Results of Analysis

A. Strain Gradient Effect on Flexural Strength of RC Beams

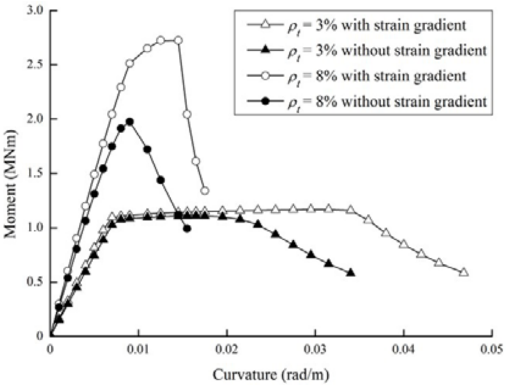
The strain gradient effect on flexural behavior of singly- and doubly-reinforced beam sections with various concrete strengths and steel ratios are shown in Fig. 3.

Two values of concrete strength (40 MPa and 80 MPa) and two values of steel ratio (3% and 8%) are chosen in Figs. 3 (a) and (b) to include the effect of concrete strength and to include both under-reinforced and over-reinforced scenarios. The improvement in flexural strength of both under- and over-reinforced beam sections can be observed from both figures. It is evident that the relative improvement in flexural strength is larger for over-reinforced section because of the larger compression zone and the extent of flexural strength improvement of RC beams due to strain gradient effect is also concrete-strength-dependent.

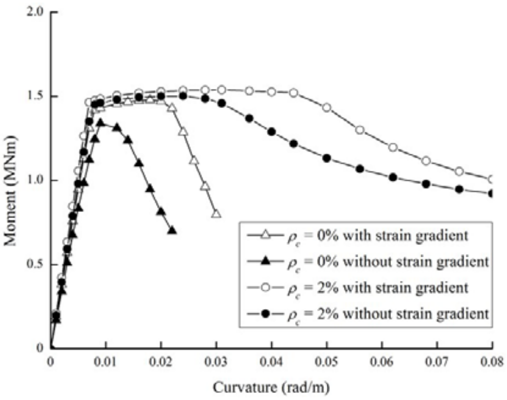
Moment-curvature curves for sections with two values of compression steel ratio (0% and 2%) are chosen in Fig. 3 (c) to investigate strain gradient effect on doubly-reinforced section. It is apparent from Fig. 3 (c) that the flexural strength improvement due to strain gradient decreases as the compression steel increases due to smaller compression zone.



(a) f_c' = 40 MPa, ρ_c = 0%



(b) $f_c' = 80$ MPa, $\rho_c = 0\%$



(c) $f_c' = 60$ MPa, $\rho_t = 4\%$

Fig. 3 Complete moment-curvature curves of singly- and doubly-RC beams with and without strain gradient effect considered

B. Derivation of Equivalent Stress Block Parameters

For practical flexural strength design of RC members, the nonlinear concrete stress distribution is usually replaced by simplified equivalent rectangular stress block with the width of $\alpha f_c'$ representing equivalent concrete stress in flexure and the height of βc representing the depth of equivalent stress block as shown in Fig. 4.

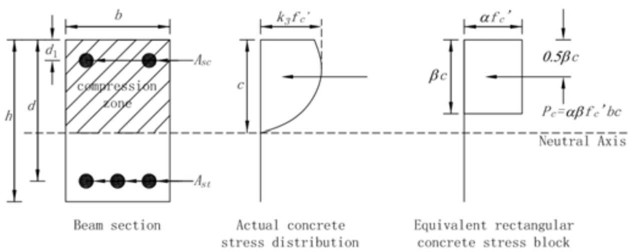


Fig. 4 Actual nonlinear concrete stress distribution and equivalent stress block

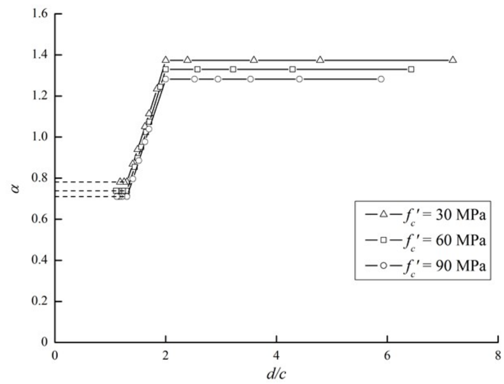
The values of equivalent stress block parameters α and β are derived based on both axial force and moment equilibrium conditions to match the force and moment obtained from the non-linear concrete stress-strain curve and the rectangular stress block as the following equations:

$$P = \alpha\beta f_c' bc + \sum_{i=1}^{n} f_{si} A_{si} \quad (2a)$$

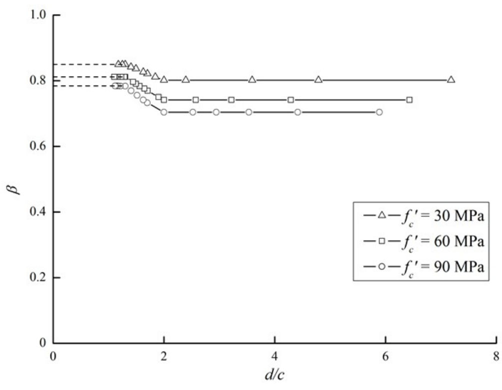
$$M = \alpha\beta f_c' bc\left(\frac{h}{2} - \frac{\beta}{2}c\right) + \sum_{i=1}^{n} f_{si} A_{si}\left(\frac{h}{2} - d_i\right) \quad (2b)$$

where b is the breadth of beam section; c is the neutral axis depth; n is the total number of steel bars; f_{si} and A_{si} are respectively the stress and area of the i^{th} steel bar; d_i is the distance of the i^{th} steel bar from the extreme concrete compressive fiber; h is the total depth of the beam section; P and M are the axial force and moment capacity calculated using non-linear analysis.

The derived equivalent rectangular stress block parameters α and β for three selected concrete strengths are plotted against strain gradient factor d/c in Figs. 5 (a) and (b), respectively. It is apparent that both concrete stress block parameters, α and β, are dependent on strain gradient and concrete strength. Accordingly, the combined effect of strain gradient and concrete strength should be considered simultaneously in the flexural strength design of RC beams.



(a) α plotted against d/c



(b) β plotted against d/c

Fig. 5 Graphs of α and β plotted against strain gradient d/c
Note: Dotted lines indicate that neutral axis depth c falls out of section

For the application to practical design of RC beams with concrete strength from 30 to 100 MPa incorporating strain gradient effect, a set of equivalent rectangular stress-block parameters α and β with equations shown in (3) and (4) are recommended based on the results obtained from the parametric study.

$$\alpha = \begin{cases} \alpha_1 & \text{for } 0 \le d/c \le 1.3 \\ (\alpha_2 - \alpha_1)\left(\dfrac{d/c - 1.3}{0.7}\right) + \alpha_1 & \text{for } 1.3 < d/c < 2.0 \\ \alpha_2 & \text{for } 2.0 \le d/c \end{cases} \quad (3a)$$

$$\alpha_1 = 0.063(f_c'/100)^2 - 0.19(f_c'/100) + 0.83 \quad (3b)$$

$$\alpha_2 = -0.077(f_c'/100)^2 - 0.06(f_c'/100) + 1.395 \quad (3c)$$

$$\beta = \begin{cases} \beta_1 & \text{for } 0 \le d/c \le 1.3 \\ (\beta_2 - \beta_1)\left(\dfrac{d/c - 1.3}{0.7}\right) + \beta_1 & \text{for } 1.3 < d/c < 2.0 \\ \beta_2 & \text{for } 2.0 \le d/c \end{cases} \quad (4a)$$

$$\beta_1 = 0.06(f_c'/100)^2 - 0.183(f_c'/100) + 0.9 \quad (4b)$$

$$\beta_2 = 0.128(f_c'/100)^2 - 0.316(f_c'/100) + 0.885 \quad (4c)$$

C. Ultimate Concrete Strain

Equivalent stress block parameters should be adopted together with appropriate ultimate concrete strain ε_{cu} in practical flexural strength design of RC beams incorporating strain gradient effect. Since the determination of ε_{cu} is quite difficult for under-reinforced beams, which show long flat plateau when reaching the maximum moment capacity in the moment-curvature curves, the authors have proposed a method to minimize the sensitivity of ε_{cu} and at the same time maintaining the accuracy in moment capacity prediction by studying the lower bound and upper bound values of ε_{cu} that may cause at most 1% error in evaluating the maximum moment capacity. It was found that a constant value of 3200 $\mu\varepsilon$ can always be adopted as the design value of ultimate concrete strain for all f_c' and ρ_t studied in this paper [9].

D. Verification of Proposal

To validate the proposed equivalent concrete stress block together with the proposed design value of ultimate concrete strain, the proposed equations (3) and (4) and ultimate concrete strain with design value of 0.0032 are used to evaluate the flexural strength of over 200 beam specimens tested by other researchers [11]-[25]. The predicted flexural strengths of beam specimens based on the new proposal are compared with the measured strengths and those calculated as per various RC design codes [26]-[28]. The accuracy of flexural strength prediction is improved by 6% on average [9].

IV. Application on Flexural Strength Design of RC Beams

A. Singly-Reinforced Beams

The proposed equivalent rectangular stress block parameters with strain gradient effect considered can be applied to flexural strength design of singly-reinforced beams with various tension steel ratios and concrete strengths based on force and moment equilibrium conditions as shown in the following equations:

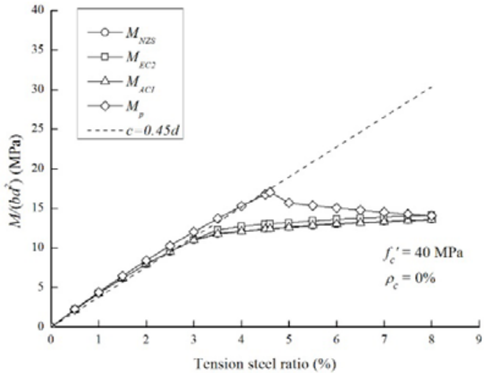
$$\alpha\beta f_c' bc = f_{st}\rho_t bd \quad (5a)$$

$$M = \alpha\beta f_c' bc(d - 0.5\beta c) \quad (5b)$$

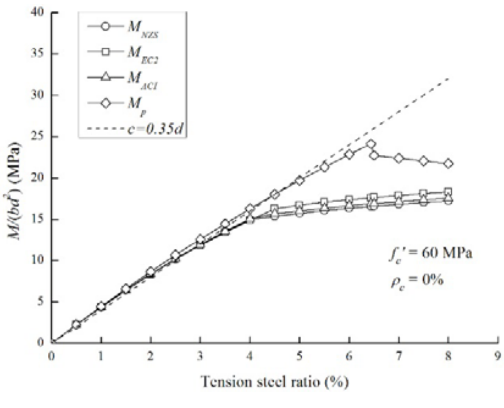
where α and β are proposed by (3) and (4) respectively.

To facilitate practical design application, the above formulas have been converted into a series of design charts for singly-reinforced beams with different concrete strength varied from 40 to 100 MPa and tension steel ratio varied from 0 to 8% as shown in Fig. 6. The flexural strength is expressed in terms of $M/(bd^2)$ to eliminate the size effect. The flexural strength improvement when strain gradient effect is considered is investigated by comparing the predicted strength using proposed method (M_p) with the theoretical strengths (M_{ACI}, M_{EC2}, M_{NZS}) calculated as per various design codes [26]-[28] respectively. The dotted line is referred to the strength calculated with allowable neutral axis depth of 0.45d for $f_c' \le 50$ MPa or 0.35d for $f_c' > 50$ MPa as stipulated in Eurocode 2 [27].

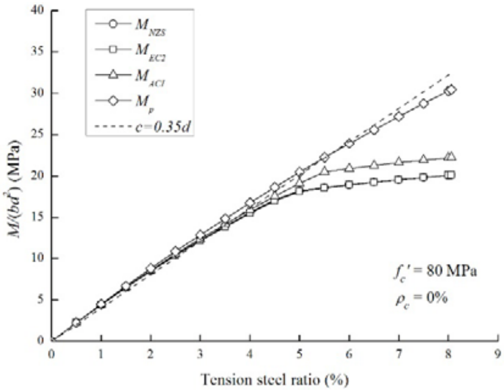
From Fig. 6, the maximum difference in flexural strength with strain gradient effect considered is 34% when $f_c' = 80$ MPa and $\rho_t = 8\%$. However, for practical design where $c \le 0.45d$ or 0.35d and $\rho_t \le 4\%$ to allow for nominal ductility requirement [27], the maximum difference reduces to 21% ($f_c' = 40$ MPa and $\rho_t = 4\%$).



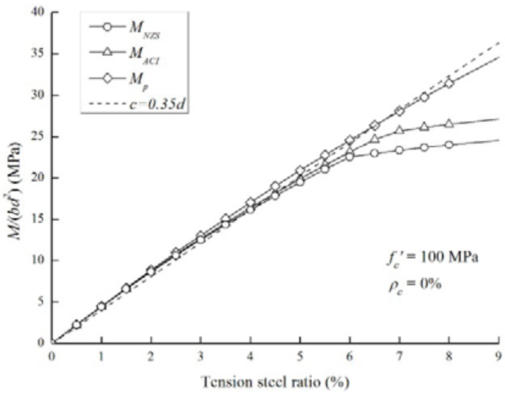
(a) $f_c' = 40$ MPa and $\rho_c = 0\%$



(b) $f_c' = 60$ MPa and $\rho_c = 0\%$



(c) f_c' = 60 MPa and ρ_c = 0%



(d) f_c' = 100 MPa* and ρ_c = 0%

Fig. 6 Design charts for flexural strength of singly-reinforced beams
Note: *: concrete strength of 100 MPa is beyond the limit specified in Eurocode 2, no data can be reported in (d)

B. Doubly-Reinforced Beams

Similarly to singly-reinforced beams, the flexural strength of doubly-reinforced beams with various tension steel ratios and concrete strengths can be evaluated using following equations:

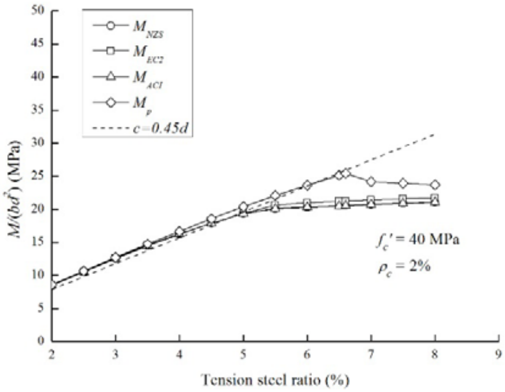
$$\alpha\beta f_c' bc + f_{sc}\rho_c bd = f_{st}\rho_t bd \qquad (6a)$$

$$M = \alpha\beta f_c' bc\left(d - 0.5\beta c\right) + f_{sc}\rho_c bd(d - d_1) \qquad (6b)$$

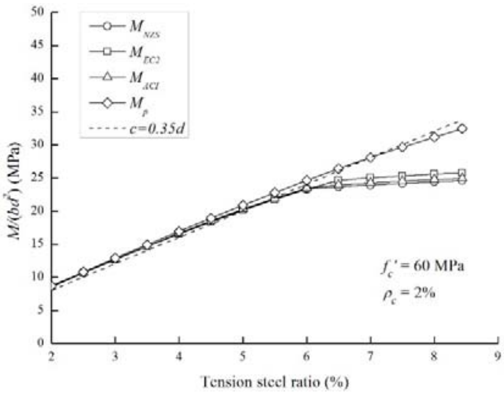
where α and β are proposed by (3) and (4) respectively.

A series of design charts is produced for flexural strength design of doubly-reinforced beams with compression steel ratio of 2%, concrete strengths from 40 to 100 MPa and tension steel ratios from 0 to 8% as shown in Fig. 7.

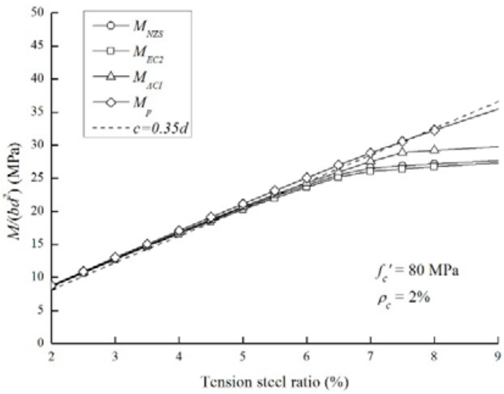
From Fig. 7, the maximum difference in flexural strength with strain gradient effect considered for doubly-reinforced beam with ρ_c = 2% is 22% when f_c' = 60 MPa and ρ_t = 8%. However, for practical design considering nominal ductility requirement, the maximum difference reduces to only 3% (f_c' = 40 MPa and ρ_t = 4%).



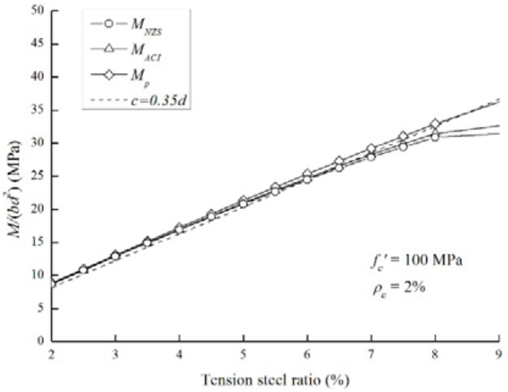
(a) f_c' = 40 MPa and ρ_c = 2%



(b) f_c' = 60 MPa and ρ_c = 2%



(c) f_c' = 80 MPa and ρ_c = 2%



(d) f_c' = 100 MPa* and ρ_c = 2%

Fig. 7 Design charts for flexural strength of doubly-reinforced beams (ρ_c = 2%) Note: *: concrete strength of 100 MPa is beyond the limit specified in Eurocode 2, no data can be reported in (d)

V. Conclusion

A strain-gradient-dependent concrete stress-strain curve is adopted in nonlinear moment-curvature analysis to extend the investigation of strain gradient effect on flexural strength of RC beams to high strength concrete up to 100 MPa. The improvement of flexural strength due to strain gradient effect can be observed from both under- and over-reinforced concrete beams with or without compression steel. It is found that the strain gradient effect on flexural strength improvement of RC beams is also concrete-strength-dependent. Therefore, two equations of equivalent rectangular stress block parameters considering the combined effects of strain gradient and concrete strength together with a constant design value of 0.0032, which were verified by comparing the predicted strengths to the measured strengths of over 200 beam specimens, are proposed. Based on that, a new flexural strength design method considering the combined effects of strain gradient and concrete strength is proposed for both singly- and doubly-reinforced beams. The proposed flexural strength design equations are further converted into a series of design charts for the purpose of facilitation of practical flexural design.

By comparing the predicted flexural strength of singly- and doubly-reinforced beams with the theoretical strength calculated as per various RC design codes, it is apparent that the improvement in flexural strength prediction due to consideration of strain gradient effect is more significant for singly-reinforced beams (with maximum of 34% improvement) than doubly-reinforced beams (with maximum of 22% improvement). However, for practical design, by limiting the neutral axis depth to be within 0.45 or 0.35 times the effective depth and tension steel ratio to be smaller than 4% to allow for the minimum ductility requirement, the flexural strength improvement reduces to the range between 3 and 21%.

Acknowledgment

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Mr. Mantai Chen obtained his BEng degree in 2012 in civil engineering from The University of Hong Kong (HKU). He is now a MPhil student in the civil engineering department in The University of Hong Kong. His research interest is strain gradient effects of flexural strength and ductility of reinforced concrete beams and columns.

Ir. Dr. Johnny Ching Ming Ho is Senior Lecturer of the School of Civil Engineering, The University of Queensland. Before joining the university in 2013, Dr Ho has been working as an Assistant Professor in The University of Hong Kong, and as a Civil Engineers in both Hong Kong and Brisbane offices of Arup on some large scale infrastructure projects such as The Stonecutters Bridge in Hong Kong and Ipswich Motorway Upgrade in Queensland, Australia. Dr Ho's research interests are on ductility and deformability of high-strength concrete beams and columns, plastic hinge analysis of reinforced concrete members and behaviour of single- and double-skinned concrete-filled-steel-tube columns with external confinement and internal concrete expansive agent.