

# Direct Transient Stability Assessment of Stressed Power Systems

E. Popov, N. Yorino, Y. Zoka, Y. Sasaki, H. Sugihara

**Abstract**—This paper discusses the performance of critical trajectory method (CTrj) for power system transient stability analysis under various loading settings and heavy fault condition. The method obtains Controlling Unstable Equilibrium Point (CUEP) which is essential for estimation of power system stability margins. The CUEP is computed by applying the CTrj to the boundary controlling unstable equilibrium point (BCU) method. The Proposed method computes a trajectory on the stability boundary that starts from the exit point and reaches CUEP under certain assumptions. The robustness and effectiveness of the method are demonstrated via six power system models and five loading conditions. As benchmark is used conventional simulation method whereas the performance is compared with and BCU Shadowing method.

**Keywords**—Power system, Transient stability, Critical trajectory method, Energy function method.

## I. INTRODUCTION

THE power system stability is defined as “that property of a power system that enables it to remain in a state of operating equilibrium under normal operating conditions and to regain an acceptable state of equilibrium after being subjected to a disturbance” [1].

Transient stability analysis has major impact on planning and operation of power systems. The recent technology solutions enable the power systems to increase the capacity of the existing transmission networks, and effectively maintain the operating margins. However, these improvements cannot keep pace with the growing demand and stress. Consequently, the related utilities operate close to their limits, and furthermore the threat from transient problems increases. Among various contingencies, fault on transmission lines, large load variations, outage of power components are more common than ever before. These conditions cause system overload and exceeding the power limits, thus, resulting in an insecure system. Hence, a fast (online) method is necessary in order to predict power system behavior in case of contingencies and presence of heavy load conditions. Such a method should also be able evaluate the degree of the power system stability and provide information

regarding the derivation of preventive control and load shedding actions.

The transient stability analysis is based mainly on two methods: the time-domain approach and the direct methods approach. The time domain approach is performed by step-by-step numerical integrations of power system models. Main advantage is that various complicated dynamic models can be easily integrated providing accurate results. The drawback of the time-domain approach is the time consuming computation, therefore, it is not suitable for online stability assessment. Recent improvements include faster techniques as in [2] and [3], as well as enhanced security in [4]–[6].

An alternative approach, based on transient energy function (TEF), is the direct methods. The power system condition is determined in terms of system energy. The stability is judged by comparison of the system energy, after a disturbance, with a critical value. For a given fault, the system trajectory might exit the stability boundary. In this case, the trajectory passes near certain type-1 unstable equilibrium point (UEP) on the boundary more closely than others UEPs. This point is called the Controlling UEP (CUEP) and, at that location, the transient energy function has a local minimum. If the trajectory owns energy greater than that of the CUEP then it will pass the stability boundary and the power system becomes unstable. The time at which the power system reaches this critical energy is called the critical clearing time (CCT). It is a conservative estimate of the clearing time which guarantees the first-swing stability of the power system.

Apart from their fast computational times, direct methods provide quantitative measure of the degree of stability and data necessary for preventive control. This information is useful when operating limits must be estimated in a fast manner, which makes them suitable for online stability assessment.

Over the last three decades, significant progress has been made on these methods [7]–[18]. However, one of the disadvantages of the direct methods is that the models are relatively simple because detailed models are difficult to be treated. The accuracy of stability assessment is highly dependent on correct determination of CUEP. Obtaining wrong CUEP leads to flawed stability judgment. Recent progress regarding this problem takes into account multi-swing stability issues in [7] and an application of stochastic approach treats uncertainties [8].

A promising approach, among the direct methods, is the controlling unstable equilibrium point (BCU) method [12] and [13] which possesses strict theoretical background for evaluation of the critical energy. It provides sequence of procedures for determination of a suitable starting point close

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enough to the CUEP. Nevertheless, the BCU method is unable to locate the exit point or detects an incorrect one in some situations. Moreover, assumptions of the method itself are questioned in [19]–[21], such as the validity of transversality condition. Improved techniques overcome certain drawbacks [22]–[27]. The accuracy is improved in [22] and [25], the dynamical method [24] delivers faster results using modified backward differential formulae. The comprehensive method [27] combines the strengths of [12], [17] and [22] in order to increase the robustness of CUEP determination. In [26] the system is evaluated with parameterized equations. Among the methods, the Shadowing method [22] has superior performance for CUEP computation.

Critical trajectory method (CTRj) [30]–[34], another our approach, computes critical condition for the ordinary differential equations of transient stability formulated as a boundary value problem. The method is developed for the computation of “exact” CCT in [30] and [31], whereas it is applied to the BCU method for “approximated” CCT as a transient energy function method in [32]–[34]. Latest version in [34] improves further the accuracy for ill-conditioned systems.

In this paper, we provide a critical assessment of our previously established method [34]. It was considered six power system models and five load conditions. As benchmark was used the time-domain approach and the efficiency was compared with the BCU Shadowing method. This assessment is mainly focused examination of the robustness, accuracy and conservativeness of the stability judgment. Graphical representations are used to summarize the accurateness of the CCTs computation whereas more detailed data provide information in regards to the conservativeness of the method. The results confirmed the robustness and superiority of our method under all conditions as well as the adequacy of the estimated CCTs.

The power system model and the transient energy function are described in chapter II. The application of the Critical Trajectory method to BCU method and the necessary procedures for CUEP computation are discussed in chapter III. The numerical examination in Chapter IV A presents the considered power system models; Chapter IV B is a summary of the simulations results with graphical and data comparisons. Conclusion is given in Chapter V.

## II. POWER SYSTEM MODEL

It is used the classical model that consists of  $nX_d'$  generators and the loads are modeled as constant impedances:

$$\begin{cases} M_i \dot{\omega}_i = P_i - \sum_{j=1, j \neq i}^n [C_{ij} \sin(\theta_i - \theta_j) + D_{ij} \cos(\theta_i - \theta_j)] \\ \dot{\theta}_i = \omega_i \end{cases} \quad (1)$$

where  $E_i$ - constant voltage behind the direct axis transient reactance,  $\omega_i$  and  $\theta_i$ - generator rotor speed and angle deviations,  $M_i$ - inertia constant,  $n$ - number of generators,  $i=1, \dots, n$ ,  $P_i = P_{mi} - E_i^2 G_{ii}$ ,  $C_{ij} = E_i E_j B_{ij}$ ,  $D_{ij} = E_i E_j G_{ij}$ ,  $G_{ii}$ - driving point conductance,  $G_{ij}$  and  $B_{ij}$ - real and imaginary components of the

$ij$ th element of the reduced admittance matrix  $Y$  of the power system,  $P_{mi}$ - mechanical power input.

The center of inertia frame is employed, (1) takes form:

$$\begin{cases} M_i \dot{\tilde{\omega}}_i = P_{mi} - P_{ei}(\delta) - \frac{M_i}{M_T} P_{COA}(\delta) \\ \dot{\tilde{\delta}}_i = \tilde{\omega}_i \end{cases} \quad (2)$$

where

$$P_{ei}(\delta) = \sum_{j=1, j \neq i}^n [C_{ij} \sin(\delta_i - \delta_j) + D_{ij} \cos(\delta_i - \delta_j)] \quad (3)$$

$$P_{COA}(\delta) = \sum_{i=1}^n (P_{mi} - P_{ei}(\delta)), M_T = \sum_{i=1}^n M_i \quad (4)$$

$$\tilde{\delta}_i = \theta_i - \frac{1}{M_T} \sum_{i=1}^n M_i \theta_i, \tilde{\omega}_i = \omega_i - \frac{1}{M_T} \sum_{i=1}^n M_i \omega_i \quad (5)$$

The post fault configuration of the power system is considered with zero transfer conductance ( $D_{ij}=0$ ). The transient energy function, defined in [29], is used in accordance with mentioned above assumption ( $D_{ij}=0$ ):

$$V = V_K(\tilde{\omega}) + V_P(\delta) \quad (6)$$

$$V_K(\tilde{\omega}) = \frac{1}{2} \sum_{i=1}^n M_i \tilde{\omega}_i^2 \quad (7)$$

$$\begin{aligned} V_P(\delta) &= \sum_{i=1}^n \int_{\delta_i^s}^{\delta_i^s} P_i(\delta) d\delta_i \approx - \sum_{i=1}^n (P_i)(\delta_i - \delta_i^s) \\ &- \sum_{i=1}^{n-1} \sum_{j=i+1}^n [E_i E_j B_{ij} (\cos(\delta_i - \delta_j) - \cos(\delta_i^s - \delta_j^s))] \end{aligned} \quad (8)$$

$$P_i(\delta) = \frac{\partial V_P(\delta)}{\partial \delta_i} = P_{mi} - P_{ei}(\delta) - \frac{M_i}{M_T} P_{COA}(\delta) \quad (9)$$

## III. APPLICATION OF CTRJ TO BCU METHOD

The importance of finding the correct CUEP has led methods such as the BCU method to derive theoretic-based algorithm for detection of the CUEP. The BCU method uses the relationship between the stability boundary of the post-fault classical power system model and the stability boundary of the post-fault reduced system:

$$\dot{\delta} = P(\delta) \quad (10)$$

The system consists of  $n$  generators and the state vector in (10) can be represented as:

$$x = \delta = [\delta_1, \delta_2, \dots, \delta_n]^T \quad (11)$$

Stability of the power system is judged by its ability to

remain stable after severe disturbance such as 3LG fault. After fault clearance, the post fault system is analyzed with an initial condition called “exit point” that is obtained along the fault trajectory. The CUEP satisfy the following condition:

$$P(\delta) = 0 \tag{12}$$

The CTTrj method in [32]–[34] obtains the critical condition of dynamic power system to directly obtain CCTs. It is used the characteristic that potential energy boundary surface (PEBS) of the BCU problem corresponds to the critical trajectory of the previously stated gradient system in (10), and proposes an improved solution of CUEP. The approach is explained as follows.

Similarly to BCU method, as an initial condition is used the exit point:

$$\delta^0 = \delta^{exit} \tag{13}$$

where  $\delta^{exit}$  is the exit point. The exit point is computed along the fault-on trajectory and obtained at the first local maxima of the potential energy,  $V_p$ . Based on theory, this maximum is on the boundary of the stability region of an associated stable equilibrium point (SEP) of the post fault system. The boundary of the stability region is consisted of the stable manifolds of all UEPs and corresponds to the ridge of the potential energy of (10), referred as PEBS. For the end point condition (UEP) is used the equilibrium in (12). The equilibrium equation in the minimization problem takes the following form.

$$\mu^{m+1} = P(\delta^{m+1}) = 0 \tag{14}$$

The BCU problem for the CUEP is re-formulated by the CTTrj method in [34] as minimization problem:

$$\min_X S(X) = \mu(X)^T \cdot \mu(X) = \sum_{k=0}^{m+1} |\mu^k|^2 \tag{15}$$

$$X = [\delta^1, \delta^2, \dots, \delta^{m+1}, d]^T = [\delta_1^1 \dots \delta_n^1 \mid \dots \mid \delta_1^{m+1} \dots \delta_n^{m+1} \mid d]^T \tag{16}$$

$$\mu(X) = \begin{pmatrix} \mu^1 \\ \vdots \\ \mu^{2k-1} \\ \mu^{2k} \end{pmatrix} = \begin{pmatrix} \delta^1 - \delta^{exit} - \frac{\delta^1 + \delta^{exit}}{|\delta^1 + \delta^{exit}|} d \\ \vdots \\ \delta^m - \delta^{m-1} - \frac{\delta^m + \delta^{m-1}}{|\delta^m + \delta^{m-1}|} d \\ \delta^{m+1} - \delta^m - \frac{\delta^m}{|\delta^m|} d \\ P(\delta^{m+1}) \\ \delta^{1T} (\delta^1 - \delta^s) \\ \vdots \\ \delta^{mT} (\delta^m - \delta^s) \end{pmatrix} \tag{17}$$

where

$$\dot{\delta}^{exit} = P(\delta^{exit}) \tag{18}$$

$$\dot{\delta}^k = P(\delta^k) \tag{19}$$

$$\mu^k = \delta^k - \delta^{k-1} - \frac{\delta^k + \delta^{k-1}}{|\delta^k + \delta^{k-1}|} d \tag{20}$$

In the minimization problem, the modified trapezoidal formula (20) represents the relationship between two points ( $\delta^{k-1}, \delta^k$ ) of the trajectory,  $k=1, \dots, m+1$ . Equation (20) is derived from the trapezoidal formula, where the numerical integration with respect to time is transformed into that with distance  $d$ . Derivation of the complete formula is given in [32]. We assume the existence of solution of (20), common practice in ODE problem. The modified trapezoidal formula makes possible to represent the CTTrj by specified number of points ( $\delta^0, \delta^1, \dots, \delta^{m+1}$ ) with same distance. The parameter  $m$  defines the number of points that represent the trajectory computed by the proposed method. Note that  $m$  is initially predetermined and during the actual computation process the positions of all points are updated simultaneously. Moreover,  $m$  affects the accuracy and computation time since it is roughly proportional to the size of the proposed minimization problem. The distance  $d$  is automatically determined as a solution for specified  $m$  points.

In order to improve the CUEP determination, the CTTrj method is combined with PEBS property firstly established in [18]. We consider the following maximization problem on the trajectory:

$$\max_{\lambda} V_p(\delta(\lambda)) \tag{21}$$

$$\delta(\lambda) = \delta^s + \lambda(\delta - \delta^s) \tag{22}$$

where  $\lambda \geq 0, \delta^s$  implies SEP in this paper but any fixed point may be useful. The solution of the above problem implies the point of maximum potential energy on the line connecting SEP to the points of the trajectory as shown in Fig. 1. The optimal condition of the above problem is given as:

$$\frac{\partial V_p(\delta(\lambda))}{\partial \delta} \frac{\partial \delta}{\partial \lambda} = \left( \frac{\partial V_p(\delta(\lambda))}{\partial \delta} \right)^T (\delta - \delta^s) = \dot{\delta}^T (\delta - \delta^s) = 0 \tag{23}$$

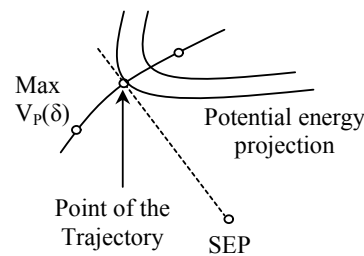


Fig. 1 Concept of the PEBS

In the proposed method the above optimal condition is

applied to all points on the trajectory except the final one.

$$\delta^i T (\delta^i - \delta^s) = 0 \quad (24)$$

where  $i=1..m$

Basic concept of the proposed method is given in Fig. 2:

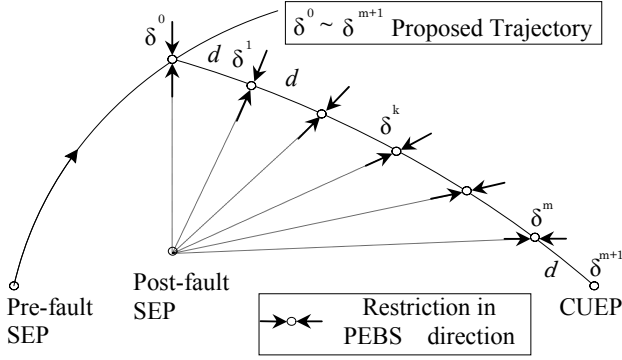


Fig. 2 Concept of the proposed method

The Newton's method is used to solve the stationary condition for the above problem.

$$\frac{\partial S}{\partial X} = 2J^T \mu(X) \approx 2J^T (J\Delta X + \mu(X_0)) = 0 \quad (25)$$

$$J \equiv \frac{\partial \mu(X)}{\partial X} \quad (26)$$

Update  $\Delta X$  is obtained by solving (25) to update solution  $X$  using (26).

$$J^T J\Delta X + J^T \mu(X_0) = 0 \quad (27)$$

$$X = X_0 + \Delta X \quad (28)$$

A good initial guess for all state variables of  $X$  can be formed in following manner:

$$\delta^k = \delta^{exit} + k \cdot d \cdot e, \quad k = 1, \dots, m+1 \quad (29)$$

where

$$d = c/m \quad (c=1 \text{ is recommended}) \quad (30)$$

$$e = \dot{\delta} / \|\dot{\delta}\| = P(\delta^{exit}) / \|P(\delta^{exit})\| \quad (31)$$

Note that  $e$  is the unit vector in gradient direction. The more points are used, the lesser initial step size is selected. A suitable prediction for the distance  $d$  can be found by the formula  $d=c/m$ , where  $c=1$  was determined to be the most efficient and used for the entire examination. The proposed method is applied after computation of exit point.

Computational Procedures:

- S1 Compute initial guess (29) from the exit point  $\delta^{exit}$ .
- S2 Compute of the post fault SEP.
- S3 Repeat (27) and (28) to obtain convergence of  $X$ .
- S4 CUEP is obtained as  $\delta^{m+1}$  in  $X$ .

#### IV. NUMERICAL EXAMINATION

##### A. Power System Models

Numerical examinations were carried out on

- 3-machine 9-bus system [35] – 9 fault locations.
- 4-machine 9-bus system [30] – 9 fault locations.
- 6-machine IEEE 30-bus system – 10 fault locations.
- 7-machine IEEE 57-bus system – 16 fault locations.
- west 10-machine IEEJ 27-bus system – 12 fault locations.
- 30-machine 115-bus system (IEE Japan West 30)– 26 fault locations.

It is assumed that every transmission line consists of double parallel circuits, and a 3LG fault occurs at a point very close to a bus on one of the parallel lines; after the fault clearance the faulted line is disconnected.

The proposed method is tested for  $m=10$  for various load conditions and fault locations. Convergence criterion for the Newton's method is used  $|dX_i| < 10^{-3}$ . The performance of the proposed method is compared with the Shadowing method. The results are given in Tables I–III and Figs. 3–8. These figures summarize the accuracy performance of both methods based on average CCT errors, whereas the tables provide more detailed information regarding the conservativeness of the estimated results. For benchmark, the obtained CCTs are compared with the conventional numerical simulation method.

##### B. Performance Assessment

The performance of the proposed method is demonstrated in comparison with the existing methods, which are the Shadowing method and the proposed method [34].

The cases with 0% error imply ideal solutions in which the transfer energy function estimations are very accurate. Under certain assumptions the computed CCT agrees with those computed by the conventional method. Such assumptions include:

- Simplified power system model.
- Zero transfer conductance.
- Non damping.

However, it is known that energy functions methods are rather conservative and even proved methods such as the Shadowing Method fail occasionally in heavily stressed systems such as West Japan system. Practical power systems are lossy, and losses occur in transmission lines, loads or transfer conductances. Mainly, the error in CCT computation is due to a common problem with the energy function, the transfer function problem. It has not been found a general energy function for multi-machine lossy stability models [28]. There is not complete representation of the transfer conductance in the energy function and this often causes problems besides the robustness of the direct methods themselves. That is the dissipation of the mechanical energy is path dependent integral

that cannot be evaluated unless the system trajectory is known. Thus, the transfer conductance is neglected in this examination for the CUEP computations in order to discuss the robustness of the proposed method. For this purpose, as benchmark is used the numerical simulation with zero transfer conductance. The approximated CCT's must agree with the simulation method results with zero transfer conductance. The conservativeness of the results for some fault locations is consequence of the discussed representation. This statement is also confirmed by the results in Table III which show the predominant number of underestimated CCTs. Theoretically, based on conservative nature of the direct methods the estimated error  $\varepsilon$  has a positive value that is:

$$\varepsilon = CCT_{ACT} - CCT_{EST} \tag{32}$$

where  $CCT_{ACT}$  is the actual CCT, (computed by the conventional simulation method) and  $CCT_{EST}$  is the computed CCT (either BCU Shadowing or the proposed method)

However, based on the method accuracy and adequacy of the power system model, also a small number of results appeared to be optimistic. Tables I and III comprise detailed information.

$$\bar{\varepsilon}^U = \frac{1}{N_{UC}} \sum_{i=1}^{N_{UC}} \varepsilon_i \tag{33}$$

$\bar{\varepsilon}^U$  - Average estimated error of the conservative results.

$$\bar{\varepsilon}^O = \frac{1}{N_{OC}} \sum_{i=1}^{N_{OC}} |\varepsilon_i| \tag{34}$$

$\bar{\varepsilon}^O$  - Average estimated error for the optimistic results.

$$\bar{\varepsilon} = \frac{1}{N} \sum_{i=1}^N |\varepsilon_i| \tag{35}$$

$\bar{\varepsilon}$  -Average estimated error for all faults

$N_{OC}$ –number of optimistic cases.

$N_{UC}$ – number of conservative cases.

$N$ –total number of faults

The tendency of accuracy deviation increases with stability assessment of heavily stressed systems. From Fig. 6, for 30 machine system, it can be seen that the system works near its limits. In this power system model, further load increase leads to loss of stable operating condition. The examination showed that the system cannot handle load factor of 120% and 140%. In Table II is given similar example in which the 10 machine system operates under 140% load factor. The system also becomes very insecure and some of the faults lead to immediate instability. The reasonable estimations of CCTs show that even under such heavy conditions our method is reliable. The column next to last in Table II show the CCTs obtained by the numerical simulation related to the used representation. These results are acceptable since the transient energy function methods inherently cannot take into account the transfer

conductance correctly.

The expression of “0.05–0.06” means that the system is stable with clearing time of 0.05 [s] and unstable at 0.06 [s] and the exact value of CCT exists between 0.05 and 0.06 [s].

The desirable CUEPs were attained for most of the cases. This implies that the proposed procedures S3 and S4 are robust enough to provide convergence of the final point sufficiently close to the CUEP. The proposed method showed similar accuracy as the Shadowing method for light load condition cases and superior for heavy load conditions. Although improved cases may look few, this is regarded as very meaningful achievement since the system conditions are heavily ill-conditioned and very difficult for analysis by the existing methods.

The computational times and efficiency are discussed in this in [32] and [34].

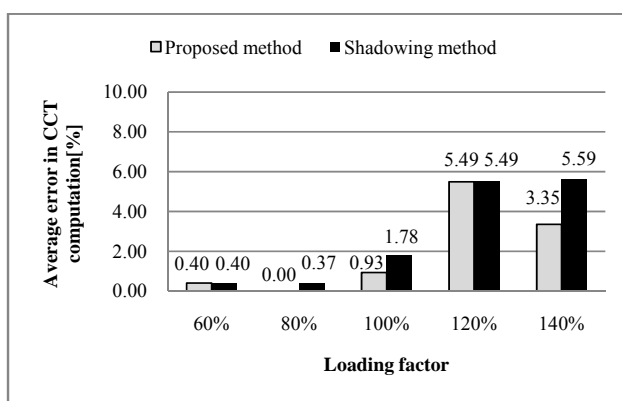


Fig. 3 Performance of the BCU shadowing and proposed methods based on loading conditions – 3 machine system

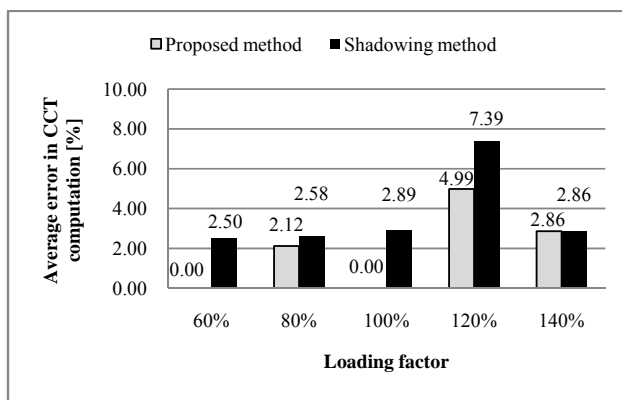


Fig. 4 Performance of the BCU shadowing and proposed methods based on loading conditions – 4 machine system

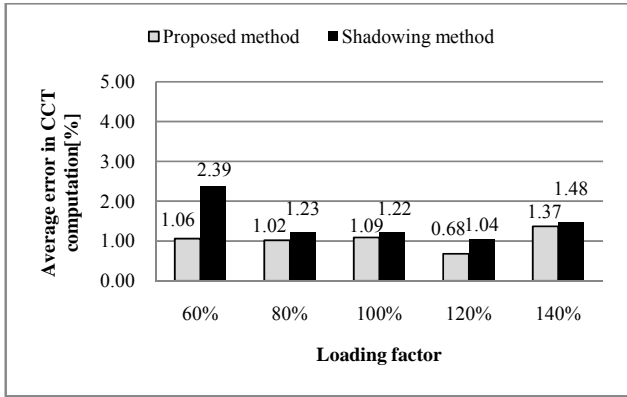


Fig. 5 Performance of the BCU shadowing and proposed methods based on loading conditions –IEEE 6 machine system

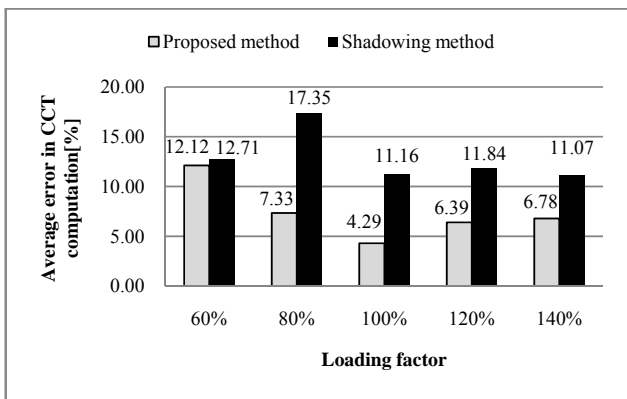


Fig. 6 Performance of the BCU shadowing and proposed methods based on loading conditions –IEEE 7 machine system

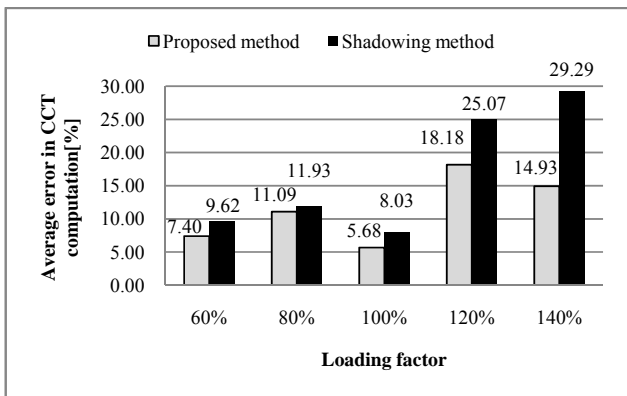


Fig. 7 Performance of the BCU shadowing and proposed methods based on loading conditions –IEEJ west10 machine system

V.CONCLUSION

A critical evaluation of the proposed method for transient stability analysis is shown in this paper on six power system models under five load conditions. The assessment proved the robust performance of the proposed method. The method estimated all CCTs with reasonable accuracy under all conditions and showed superiority to Shadowing method.

Further examination should account improvement in transient energy function and more detailed power system model.

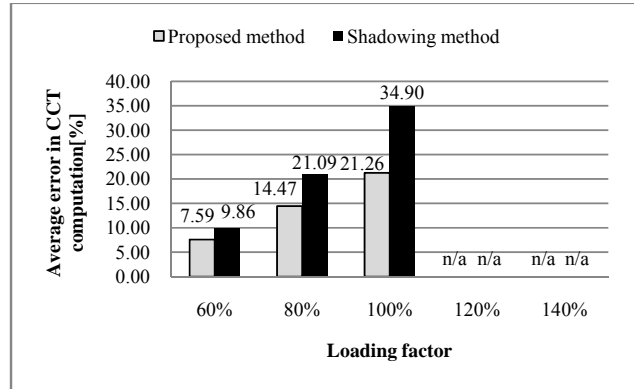


Fig. 8 Performance of the BCU shadowing and proposed methods based on loading conditions –30 machine system (IEE Japan West 30)

TABLE I  
DETAILED INFORMATION OF THE CCT ERRORS

system	method	Error type	loading factor				
			60%	80%	100%	120%	140%
3 generators	Proposed	$\bar{\varepsilon}^U$	0.0	0.0	4.2	6,18	7.5
		$\bar{\varepsilon}^O$	1.8	0.0	0.0	0.0	0.0
	Shadowing	$\bar{\varepsilon}^U$	0.0	3.3	5.4	6.2	10.1
		$\bar{\varepsilon}^O$	1.8	0.0	0.0	0.0	0.0
4 generators	Proposed	$\bar{\varepsilon}^U$	0.0	3.7	0.0	7.6	12.9
		$\bar{\varepsilon}^O$	0.0	5.9	0.0	7.1	0.0
	Shadowing	$\bar{\varepsilon}^U$	5.9	5.8	13.0	8.3	12.9
		$\bar{\varepsilon}^O$	2.5	0.0	0.0	0.0	0.0
6 generators	Proposed	$\bar{\varepsilon}^U$	1.7	2.6	2.0	2.8	2.5
		$\bar{\varepsilon}^O$	2.8	2.3	1.2	1.2	1.3
	Shadowing	$\bar{\varepsilon}^U$	4.1	2.6	2.2	2.8	2.8
		$\bar{\varepsilon}^O$	1.9	1.1	1.2	1.2	1.3
7 generators	Proposed	$\bar{\varepsilon}^U$	11.7	6.9	5.7	12.3	9.9
		$\bar{\varepsilon}^O$	4.6	0.0	0.0	0.0	0.0
	Shadowing	$\bar{\varepsilon}^U$	12.7	16.3	11.9	13.5	12.7
		$\bar{\varepsilon}^O$	0.0	0.0	0.0	0.0	0.0
10 generators	Proposed	$\bar{\varepsilon}^U$	10.5	13.3	13.6	18.5	22.4
		$\bar{\varepsilon}^O$	5.1	0.0	0.0	33.3	0.0
	Shadowing	$\bar{\varepsilon}^U$	11.5	15.9	13.8	28.9	35.2
		$\bar{\varepsilon}^O$	0.0	0.0	0.0	66.7	0.0
30 generators	Proposed	$\bar{\varepsilon}^U$	9.4	17.9	30.1	n/a	n/a
		$\bar{\varepsilon}^O$	0.0	0.0	25.4	n/a	n/a
	Shadowing	$\bar{\varepsilon}^U$	10.8	23.6	46.1	n/a	n/a
		$\bar{\varepsilon}^O$	4.6	4.9	46.3	n/a	n/a

TABLE II  
DETAILED INFORMATION IEEEJ WEST 10 MACHINE SYSTEM WITH 140%  
LOADING FACTOR

Fault Point	Open Line	proposed method [s]	Shadowing method [s]	Simulation Method zero transfer conductance [s]	Simulation Method [s]
A	1-2	0.05	0.05	0.05 - 0.06	0.00
B	2-1	0.08	0.06	0.07 - 0.08	0.00
C	2-3	0.05	0.05	0.06 - 0.07	0.00
D	3-4	0.06	0.06	0.07 - 0.08	0.00
E	4-5	0.07	0.07	0.08 - 0.09	0.05 - 0.06
F	5-6	0.07	0.07	0.10 - 0.11	0.08 - 0.09
G	6-7	0.07	-	0.12 - 0.13	0.07 - 0.08
H	7-8	0.08	0.08	0.14 - 0.15	0.07 - 0.08
I	8-9	0.13	0.91	0.13 - 0.15	0.00
J	9-8	0.03	0.03	0.02 - 0.03	0.02 - 0.03
K	2-10	0.08	0.08	0.09 - 0.10	0.03 - 0.04
L	10-2	0.09	0.09	0.10 - 0.11	0.03 - 0.04

" - " indicates convergence to incorrect UEPs.

TABLE III  
CLASSIFICATION OF THE CCT ERRORS

system	method	Error type	Number of faults				
			loading factor				
			60%	80%	100%	120%	140%
3 generators	Proposed	$\bar{\epsilon}^U$	0	0	2	8	4
		$\bar{\epsilon}^O$	2	0	0	0	0
	Shadowing	$\bar{\epsilon}^U$	0	1	3	8	5
		$\bar{\epsilon}^O$	2	0	0	0	0
4 generators	Proposed	$\bar{\epsilon}^U$	0	2	0	5	2
		$\bar{\epsilon}^O$	0	2	0	1	0
	Shadowing	$\bar{\epsilon}^U$	3	4	2	8	2
		$\bar{\epsilon}^O$	2	0	0	0	0
6 generators	Proposed	$\bar{\epsilon}^U$	3	3	4	2	5
		$\bar{\epsilon}^O$	2	1	1	1	1
	Shadowing	$\bar{\epsilon}^U$	4	3	5	2	4
		$\bar{\epsilon}^O$	4	3	1	4	3
7 generators	Proposed	$\bar{\epsilon}^U$	15	15	13	9	11
		$\bar{\epsilon}^O$	1	0	0	0	0
	Shadowing	$\bar{\epsilon}^U$	16	15	15	14	14
		$\bar{\epsilon}^O$	0	0	0	0	0
10 generators	Proposed	$\bar{\epsilon}^U$	8	10	5	10	8
		$\bar{\epsilon}^O$	1	0	0	1	0
	Shadowing	$\bar{\epsilon}^U$	10	9	7	11	10
		$\bar{\epsilon}^O$	0	0	0	1	0
30 generators	Proposed	$\bar{\epsilon}^U$	19	21	12	n/a	n/a
		$\bar{\epsilon}^O$	0	0	7	n/a	n/a
	Shadowing	$\bar{\epsilon}^U$	23	23	16	n/a	n/a
		$\bar{\epsilon}^O$	2	1	6	n/a	n/a

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